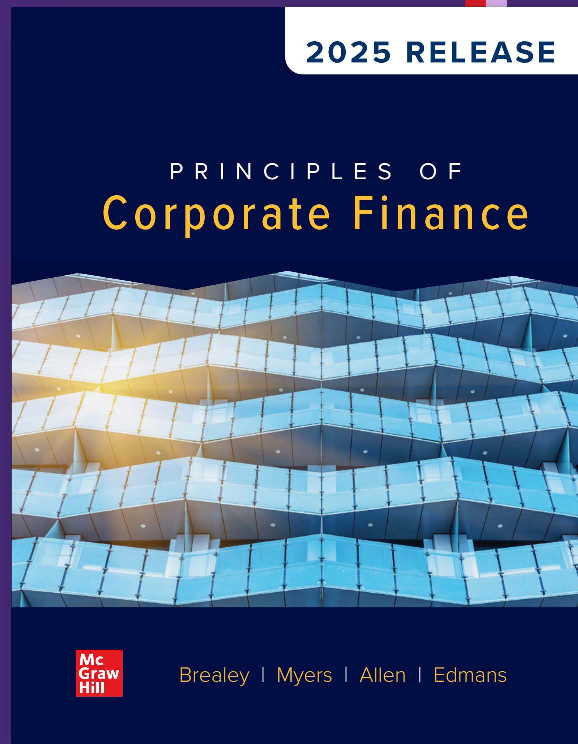
Principles of Corporate Finance

Evergreen Release

Richard Brealey, Stewart Myers,
Franklin Allen and Alex Edmans



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2025 RELEASE

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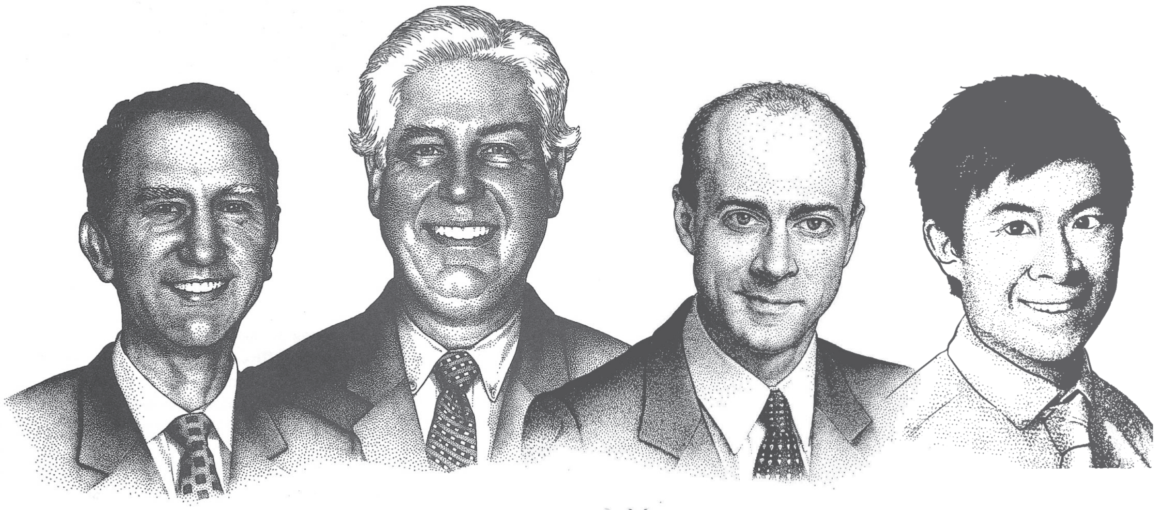
Dedication

To our parents.

About the Authors



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Preface

This book describes the theory and practice of corporate finance. It's clear why financial managers have to master the practical aspects of their job, but why do practitioners need to bother with theory?

Managers learn from experience how to cope with routine problems. But the best managers are also able to respond to change and deal with non-routine situations. To do so you need more than rules of thumb and gut feel; you must understand why companies and financial markets behave the way they do. In other words, you need a *theory* of finance.

That should not sound intimidating. Good theory helps you grasp what is going on in the world around you. It helps you ask the right questions when times change and new problems need to be analyzed. It also tells you which things you do *not* need to worry about. Throughout this book, we show how managers use financial theory to solve practical problems.

Of course, the theory presented in this book is not perfect and complete—no theory is. There are some famous controversies where financial economists cannot agree. We have not glossed over these disagreements. We set out the arguments for each side and tell you where we stand.

Much of this book is concerned with understanding what financial managers do and why. But we also say what financial managers *should* do to increase company value. Where theory suggests that financial managers are making mistakes, we say so. In brief, we have tried to be fair but to pull no punches.

This book may be your first view of the world of modern finance. If so, you will read first for new ideas, and for an understanding of how finance theory translates into practice. But eventually you will be in a position to make financial decisions, not just study them. At that point, you can turn to this book as a reference and guide.

Changes in the 2025 Release

The fourteenth edition already involved major changes. The author team was enhanced by the addition of Alex Edmans, a global authority in corporate finance with substantial practitioner expertise. Alex's research expertise is in responsible business, corporate governance, and behavioral finance. Accordingly, we added a new chapter, Chapter 20, on responsible business—companies that have objectives beyond shareholder value. We substantially rewrote Chapter 19, on corporate governance and agency issues, and Chapter 12, on market efficiency and behavioral finance. In addition, we revamped Chapters 7 and 8 to make a clearer separation between portfolio choice and asset pricing, Chapter 13 to include financial technology, and Chapter 14 to add international differences in financing.

Despite these major changes, we have not rested on our laurels but have pressed on. While the changes in the fourteenth edition were concentrated on specific chapters, for this 2025 release **we have upgraded the whole book. Every chapter has been rewritten**, some significantly so. We fully leveraged Alex's expertise in explaining complex concepts for a general audience, developed through delivering talks and public lectures on finance and writing articles and popular books, and evidenced by a slew of teaching awards at MIT, Wharton, and London Business School. The overriding goal of the new edition is to make a step-change improvement to the pedagogy, making the book much clearer, simpler, and punchier, while at the same time more accurate and more precise.

Terms are all clearly defined the first times they are introduced, rather than having to be inferred from the narrative. Long swathes of dense text have been broken up into bite-sized chunks. Laundry lists of important considerations are subdivided into bullet points, each starting with a one- or two-word summary of the point to make it easy to remember. Key terms and results are highlighted in bold so that they pop out on the page.

Extraneous, "nice-to-have" but peripheral material has been moved to footnotes or deleted so that the reader can focus on what really matters. On the other hand, extra explanations have been added to walk the reader through a difficult concept step-by-step, removing the need for logical leaps. We have replaced mathematical and theoretical explanations with commonsense intuitions, often drawing parallels with everyday ideas outside finance to make the concepts as clear as possible.

When students are allowed "cheat sheets" or "study aids" for an exam, they almost always consist entirely of formulas. We indeed make the key formulas clear, both within the main text and the key takeaways at the end of each chapter. But the goal of the book is to help you not only ace a finance exam, but practice finance in the real world (or understand finance even if you do not end up in a finance job). For this, concepts may be even more important than formulas. Since this is the *Principles of Corporate Finance,* we have highlighted key principles throughout the book, which are broken out clearly in Principle boxes. Many of these principles run through the book. For example, the DIY Principle is that companies cannot add value by achieving outcomes that shareholders can achieve costlessly themselves. We first encounter it in Chapter 2 when discussing why financial managers do not need to consider shareholders' time or risk preferences when choosing projects. It also explains why companies do not need to pay dividends to satisfy shareholders' liquidity needs (Chapter 15), increase leverage to offer shareholders a high expected return (Chapter 16), or merge to diversify risk (Chapter 32).

In addition to the Principle boxes, we have added two other types of boxes. One is titled Common Confusions: a single concept that is frequently misunderstood, or two separate concepts that are often conflated. A second is called Sense Checks. These enhance the book's general mission of helping the reader understand how finance works, rather than how to plug numbers into formulas. The Sense Check boxes encourage the reader to pause and reflect on whether an answer given by a formula makes sense. In an exam, students may enter a number incorrectly into a calculator; in a job, financial analysts may mislink a cell in Excel. Sense Checks allow the reader to "be their own boss" and apply the same checks their manager would.

The Beyond the Page digital extensions and applications provide additional examples, anecdotes, spreadsheet programs, and more detailed explanations and practice examples of some topics. This extra material makes it possible to escape from the constraints of the printed page by providing more explanation for readers who need it and additional material for those who would like to dig deeper. There are now more than 150 of these apps. They are seamlessly available with a click on the ebook, but they are also readily accessible in the traditional hard copy of the text using the shortcut URLs provided in the margins of relevant pages.

Examples of these applications include

- **Chapter 2** Would you like to learn more about how to use Excel spreadsheets to solve time value of money problems? A Beyond the Page application shows how to do so.
- **Chapter 3** Do you need to calculate a bond's duration, see how it predicts the effect of small interest rate changes on bond price, calculate the duration of a common stock, or learn how to measure convexity? The duration app allows you to do so.
- **Chapter 5** Want more practice in valuing annuities? There is an application that provides worked examples and hands-on practice.
- **Chapter 7** Ever wondered how COVID-19 has affected the risk of stocks in the travel industry? An app provides the answer.
- **Chapter 12** Want an example of how speculative trading can swamp the actions of arbitrageurs? The app on the explosion in the price of GameStop shares provides one.
- **Chapter 18** The text briefly describes the flow-to-equity method for valuing businesses, but using the method can be tricky. We provide an application that guides you step-by-step.
- **Chapter 22** The Black–Scholes Beyond the Page application provides an option calculator. It also shows how to estimate the option's sensitivity to changes in the inputs and how to measure an option's risk.

Chapter Structure

Each chapter of the book includes an introductory preview, a list of key takeaways, and suggested further reading. The list of candidates for further reading is significantly updated. Rather than trying to include every important article, we largely list survey articles or general books. We give more specific references in footnotes.

In addition to the self-test questions within the chapter, each chapter is followed by a set of problems on both numerical and conceptual topics, together with a few challenge problems.

We include a Finance on the Web section in chapters where it makes sense to do so. These exercises seek to familiarize the reader with some useful websites and to explain how to download and process data from the web.

The book also contains 12 end-of-chapter Mini-Cases. These include specific questions to guide the case analyses. Answers to the mini-cases are available to instructors on the book's website.

Spreadsheet programs such as Excel are tailor-made for many financial calculations. Several chapters include boxes that introduce the most useful financial functions and provide some short practice questions. We give screenshots from Excel showing the exact

Excel formula to use to solve a problem and the inputs that the formula draws from.

We conclude the book with a glossary of financial terms.

The 34 chapters in this book are divided into 12 parts. Parts 1, 2, and 3 cover valuation and capital investment decisions, including portfolio theory, asset pricing models, and the cost of capital. Parts 4 through 9 cover financing decisions, payout policy and capital structure, corporate objectives and governance, options, debt financing, and risk management. Part 10 covers financial analysis, planning, and working capital management. Part 11 covers mergers and acquisitions, and corporate restructuring. Part 12 concludes.

We realize that instructors will wish to select topics and may prefer a different sequence. We have therefore written chapters so that topics can be introduced in several logical orders. For example, there should be no difficulty in reading the chapters on financial analysis and planning before the chapters on valuation and capital investment.

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Stewart C. Myers
Franklin Allen
Alex Edmans

The Principles

Below we list several fundamental principles of corporate finance that run through many chapters in the book, along with the section that first introduces them.

The Do-It-Yourself (DIY) Principle: A company can never add value by achieving outcomes that share-holders can achieve costlessly themselves.

The Time Value Principle: A dollar today is worth more than a dollar tomorrow because it can be invested to earn a return.

The Risk Principle: A safe dollar is worth more than a risky dollar.

The Discount Rate Principle: The discount rate for a project is the opportunity cost of investing in that project, which depends on the risk of that project. It does not depend on the cost of capital or risk of the company undertaking the project.

The Price-Yield Principle: The higher the yield on a bond, the lower its price.

The Market Value Principle: In almost all financial applications, use market values rather than book values.

The Forward-Looking Principle: Cash flows and discount rates should reflect what is expected to happen in the future, not what has happened in the past.

The Growth Principle: Growth only adds value to a company if the rate of return on reinvested capital exceeds the cost of capital.

The Stable Risk Premium Principle: When forecasting future market returns, it is more realistic to assume a stable market risk premium than a stable market return.

The Diversification Principle: The standard deviation of a portfolio is lower than the average standard deviation of the stocks in that portfolio, because diversification reduces risk.

The Limits to Diversification Principle: You can only diversify away specific risk, not systematic risk.

The Common Portfolio Principle: If all investors have the same information and form the same expectations, all should hold the same portfolio of stocks.

The Portfolio Beta Principle: The beta of a portfolio is the weighted average of the betas of the stocks in that portfolio.

The Systematic Risk Principle: The cost of capital is only affected by systematic risk, not specific risk.

The Downside Risk Principle: Downside risk should always be incorporated by reducing expected cash flows, not by adding fudge factors into the discount rate. The discount rate depends only on systematic risk, and is unaffected if the downside risk is idiosyncratic.

The Market Prices Principle: Wherever possible, use market prices to check either your assumptions or the valuations that result from your assumptions.

The Replication Principle: Any set of contingent payoffs—that is, payoffs that depend on the value of some other asset—can be replicated with a mixture of simple options on that asset.

The Project Leverage Principle: The leverage used when calculating the discount rate for a project should depend on the project's debt capacity, not how the project is initially financed.

The Currency Risk Principle: When evaluating international investment opportunities, ignore currency risk if it can be hedged.

The Investors' Portfolio Principle: The cost of capital for an international investment depends on its beta relative to your investors' portfolio, not where your investors are located nor where the investment is located.

The Country Risk Principle: Country risk should be incorporated by adjusting expected cash flows, not by adding a fudge factor such as a "country risk premium" to the discount rate.

The Synergy Principle: A merger only creates synergies if it achieves outcomes that the merging companies, or their shareholders, could not achieve themselves.

Guided Tour

Pedagogical Features

Finance in Practice Boxes

Relevant news articles, often from financial publications, appear in various chapters throughout the text. Aimed at bringing real-world flavor into the classroom, these boxes provide insight into the business world today.

> **FINANCE IN PRACTICE**
>
> **Arithmetic Averages and Compound Annual Returns**
>
> The average returns shown in Table 7.1 are **arithmetic averages**. In other words, we simply added the 124 annual returns and divided by 124 to get our average return of 11.5%. However, financial analysts may also quote the **geometric average** (also known as the **compound rate of return**). Over the 124-year period, stock values multiplied 87,620 times. The geometric average return is calculated by taking the 124th root of 87,620. This gives 9.6%, 1.9 percentage points below the arithmetic average of 11.5%.[5]
>
> capital for investments that have the same degree of risk as Big Pharma.
>
> Now suppose that we observe the returns on Big Pharma stock over a large number of years. If the odds are unchanged, the return will be –10% in a third of the years, +10% in a further third, and +30% in the remaining years. The arithmetic average of these yearly returns is
>
> $$\frac{-10+10+30}{3} = +10\%$$

Numbered Examples

Numbered and titled examples are called out within chapters to further illustrate concepts. Students can learn how to solve specific problems step-by-step and apply key principles to answer concrete questions and scenarios.

> **EXAMPLE 9.1 • Operating leverage and risk**
>
> The Stamboard Group is comparing two technologies for producing prefabricated housing. Table 9.1 shows the forecast cash flows. Both technologies generate the same *expected* level of sales and costs, but the costs of Technology A are a constant proportion (10%) of sales. By contrast the costs of Technology B are fixed at $135 million. This higher operating leverage results in a higher asset beta.

Self-Test Questions

Each chapter includes a number of self-test questions that allow students to check their understanding. Answers to these questions are given at the end of the chapter.

> **6.4 Self-Test**
>
> A firm is considering investment in a new manufacturing plant. The site is owned by the company, but existing buildings would need to be demolished. Which of the following should be treated as incremental cash flows?
>
> a. The market value of the site and existing buildings.
>
> b. Demolition costs and site clearance.
>
> c. The cost of a new access road put in last year.
>
> d. Lost cash flows on an existing product that will be replaced by the new

Sense Check Boxes

These show how a reader can do a quick intuitive check, without conducting any calculations, to verify whether an answer makes sense.

> **Sense Check: Annual Percentage Rates and Effective Annual Rates**
>
> The effective annual rate can never be lower than the annual percentage rate due to the power of compounding. The EAR assumes annual compounding. Thus, when the APR is annually compounded, the two rates are identical. When the APR is compounded more frequently, it is equivalent to a higher EAR, and so the EAR is higher than the APR.
>
> The two examples we've seen earlier pass this sense check: A 10% APR, compounded semiannually, corresponds to a 10.25% EAR; a 12% APR, compounded monthly, corresponds to a 12.68% APR.

Common Confusion Boxes

These clarify a single concept that is frequently misunderstood, or two separate concepts that are often conflated.

> **COMMON CONFUSION: COUPONS AND YIELDS**
>
> The **coupon** is the regular payment that a bondholder receives. It appears in the numerator of a pricing equation and is fixed over the lifetime of a bond.
>
> The **yield** is the total return that a bondholder receives from both coupons and capital gains or losses. It appears in the denominator of the pricing equation and changes over the lifetime of a bond as the price changes.

Beyond the Page Interactive Content and Applications

Additional resources and hands-on applications are just a click away. Students can use the web address or click on the icon in the eBook to learn more about key concepts and try out calculations, tables, and figures when they go Beyond the Page.

> **BEYOND THE PAGE**
>
> Try it! Figure 10.4: Decision tree for the pharmaceutical project

Excel

Spreadsheet Functions Boxes

These boxes provide detailed examples of how to use Excel spreadsheets when applying financial concepts. Questions that apply to the spreadsheet follow for additional practice.

USEFUL SPREADSHEET FUNCTIONS

Internal Rate of Return

Spreadsheet programs such as Excel provide built-in functions to solve for internal rates of return. You can find these functions by pressing *fx* on the Excel toolbar. If you then click on the function that you wish to use, Excel will guide you through the inputs that are required. At the bottom left of the function box, there is a Help facility with an example of how the function is used.

Here is a list of useful functions for calculating internal rates of return, together with some points to remember when entering data:

- **IRR**: Internal rate of return on a series of regularly spaced cash flows.

=internal rate of return(investment made today, cash flow year 1, cash flow year 2, cash flow year 3, cash flow year 4, cash flow year 5)

=IRR(values)

For example, for a project that requires an initial investment of $7,000 that then generates $1,500 at the end of each year for a period of 5 years, we can solve for the internal rate of return using the IRR formula. First, set up the numbers in your spreadsheet as follows:

	A	B
1	Initial investment	$ (7,000)
2	Cash flow year 1	$ 1,500
3	Cash flow year 2	$ 1,500
4	Cash flow year 3	$ 1,500
5	Cash flow year 4	$ 1,500
6	Cash flow year 5	$ 1,500

Now, enter the formula as =IRR(B1:B6) to find IRR = 2.34%.

- **XIRR**: The same as IRR, but for irregularly spaced flows.

=X internal rate of return(investment made today, cash flow year 1, cash flow year 2, cash flow year 3, cash flow year 4, cash flow year 5, date of initial investment, date of investment year 1, date of investment year 2, date of investment year 3, date of investment year 4, date of investment year 5)

=XIRR(values, dates)

For example, for a project that requires an initial investment of $4,000 that then generates $1,500 at the end of year 1, $2,000 at the end of year 3, and $1,000 at the end of year 5, and no cash flows in any other year, we can solve for the internal rate of return using the XIRR formula. First, set up the numbers and dates in your spreadsheet as follows:

	A	B	C
1	Initial investment	1/31/2025	$ (4,000)
2	Cash flow year 1	1/31/2026	$ 1,500
3	Cash flow year 2	1/31/2027	$ –
4	Cash flow year 3	1/31/2028	$ 2,000
5	Cash flow year 4	1/31/2029	$ –
6	Cash flow year 5	1/31/2030	$ 1,000

Now, enter the formula as =XIRR(C1:C6,B1:B6) to find XIRR = 4.41%.

Note the following:

- For these functions, you must enter the addresses of the cells that contain the input values.
- The IRR functions calculate (at most) only one IRR even when there are multiple IRRs.

Spreadsheet Questions

The following questions provide an opportunity to practice each of the above functions:

1. (IRR) Check the IRRs for project F in Section 5-3.

Excel Exhibits

Select tables are set as spreadsheets, and the corresponding Excel files are also available in Connect and through the Beyond the Page features.

		Year							
		0	1	2	3	4	5	6	7
1	Capital investment	$12,000							
2	Post-tax salvage value								–$1,949[1]
3	Accumulated depreciation		$ 2,000	$ 4,000	$ 6,000	$ 8,000	$10,000	$12,000	$ 0
4	Year-end book value	$ 0	–$10,000	–$ 8,000	–$ 6,000	–$ 4,000	–$ 2,000	–$ 0	$ 0
5	Working capital		$ 550	$ 1,289	$ 3,261	$ 4,890	$ 3,583	$ 2,002	$ 0
6	Revenues		$ 523	$12,887	$32,610	$48,901	$35,834	$19,717	$
7	Expenses	$ 4,000	$ 3,037	$ 8,939	$20,883	$30,809	$23,103	$13,602	$
8	Depreciation[2]		$ 2,000	$ 2,000	$ 2,000	$ 2,000	$ 2,000	$ 2,000	$ 0
9	Pre-tax profit (6 – 7 – 8 – 2)	–$ 4,000	–$ 4,514	$ 1,948	$ 9,727	$16,092	$10,731	$ 4,115	$ 1,949
10	Tax at 21%	–$ 840[3]	–$ 948	$ 409	$ 2,043	$ 3,379	$ 2,254	$ 864	$ 409
11	Profit after tax (9 – 10)	–$ 3,160	–$ 3,566	$ 1,539	$ 7,684	$12,713	$ 8,477	$ 3,251	$ 1,540

TABLE 6.4 Income Statement for Guano Project ($ thousands)

[1]The asset has been entirely depreciated for tax purposes and so the entire sales price is subject to tax.
[2]The initial investment of $12 million is depreciated straight-line over the six years.
[3]A negative tax payment means a cash inflow, assuming that IM&C can use the tax loss on the guano project to shield income from the rest of its business.

End-of-Chapter Features

Problem Sets

Beside each end-of-chapter problem we note the section of the chapter to which the question relates. This helps instructors create assignments and makes it simpler for students to look back for help. These end-of-chapter problems give students hands-on practice with key concepts and applications.

connect — Select problems are available in McGraw Hill's *Connect*. Please see the preface for more information.

PROBLEM SETS

1. **Behavioral biases (S11-1)** Explain why setting a higher discount rate is not a cure for overly optimistic cash-flow forecasts.
2. **Behavioral biases (S11-1)** Look back to the cash flows for projects F and G in Section 5-3. The cost of capital was assumed to be 10%. Assume that the forecast cash flows for projects of this type are overstated by 8% on average. That is, the forecast for each cash flow from each project should be reduced by 8%. But a lazy financial manager, unwilling to take the time to argue with the projects' sponsors, instructs them to use a discount rate of 18%.

CHALLENGE PROBLEMS

20. **Economic rents (S11-3)** Accidental setbacks can result in negative rents in any year. But can a project have positive *expected* rents in some years and negative expected rents in other years? Explain.
21. **Economic rents (S11-3)** The manufacture of polysyllabic acid is a competitive industry. Most plants have an annual output of 100,000 tons. Operating costs are \$0.90 a ton, and the sales price is \$1 a ton. A 100,000-ton plant costs \$100,000 and has an indefinite life. Its current scrap value of \$60,000 is expected to decline to \$57,900 over the next two years. Phlogiston Inc. proposes to invest \$100,000 in a plant that employs a new low-cost process to manufacture polysyllabic acid. The plant has the same capacity as existing units, but operating costs are \$0.85 a ton. Phlogiston estimates that it has two years' lead over each of its rivals in use of the process but is unable to build any more plants itself before year 2. Also it believes that demand over the next two years is likely to be sluggish and that its new plant will therefore cause temporary overcapacity. You can assume that there are no taxes and that the cost of capital is 10%.
    a. By the end of year 2, the prospective increase in acid demand will require the construction of several new plants using the Phlogiston process. What is the likely NPV of such plants?

Finance on the Web

These web exercises give students the opportunity to explore financial websites on their own. The web exercises make it easy to include current, real-world data in the classroom.

FINANCE ON THE WEB

You can download data for Questions 1 and 2 from **finance.yahoo.com**. Refer to the Useful Spreadsheet Functions box near the end of Chapter 9 for information on Excel functions.

1. Download to a spreadsheet the last three years of monthly adjusted stock prices for Coca-Cola (KO), Citigroup (C), and Pfizer (PFE).
    a. Calculate the monthly returns.
    b. Calculate the monthly standard deviation of those returns (see Section 7-2). Use the Excel function STDEV.P to check your answer. Find the annualized standard deviation by multiplying by the square root of 12.
    c. Use the Excel function CORREL to calculate the correlation coefficient between the monthly returns for each pair of stocks. Which pair provides the greatest gain from diversification?
    d. Calculate the standard deviation of returns for a portfolio with equal investments in the three stocks.
2. A large mutual fund group such as Fidelity offers a variety of funds. They include *sector funds* that specialize in particular industries and *index funds* that simply invest in the market index. Log on to **fidelity.com** and find first the standard deviation of returns on the Fidelity Spartan 500 Index Fund, which replicates the S&P 500. Now find the standard deviations for different sector funds. Are they larger or smaller than the figure for the index fund? How do you interpret your findings?

Mini-Cases

Mini-cases are included in select chapters so students can apply their knowledge to real-world scenarios.

MINI-CASE

Waldo County

Waldo County, the well-known real estate developer, worked long hours, and he expected his staff to do the same. So George Chavez was not surprised to receive a call from the boss just as George was about to leave for a long summer's weekend.

Mr. County's success had been built on a remarkable instinct for a good site. He would exclaim "Location! Location! Location!" at some point in every planning meeting. Yet finance was not his strong suit. On this occasion, he wanted George to go over the figures for a new \$90 million outlet mall designed to intercept tourists heading downeast from Bar Harbor through southern Maine. "First thing Monday will do just fine," he said as he handed George the file. "I'll be in my house

Key Takeaways

At the end of each chapter we list the key conceptual takeaways and most important formulas.

KEY TAKEAWAYS

Financial decisions usually involve a comparison of cash flows at different points in time. In this chapter, we have shown how to convert between present and future values of a cash flow.

- **Future values** An investment of \$1 earning an interest rate of r will increase in value each period by the factor $(1 + r)$. The future value after t periods is

$$\text{Future value} = \text{present value} \times (1 + r)^t$$

- **Present values** The present value (PV) of a future cash payment is the amount that you would need to invest today to produce that future payment. To calculate present value, discount future cash flows (C_t) by an appropriate rate r, usually called the *discount rate* or *opportunity cost of capital*:

$$\text{Present value} = \text{PV} = \sum_{t=1}^{T} \frac{C_t}{(1 + r)^t}$$

Instructor Resources

The *Connect* Instructor Library provides additional resources to improve student engagement in and out of class. This library contains information about the book and the authors, as well as all of the instructor supplements, many of which were carefully updated for this edition by Nicholas Racculia, St. Vincent College.

- **Instructor's Manual** The Instructor's Manual contains an overview of each chapter, teaching tips, learning objectives, challenge areas, key terms, and an annotated outline that provides references to the PowerPoint slides.
- **Solutions Manual** The Solutions Manual contains solutions to all basic, intermediate, and challenge problems found at the end of each chapter.
- **Test Bank** The Test Bank contains hundreds of multiple-choice and short answer/discussion questions, updated based on the revisions of the authors. The level of difficulty varies, as indicated by the easy, medium, or difficult labels.
- **PowerPoint Presentations** The PowerPoint presentations contain exhibits, outlines, key points, and summaries in a visually stimulating collection of slides. The instructor can edit, print, or rearrange the slides to fit the needs of his or her course.
- **Beyond the Page** The authors have created a wealth of additional examples, explanations, and applications, available for quick access by instructors and students. Each Beyond the Page feature is called out in the text with an icon that links directly to the content.
- **Excel Solutions and Templates** There are templates for select exhibits, as well as various end-of-chapter problems that have been set as Excel spreadsheets—all denoted by an icon. They correlate with specific concepts in the text and allow students to work through financial problems and gain experience using spreadsheets. Useful Spreadsheet Functions boxes are sprinkled throughout the text to provide helpful prompts on working in Excel.

Test Builder within Connect

Available within Connect, Test Builder is a cloud-based tool that enables instructors to format tests that can be printed or administered within an LMS. Test Builder offers a modern, streamlined interface for easy content configuration that matches course needs, without requiring a download.

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- Easily pinpoint the most relevant content through robust filtering options.
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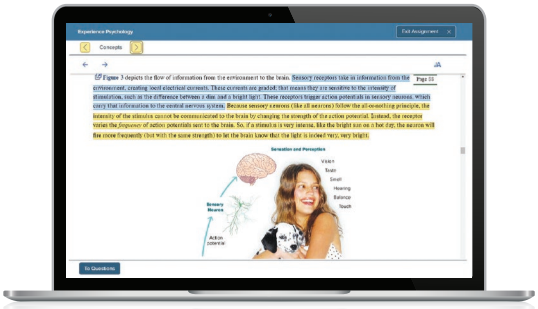

Laptop: Getty Images; Woman/dog: George Doyle/Getty Images

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Brief Contents

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Part Twelve: Conclusion

Part 1 Value

CHAPTER 4

Valuing Stocks

On a typical day in the New York Stock Exchange, investors trade over 1 trillion shares. What determines the prices at which these shares are bought and sold? This chapter explains the fundamental tools and concepts that investors and financial analysts use to value stocks.

Why should you care about valuation fundamentals? If you want to know what a stock is worth, why can't you just look up the price on the Internet?

There are several reasons you should care:

1. The stock price may not always be right. Before electric vehicle stocks plummeted in 2022, their prices were unsustainably high. On the other hand, hindsight tells us that most stock prices were abnormally low in the Global Financial Crisis of 2007–2008. What a stock *costs* may not be what it's *worth*. Financial analysts use the tools presented in this chapter as they try to identify which stocks to buy and which to avoid.
2. Many companies are not public. A private company may want to understand what its stock is worth or what it would be worth if it were traded.
3. Changes in the price of a public company's shares reveal how well the company is performing, and usually determine a large fraction of a CEO's pay and even whether she keeps her job. If your company is going to use its stock price to assess performance, you should understand what determines this price.

Section 4-1: How Stocks are Traded

We begin with a brief look at how stocks are traded. Then we introduce some key statistics and valuation ratios that investors use to evaluate stocks, including dividend yields, price-earnings (P/E) ratios, and market-to-book ratios.

Section 4-2: Valuation by Comparables

We start our discussion of share valuation by looking at the popular method of *comparable companies analysis*. That approach rests on the commonsense idea that similar stocks ought to sell at similar valuation ratios, for example, similar P/Es.

Section 4-3: Valuation by Fundamentals

A second common valuation method is to focus on the fundamental cash flows provided by a stock. *Dividend discount models* all start with a present value equation: The price of a share equals the present value of all future dividends paid to the owner of that share. This fundamental PV equation can be written as

$$P_0 = \sum_{t=1}^{\infty} \frac{\text{DIV}_t}{(1+r)^t}$$

where P_0 is today's price, DIV_t the expected dividend per share at future date *t*, and *r* the discount rate, the *cost of equity*.

If the expected growth rate of dividends is steady and perpetual, the present value formula simplifies to

$$P_0 = \frac{\text{DIV}_1}{r - g}$$

where DIV_1 is next year's expected dividend, *g* is the growth rate, and *r* is the cost of equity.

Section 4-4: Terminal Value

The above two equations capture the two methods for valuing stocks. While the second method is convenient, it can only be used once a company's dividends are growing at a constant rate. In reality, many companies may pay no dividends for several years. Then, when they start paying dividends, they ramp up rapidly. Only once a company is mature will its dividends grow at a roughly consistent pace.

As a result, financial analysts typically use a two-stage or multistage method that combines the first approach with either the second or with comparable companies analysis. They forecast dividends up to a horizon date *H* and then calculate the **terminal value** of that company at that date.

This section works through valuation exercises using dividend discount models with two stages of growth. It also considers how the terminal value can be found using both fundamentals and comparables.

Section 4-5: The Determinants of Price-Earnings Ratios

How does growth affect price-earnings ratios? We show that if the growth is in zero-NPV projects then a company's price-earnings ratio is unaffected. However, if there are positive-NPV growth opportunities then this increases the price-earnings ratio. We explain the fundamental difference between *growth stocks* and *income stocks*. Growth stocks are ones where the present value of growth opportunities (PVGO) is a large proportion of the stock price; income stocks are ones where PVGO is a small proportion.

Section 4-6: Valuation Based on Free Cash Flow

Sometimes it is easier and more practical to value the company as a whole rather than a single share. Section 4-6 explains why and how. The present value equation is the same, except that it now determines the *market capitalization*—the market value of all outstanding shares. The market capitalization equals the present value of all future *free cash flow.* Free cash flow is the post-tax cash flow generated by the company's operations after subtracting investment. It is the amount of cash available for payout to shareholders.

It is still early in this book, so we have deferred several important issues. For example, we treat the cost of equity *r* as given. In fact, it depends on the level of interest rates, as one would expect from Chapter 3, and also on risk (Chapters 7, 8, 9) and on the choice between debt and equity financing (Chapters 16, 17). We assume the firm has no debt and so the return on equity and the return on assets are the same. Chapter 18 covers valuation models taking all these issues into account.

Appendix: Using Stock Prices to Estimate the Cost of Equity

This chapter considers how the price of a stock can be calculated given the expected growth of dividends and the cost of equity *r.* The appendix considers how we can turn around the various formulas to infer the cost of equity *r* from the stock price. This method for calculating the cost of equity has been used in the United States to set regulated prices such as for water, gas, and electricity. An example is given for water companies.

4-1 How Stocks Are Traded

Nike, the world's largest manufacturer of athletic footwear and a maker of clothing and sports equipment, had 1.52 billion shares outstanding in February 2024. Shareholders included large pension funds and insurance companies that each owned millions of shares, as well as individuals who owned a handful. If you owned one Nike share, you would own 0.00000007% of the company and have a claim on the same tiny fraction of its profits. Of course, the more shares you own, the larger your share of the company.

If Nike wishes to raise new capital, it can sell new shares to investors. These share issues occur in the **primary market**. But most trades in Nike take place on the stock exchange, where investors buy and sell existing Nike shares. Stock exchanges are **secondary markets**, where investors trade secondhand shares that have already been issued.

The two principal U.S. stock exchanges are the New York Stock Exchange (NYSE) and Nasdaq. Both compete vigorously for business and just as vigorously tout the advantages of their trading systems. In addition to the NYSE and Nasdaq, there are *electronic communication networks (ECNs)* that connect traders with one another. Large U.S. companies may also arrange for their shares to be traded on foreign exchanges, such as the London Stock Exchange or the Euronext exchange which serves seven European countries. At the same time, many foreign companies are listed on the U.S. exchanges. For example, the NYSE trades shares in Sony, Royal Dutch Shell, Canadian Pacific, Tata Motors, Deutsche Bank, Telefonica Brasil, China Eastern Airlines, and more than 500 other companies.

Suppose that Ms. Almeida, a long-time Nike shareholder, no longer wishes to hold her shares. She can sell them via the NYSE to Mr. Saetang, who wants to increase his stake in the firm. The transaction merely transfers partial ownership of the firm from one investor to another. No new shares are created, and Nike will not know that the trade has taken place.

Ms. Almeida and Mr. Saetang do not show up on the floor of the NYSE to execute their sell and buy orders personally. Their stockbrokers convey their orders to the NYSE, which operates a huge **auction market** matching up orders from thousands of investors. Buyers and sellers trade directly with no intermediary. Most major exchanges around the world, including the Tokyo, Shanghai, and London stock exchanges and the Deutsche Börse, are also auction markets, but the auctioneer in these cases is a computer.[1] This means that, unlike in the past, there is no stock exchange floor to show on the evening news and no one needs to ring a bell to start trading.

Nasdaq is not an auction market but a **dealer market**. All trades on Nasdaq take place between the investor and one of a group of professional dealers or "market makers" who buy and sell stock to create liquidity in the market. Dealer markets are common for other financial instruments. For example, most bonds are traded in dealer markets.

Most of the trading on the NYSE and Nasdaq is in ordinary "common" stocks, but other securities are traded also. These include the following:

- **Preferred stocks**, which we cover in Chapter 13.
- **Warrants**, which we cover in Chapter 22.
- **Exchange-traded funds (ETFs)**, which are portfolios of stocks that can be bought or sold in a single trade. Most ETFs are not actively managed; they simply aim to track a well-known market index such as the Dow Jones Industrial Average or the S&P 500. Others track specific industries or commodities. ETFs may also invest in bonds or other investments. We discuss ETFs more fully in Chapter 13.
- **Closed-end funds**[2] that invest in portfolios of securities. These include country funds, such as the Mexico and India funds, that invest in portfolios of stocks in specific countries. Unlike ETFs, most closed-end funds are actively managed and seek to "beat the market."

Trading Results for Nike

You can track trades in Nike and other public companies on the Internet. For example, if you go to **finance.yahoo.com** and enter the ticker symbol NKE, you will see trading results displayed as in Table 4.1.[3]

[1] Trades are still made face-to-face on the floor of the NYSE, but computerized trading is taking over. In 2006, the NYSE merged with Archipelago, an electronic trading system, and transformed itself into a public corporation. The NYSE is now owned by Intercontinental Exchange Inc., a U.S.-based network of exchanges and clearinghouses.

[2] *Closed-end* mutual funds issue shares that are traded on stock exchanges. *Open-end* funds are not traded on exchanges. Investors in open-end funds transact directly with the fund. The fund issues new shares to investors and redeems shares from investors who want to withdraw money from the fund.

[3] Some of these figures are taken from the "Summary" page, others from other pages such as "Statistics" and "Analysis." This table reports a small slice of what Yahoo! Finance provides. For example, you can dig deeper into trading history, financial statements, and analyst forecasts. Other good sources of trading and financial information include moneycentral.msn.com, finance.google.com, and the online edition of the *Wall Street Journal* at wsj.com (look for the "Market" and then the "Market Data" tabs).

Nike, Inc. (NKE)			
103.77 –0.02 (–0.02%) Feb. 8 4:00 pm EST			
Previous close	103.79	Market cap	157.224 B
Open	104.43	EPS (TTM)	3.43
Day's range	103.04–104.96	EPS (2023A)	3.23
52-week range	88.66–128.68	EPS (2024E)	3.59
Volume	6,324,489	EPS (2025E)	4.25
Average volume	9,324,623	P/E (TTM)	30.25
Book value/share	9.32	Dividend (yld)	1.48 (1.43%)
Price/book value	11.11	1y Target Est	123.23

TABLE 4.1 Trading Data for Nike Inc.

Data as of February 8, 2024.

Source: finance.yahoo.com.

Nike's closing price on February 8, 2024, was \$103.77, down \$0.02, or 0.02%, from the previous close of \$103.79. Nike stock started the trading day at \$104.43 and traded in a range of \$103.04 to \$104.96. Trading volume was 6,324,489 shares, lower than average volume.

Nike had 1.52 billion shares outstanding, so its **market capitalization** (also known as **market cap** or **market value**) was $1.52 \times 103.77 = \$157.224$ billion. This is how much the market is valuing the entire equity of Nike.

How do we know whether Nike's stock price is too high or too low? One benchmark is to compare it to Nike's book value per share. This gives a **price-to-book ratio**, or P/B, of 11.11. It is also known as the **market-to-book ratio**, or M/B. We will shortly explain the differences between market values and book values.

A second benchmark is to compare it to Nike's **earnings per share (EPS)**. EPS equals net income divided by the number of shares outstanding, where **net income** is the company's "bottom-line" profit after deducting operating expenses, interest, and taxes. After everyone else has been paid, what's left belongs to shareholders. Net income measures how much shareholders have earned over the course of a year.

The **price-earnings multiple**, or P/E, compares Nike's stock price to its earnings. There are several different price-earnings multiples, depending on which year's earnings you compare the stock price to:

- The **trailing P/E** uses the trailing twelve months (TTM) of earnings, that is, the last 12 months of earnings available as of February 8, 2024. Nike's fiscal year end is May 31, and companies report earnings quarterly, so Nike reports earnings for the four quarters ending August 31, November 30, February 28, and May 31. As of February 8, 2024, the last quarter ended on November 30, 2023, and so the TTM earnings covered the period December 1, 2022, to November 30, 2023. These earnings were \$3.43, leading to a trailing or TTM P/E of $\$154.46/\$3.43 = 30.25$.
- The **historic P/E** uses the earnings reported for the last fiscal year (2023A), which was June 1, 2022, to May 31, 2023, and thus different from the trailing twelve months. The "A" stands for "actual" since these earnings have been reported. Last fiscal year earnings were \$3.23, leading to a historic or "LFY" P/E of $\$103.77/\$3.23 = 32.13$.
- The **current P/E** uses the earnings for the current fiscal year (2024E) ending May 31, 2024. The "E" stands for "estimated," since on February 8, 2024, investors did not yet know what Nike's full-year earnings would be. These estimates are taken from equity

analysts at firms such as Goldman Sachs or Morgan Stanley. Analysts take Nike's already-reported earnings from June 1, 2023, to November 30, 2023, and add their forecasts for the remaining two quarters to May 31, 2024. The average analyst forecast was \$3.59, leading to a current P/E of \$103.77/\$3.59 = 28.91.

- The **forward P/E** uses next year's (estimated) earnings. These are the 2025E estimated earnings of \$4.25. For the 2025 fiscal year, analysts need to estimate earnings for all four quarters. The average analyst forecast was \$4.25, leading to a forward P/E of \$103.77/\$4.25 = 24.42.

The forward P/E is most relevant because investors buy stocks for their future earnings. Someone who bought Nike shares on February 8, 2024, would receive neither the TTM or LFY earnings, and only the future portion of the current year's earnings.

A company can either reinvest its earnings per share internally or pay it out to shareholders as **dividends**. The **plowback ratio** is the fraction of EPS that is reinvested, and the **payout ratio** is the fraction that is paid out. We have

$$\text{Payout ratio} = \frac{\text{DIV}}{\text{EPS}}$$

$$\text{Plowback ratio} = 1 - \text{payout ratio} = 1 - \frac{\text{DIV}}{\text{EPS}}$$

Nike's forecast dividend for the 2024 fiscal year was \$1.48, so its forecast payout ratio was \$1.48/\$3.59 = 41.2%. This corresponds to a plowback ratio of 58.8%. You can also compare the dividend to the stock price, which leads to a **dividend yield** of \$1.48/\$103.77 = 1.43%.

In addition to dividends, the second source of returns to an investor is **capital gains**, the difference between the price at which an investor sells and the price at which he bought. The "1y Target Est" in Table 4.1 shows that the average analyst forecast for Nike's stock price in one year's time was \$123.23, corresponding to an expected capital gain of \$123.23 – \$103.77 = \$19.46.

Market Price versus Book Value

Nike, like all public companies, publishes quarterly and annual financial statements. These include the balance sheet, which reports the **book values** of all assets and liabilities—in other words, the values reported on the company's accounts. The book value of an asset captures what you paid for it, minus **depreciation** (an allowance for wear and tear that we'll cover in Section 6-1).

At the end of the most recent quarter (November 30, 2023), the total book value of all Nike assets—plant and machinery, inventories, accounts receivable from customers, cash, etc.—was \$37.203 billion. Its total liabilities—outstanding bonds and bank loans, accounts payable to suppliers, taxes due and other obligations—was \$23.057 billion. The difference of \$14.146 billion was the **book value of equity**, or simply **book equity** for short; it can also be referred to as a company's **net assets**. Book value per share was \$9.32.

Nike's ratio of stock price to book value per share (P/B ratio) of 11.11 means that stock price was over 11 times the most recent book value per share. Does that mean that investors were 11 times as optimistic as the accountants who prepared the financial statements? No, because book values and market values measure different things. The book value of an asset is what it *cost;* the market value is what it's *worth.* These are different for several reasons:

- *Historic vs. current.* What an asset cost historically may be different from what it's worth today. Even if an oil rig bought 10 years ago suffered no wear and tear, its value will have fallen due to declining prospects for the oil industry. Similarly, book values do not incorporate inflation. (Countries with high or volatile inflation often require inflation-adjusted book values, however.)

- *Intangible assets.* Book values often exclude intangible assets such as trademarks and patents. Many intangible assets are developed internally, such as human capital and corporate culture, rather than bought and so there is no purchase price to record as a historic cost. Nike's most valuable intangible asset is its brand name, which was internally developed. However, if trademarks and patents are purchased from another company, they can be included.
- *Synergies.* Accountants simply add up the book values of individual assets, and thus do not capture **going-concern value**. Going-concern value is created when a collection of assets is organized into a healthy operating business. This leads to synergies, where the whole is worth more than the sum of its parts.

For all of these reasons, financial managers almost always care about market value. For example, in an efficient market, the value of a company is given by its market value, not its book value. The performance of a CEO is assessed using her company's market value, not its book value. When we calculate a company's leverage (see Chapters 16 through 18), we will use market values, not book values. The following principle highlights this concept:

> ***The Market Value Principle**: In almost all financial applications, use market values rather than book values.*

Even though we care about market values, book values can nevertheless be a useful benchmark against which to compare market value. Suppose, for example, that Wind Energy's market cap is $900 million. Its book value of equity is $450 million. A financial analyst might say, "Wind Energy sells for two times book value. It has doubled shareholders' cumulative past investment in the company."

Book values may also be useful clues about **liquidation value**. Liquidation value is what investors get when a failed company is shut down and its assets are sold off. It thus represents a worst-case scenario; in the absence of bankruptcy costs (see Section 17-4), shareholders are unlikely to recover much less than book value. Book values of "hard" assets like land, buildings, vehicles, and machinery can indicate possible liquidation values.

Intangible "soft" assets can be important even in liquidation, however. Eastman Kodak provides a good example. Kodak, which was one of the "Nifty Fifty" U.S. growth stocks of the 1960s, suffered a long decline and finally filed for bankruptcy in January 2012. What was one of its most valuable assets in bankruptcy? Its portfolio of patents. It sold approximately 1,100 digital imaging and processing patents for $525 million after bankruptcy.

Expected Returns

Consider a shareholder who plans to invest for one year only. As discussed, his return comprises the dividend per share DIV_1 plus the expected capital gain per share of $P_1 - P_0$, The sum of the two, divided by the price at the start of the year P_0, is known as the expected **total shareholder return**, or "return" for short:

$$\text{Expected return} = r = \frac{\text{DIV}_1 + P_1 - P_0}{P_0} \qquad \textbf{(4.1)}$$

Suppose Establishment Electronics stock is selling for $100 a share ($P_0 = 100$). Investors expect a $5 dividend over the next year ($\text{DIV}_1 = 5$). They also expect the stock to sell for $110 a year, hence ($P_1 = 110$). Then the expected return to shareholders is 15%:

$$r = \frac{5 + 110 - 100}{100} = 0.15 \text{ or } 15\%$$

Today's Price? You can instead predict today's price if you are given investors' forecasts of year-end dividend and price and the expected return offered by other equally risky stocks:

$$\text{Price} = P_0 = \frac{\text{DIV}_1 + P_1}{1 + r} \tag{4.2}$$

For Establishment Electronics, $\text{DIV}_1 = 5$ and $P_1 = 110$. If r, the expected return for Establishment is 15%, then today's price should be $100:

$$P_0 = \frac{5 + 110}{1.15} = \$100$$

What determines the expected return of 15%? Among the thousands of traded stocks, some riskier than Establishment and some safer, there will be a group with similar risks which we will call Establishment's **risk class**. Suppose that the other stocks in Establishment's risk class all offer a 15% return. Then, Establishment has to be priced to offer the same 15% return as the stocks in its risk class, which leads to the above price of $100 per share.

- What if Establishment's price were above $P_0 = \$100$? In this case, its expected return would be less than 15%. Investors would shift their capital to the other stocks and, in the process, would force down the price of Establishment stock.
- If P_0 were less than $100, the process would reverse. Establishment would offer an expected return of higher than 15%. Investors would rush to buy it, forcing its price up to $100.

Therefore, at each point in time, *all stocks of equivalent risk are priced to offer the same expected return.* Establishment's cost of equity is thus determined by the opportunity cost for its shareholders, which is the return that they could earn elsewhere by buying stocks in the same risk class. In Chapter 7, we will introduce a stock's beta, which determines the risk class that it's in.

4.1 Self-Test

a. The stock of Thor Hammer Co. offers an expected one-year rate of return of 11%. How is that possible if the company pays no dividends?

b. Steamtool Inc. pays a generous dividend, but investors regard its stock as much riskier than Thor stock. What can you conclude about the expected rate of return on Steamtool stock?

4-2 Valuation by Comparables

Valuing a stock may seem complicated, but it uses the same principles that you use to value any asset.

Consider the following scenario. After working hard for a few years after university, you've saved a tidy sum of money. While you're tempted to blow it on a round-the-world cruise, you've decided to use it to step onto the first rung of the property ladder. You've found your dream house, a two-bedroom property. It's only five years old so you shouldn't need to worry about leaking pipes or faulty boilers, its decor is minimalist but characterful, and it's close to the office meaning you can roll out of bed and punch in within half an hour. The current tenants move out next month and the owner's looking to sell after that. But how do you know whether she's taking you for a ride with the asking price?

There are two approaches to valuing the house:

- One is to look at similar houses, such as other properties in the same neighborhood. Those houses might be quite different, so rather than examining their raw price, we'd compare the price to a **value driver**—what drives the value of a house. For a house, the key value driver is its size. (Even if you also care about other factors, they're less important. You don't compare the house to other five-year-old houses because you can always repair damage, or other two-bedroom houses because you can always add or remove walls to change the number of bedrooms.) If similar houses are trading at $500 per square foot, our house should trade at a similar multiple.
- The second is to consider what a house is fundamentally worth: the rent you'd get by letting it out, or that you'd save by living in it rather than having to rent yourself. You could start with the rent paid by the current tenants, forecast how this rent will grow over time, and discount this stream of rents back to today.

The same two methods are used to value a stock.

- One is to use **comparable companies analysis**. You look at similar companies, such as those in the same industry. For a company, the value driver is not its size, but other factors such as its earnings per share (EPS) or book value. Thus, the relevant multiples might be price-to-earnings (P/E) or price-to-book (P/B), as we saw with Nike. Other ratios are also used in practice, but for now we'll focus on these two ratios only.[4]
- The second is to use **fundamentals analysis**. This involves forecasting and discounting the business's future dividends or future cash flows.

In practice, both methods are widely used, and, when estimating the value of a business, an analyst will typically try both techniques.[5] We start with valuation by comparables.

Valuation by comparables works best if the relevant financial ratios cluster together by industry and for similar companies in the same industry. Table 4.2 illustrates this clustering, sometimes imperfect, sometimes close. The top line of the table reports P/Es and P/Bs for Coca-Cola and PepsiCo, the two largest U.S. soft-drinks companies. It can be seen that neither the P/Es nor the P/Bs match closely. Using just one comparable can be problematic.

Coca-Cola has only one close competitor, PepsiCo. However, most companies have multiple comparables, and it's best to use several benchmarks in case one happens to be mispriced. The second part of Table 4.2 compares ratios for the Union Pacific Railroad (UNP) with three other railroads. UNP's P/E was close to the average P/E of the other three. If you didn't know UNP's stock price, you could get a reasonable estimate by multiplying UNP's forecast earnings per share by the average P/E of 19.8. On the other hand, UNP's P/B ratio was higher than all the comparables. A stock price estimate based on average P/B would be too low.

The bottom block of the table compares Bank of America (BAC) with six other U.S banks. Here BAC's ratios are close to the average ratios. In general, the more comparables, the better.

Of course, investors did not need comparables to value any of the firms in Table 4.2. They are all public companies with actively traded shares. However, the price of a company may not reflect what it's worth, for example if it's mispriced. Just like with houses, comparables provide a useful benchmark to assess whether a price is too high or too low.

Comparable companies analysis is particularly useful when you don't have a stock price. For example, suppose a European bank is investigating a sale of its U.S. business, Ameribank, which has earnings of $200 million and book value of $1.5 billion. The bank could estimate the value of Ameribank by applying the ratios in Table 4.2 to these numbers:

[4] We discuss financial ratios in Chapter 29.

[5] For an analysis of the valuation techniques that are used by practitioners, see L. Mukhlynina and K. G. Nyborg, "The Choice of Valuation Techniques in Practice: Education versus Profession," *Critical Finance Review* 9 (2020) pp. 201–265. The authors find systematic differences between professional groups in the way that the two valuation methods are applied.

Company	Stock Price	P/E	P/B	Comparable	P/E	P/B
Coca-Cola (beverages)	$ 61.20	26.7	10.5	PepsiCo	38.7	16.0
Union Pacific (railroad)	$200.03	17.7	9.8	CPKC	26.3	2.5
				CSX	16.1	5.4
				Norfolk Southern	17.1	3.9
				Average	*19.8*	*3.9*
Bank of America (bank)	$ 28.14	8.5	0.9	JPMorgan Chase	10.2	1.5
				Wells Fargo	11.8	0.9
				Citigroup	6.4	0.5
				Capital One	6.7	0.7
				PNC Financial	8.5	1.0
				US Bancorp	8.6	1.1
				Average	*8.7*	*1.0*

TABLE 4.2 **Comparable Companies Analysis**

Stock price, price-earnings (P/E), and price-book (P/B) in June 2023 for three companies and their comparables.

Source: finance.yahoo.com.

- *P/E.* The average P/E of peers is 8.7. When applied to the earnings of $200 million, this gives an estimated value of $17.4 billion.
- *P/B.* The average P/B of peers is 1.0. When applied to the book value of $1.5 billion, this gives an estimated value of $15 billion.

Overall, this would suggest a value in the range of $15–17.4 billion for Ameribank.

While obtaining competitor P/Es and calculating their average seems to be simple, valuation by comparables involves much more investigation and judgment than we have deployed here:

- *Which comparables?* There are dozens of publicly traded banks. How do we know which ones are "similar enough" to be included, and which are sufficiently different that they should not be? For example, should we include Canadian banks, or is Canada sufficiently different from the United States? Should we include Goldman Sachs, because it offers a savings account (Marcus), or not because it has no branches or checking accounts and most of its value comes from investment banking activities?
- *How to weight them?* Even if you've agreed on a set of comparables, they won't all be equally relevant. The European owner of Ameribank would probably put more weight on ratios for Capital One, Keycorp, and U.S. Bancorp, which are more focused on U.S. banking, than much larger global banks such as Citigroup and JPMorgan Chase. It might also look for smaller, regional banks that would be better matches to its own U.S. operations.
- *Which ratios?* Table 4.2 contains both P/E and P/B, but which is more relevant? Even if we've decided on P/E, should it be trailing, historic, current, or forward? In addition, other ratios are used in practice, which we'll consider in Chapter 18.

Sections 4-1 and 4-2 *used* stock market prices but did not attempt to *explain* them. We do so in the next section by connecting stock prices to dividends.

4-3 Valuation by Fundamentals

The second method to value a stock is to consider what it's fundamentally worth. The fundamental value of a house depends on the rental income you'd receive by letting it out. In Chapter 3, we saw that the fundamental value of a bond is the discounted present value of the cash flows (interest and principal) that the bond will pay out over its lifetime. Similarly, the fundamental value of a stock is the discounted future dividends that the stock will generate. We thus have

$$\text{PV(share)} = \text{PV(expected future dividends per share)}$$

This price of a stock is thus given by

$$\text{Price} = P_0 = \sum_{t=1}^{\infty} \frac{\text{DIV}_t}{(1+r)^t} \quad \textbf{(4.3)}$$

where P_0 is today's price, DIV_t is the expected dividend per share at date t, and r is the discount rate. The formula's summation sign Σ says that the value of a stock includes all future dividends from date $t = 1$ to infinity (∞). Why infinity? Because a stock has no maturity date and could last forever. This point is particularly important for "sustainable" companies such as clean energy, biotechnology, and artificial intelligence. Even if they're not expected to pay dividends for a decade, their stock price will be high today, and so they'll be able to raise capital and invest if they have good future prospects.

The discount rate r is the expected rate of return required by shareholders. It is known as the **cost of equity** and determined by the opportunity cost of investing in the company. Shareholders invest in two ways: (1) when they buy shares, and (2) when the firm retains and reinvests earnings that otherwise could be paid out. As discussed in Section 4-1, the opportunity cost is the expected return on other securities with the same risks as the company's stock.

This PV equation (Equation 4.3) is called the **DCF** (for **discounted cash flow**) or **dividend discount model** of stock prices. But it seems to apply only if an investor intends to hold the stock forever and collect all future dividends until the end of time. What if he intends to sell in a year and will only receive the first year's dividend? In fact, he will still care about *all* future dividends—even those he won't receive—as we'll now see.

Dividends and Capital Gains

Assume that Giuseppe—or Beppe as his friends call him—owns shares in Establishment Electronics. Beppe is excited by the future growth prospects of the electronics industry, but knows that he'll have to sell his shares in a year's time because his oldest daughter will be starting college. Does this mean that Beppe will miss out on Establishment Electronics' bright future?

To Beppe, one share in Establishment Electronics is worth the following:

$$P_0 = \frac{\text{DIV}_1}{1+r} + \frac{P_1}{1+r}$$

At the end of one year, he'll receive the dividend, and immediately sell his shares on the market for the price at that time, P_1. This is the same as Equation 4.2 earlier, but now we'll explore what determines the date 1 price. Beppe won't know who he'll sell his shares to in one year's time; he'll trade them on the market and anyone can buy them. Let's say they're picked up by Suzanne. How much will Suzanne be willing to pay? She knows that, if she buys Establishment Electronics at date 1, she'll receive all future dividends from date 2 onward. Thus, to her those shares are worth

$$P_1 = \sum_{t=2}^{\infty} \frac{\text{DIV}_t}{(1+r)^{t-1}}$$

where the summation starts from $t = 2$ because the first dividend will be received at date 2. Each dividend is discounted by $t - 1$ periods since Suzanne will discount it back to $t = 1$, which is when she buys the shares, not to $t = 0$.

Substituting this into Equation 4.3 gives

$$P_0 = \frac{\text{DIV}_1}{1+r} + \frac{\sum_{t=2}^{\infty} \frac{\text{DIV}_t}{(1+r)^{t-1}}}{1+r}$$

$$P_0 = \frac{\text{DIV}_1}{1+r} + \sum_{t=2}^{\infty} \frac{\text{DIV}_t}{(1+r)^t}$$

$$P_0 = \sum_{t=1}^{\infty} \frac{\text{DIV}_t}{(1+r)^t}$$

exactly as we had earlier. Thus, to Beppe, one share in Establishment Electronics is still worth the present value of *all* future dividends, even though he intends to sell in one year. He *explicitly* receives the date 1 dividend, and he *implicitly* receives all dividends from date 2 onward, since they'll affect the price Suzanne pays him when buying his shares. In short, ***capital gains depend on expected future dividends***.

This observation is important. Commentators have highlighted the shortening holding periods of many investors—the average holding period of a typical stock in the United States in recent years is 10 months, compared with five years in the 1970s. They argue that such "short-termism" means that shareholders don't care about a company's prospects beyond a year, and will lobby it to turn down long-term investments in search of a quick buck. If true, this would be particularly problematic for ESG investments, such as clean energy, decarbonization, and workforce upskilling, which take many years to pay off.

The above analysis shows that these fears are unfounded in an efficient market (which we'll define more precisely in Chapter 12). Shareholders care about *all* future dividends, including those they won't receive, because they'll affect the stock's resale value. This is true not just in theory, but also in practice: some of the most valuable companies are tech firms, such as Alphabet, Amazon, Nvidia, and Tesla, which pay no or very few dividends.

4.2 Self-Test

Kevin Devon plans to hold his position in Steamtool shares for two years. His Aunt Cleo has also purchased Steamtool shares and contributed them to a trust for her granddaughter. The trust will probably hold these shares for at least a decade. Could you apply the DCF valuation formula to both investments? Explain how briefly.

Prices and Earnings

Notice that it is *not* correct to say that the value of a share is equal to the sum of the discounted stream of *earnings* per share. Earnings are generally larger than dividends because some of those earnings are plowed back into the company. Discounting earnings would recognize the rewards of that investment (higher *future* earnings and dividends) but not the sacrifice (a lower dividend *today*). The share price is connected to future earnings, but by a different formula, which we introduce in Section 4-5.

Given the importance of dividends, you may wonder why stocks trade at price-to-earnings rather than price-to-dividend ratios. This is because a share is worth the present value of *all* future dividends, but the denominator of a ratio can only contain a single year's dividends or earnings. Dividends in a single year are often a poor guide to dividends in the future; a growing company may pay no dividends today, even if it has positive earnings, because it is reinvesting all of these earnings to generate future dividends. Earnings are a better measure of the profitability of a company's operations and are unaffected by whether a company chooses to pay them out as dividends or reinvest them.

Perpetual Growth Methodology

We've established that the fundamental value of a stock is the present value of all future dividends, irrespective of how long an investor intends to hold it. But calculating this fundamental value requires you to forecast all dividends until the end of time, which seems impossible. However, recall that in Chapter 2 we encountered some simplified versions of the basic present value formula. Let's see whether they can be used for stocks.

Suppose you think that a company's dividends will grow at a constant rate. This doesn't rule out year-to-year deviations from the forecast, only that dividends are *expected* to increase at a steady rate. Such an investment would be another example of the growing perpetuity that we valued in Chapter 2. To find its present value, we divide the first year's dividend by the difference between the discount rate r and the growth rate g:

$$P_0 = \frac{\text{DIV}_1}{r - g} \tag{4.4}$$

This is known as the **constant-growth DCF model** or the **Gordon growth model** after the person who is credited with discovering it. Remember that we can use the model only when g is less than r. As g approaches r, the stock price becomes infinite. Obviously, r must be greater than g if growth really is perpetual.

How would you estimate the dividend growth rate in practice? One way is to use the following formula:

$$g = \text{plowback ratio} \times \text{return on equity} \tag{4.5}$$

The **return on equity (ROE)** is the ratio of net income to book equity, and shows how much additional earnings are generated by reinvesting cash inside the company. For simplicity, we assume that investment produces a constant return every year.[6] The **plowback ratio** is the fraction of earnings that a company reinvests internally rather than being paid out as dividends. If the plowback ratio is 10% and the return on equity is 20%, then every year a company's book equity increases by 10%, and this extra book equity generates $10\% \times 20\% = 2\%$ of additional earnings compared to the previous year. This in turn allows a company to pay out 2% of additional dividends, and so dividends grow by 2%.[7]

Non-Dividend-Paying Stocks

Although mature companies generally pay dividends, thousands of companies do not. For example, Amazon has never paid a dividend, yet it's a successful company with a market capitalization in February 2024 of $1.8 trillion. Why would a successful company decide *not* to pay dividends? There are two main reasons.

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[6]If the return is non-constant—for example, if it is low in early years and high in later years—the relevant measure is the internal rate of return. Loosely speaking, this is an average return that takes into account the time value of money. Section 5.3 will show how to calculate the internal rate of return.

[7]Why is the return on book equity relevant, when the Market Value Principle stated that market equity is relevant for nearly all finance applications? Because reinvesting money in the company directly increases book value. Market value, and thus the return on market equity, can be affected by many factors other than reinvestment, such as market expectations.

Reinvestment A growing company may maximize value by reinvesting all its earnings. The shareholders are better off with this policy, provided that the investments offer an expected rate of return higher than shareholders could get by investing on their own. In other words, shareholder value is maximized if the firm invests in projects that can earn more than the opportunity cost of capital. If such projects are plentiful, shareholders will be prepared to forego immediate dividends. They will be happy to wait and receive larger dividends later.[8]

The dividend discount model is still correct for growing companies, but difficult to use when dividends are far in the future. In this case, most analysts switch to valuation by comparables, which we discussed earlier, or to earnings-based formulas, which we cover in Section 4-5.

Share Repurchases A company may pay out cash not as dividends but by repurchasing shares from existing shareholders. In March 2022, Amazon announced that it would buy back up to $10 billion of shares. The dividend discount model remains valid; we just need to take into account how repurchases reduce the number of shares and allow the company to pay higher dividends per remaining share. We cover the choice between dividends and repurchases in Chapter 15.

The dividend discount model can be difficult to deploy if repurchases are irregular and unpredictable. In these cases, it can be better to value the company as a whole rather than one individual share. We cover this valuation approach in Section 4-6.

4-4 Terminal Value

So far, we've developed two valuation formulas. One is the "exact" formula which considers every single future dividend:

$$P_0 = \sum_{t=1}^{\infty} \frac{\text{DIV}_t}{(1+r)^t}$$

The second is the "back-of-the-envelope" formula which considers only next year's dividend, and then assumes that it grows at a constant rate forever:

$$P_0 = \frac{\text{DIV}_1}{r-g}$$

While the second method is convenient, it can only be used once a company's dividends are growing at a constant rate. In reality, many companies may pay no dividends for several years. Then, when they start paying dividends, they ramp up rapidly. Only once a company is mature will its dividends grow at a roughly consistent pace.

As a result, financial analysts typically use a two-stage method that combines the first approach with either the second or with comparable companies analysis. They forecast dividends up to a horizon date H and then calculate the **terminal value** of that company at that horizon date. In other words, we have

$$P_0 = \sum_{t=1}^{H} \frac{\text{DIV}_t}{(1+r)^t} + \frac{P_H}{(1+r)^H} \tag{4.6}$$

The terminal value is what the company is worth at date H. We can calculate it using the same two methods as we use to value stocks: by comparables, or by fundamentals. We'll start with the latter.

[8]The deferred payout may come all at once if the company is taken over by another. The selling price per share is equivalent to a bumper dividend.

BEYOND THE PAGE

Try it! Value and the Investor's Horizon

Terminal Value: Fundamentals

Force Velocity Group (FVG) has just paid a dividend of \$4 per share. Analysts are forecasting that FVG will experience two years of abnormally high dividend growth of 20%, before settling down to a normal growth rate of 5% in year 3 and beyond. Equally risky investments offer a return of 19%. How would you calculate the market price of FVG stock?

We can't use the perpetual growth formula immediately, because the initial 20% growth rate doesn't last forever. Instead, we deploy the two-stage method in Equation 4.6 with a horizon date of 2. We calculate the first two dividends as follows:

$$\text{DIV}_1 = 4 \times 1.2 = 4.8$$

$$\text{DIV}_2 = 4.8 \times 1.2 = 5.76$$

We then estimate the terminal value at date 2 using the perpetual growth formula, because dividends will grow at a constant 5% from then on. The terminal value is given by

$$P_2 = \frac{\text{DIV}_3}{r - g}$$

We have $\text{DIV}_3 = 5.76 \times 1.05 = 6.048$, $r = 19\%$ and $g = 5\%$. This gives

$$P_2 = \frac{6.048}{0.19 - 0.05} = 43.2$$

We can now use Equation 4.6:

$$P_0 = \frac{4.8}{1.19} + \frac{5.76}{1.19^2} + \frac{43.2}{1.19^2} = 38.61$$

Note that the first dividend in the PV formula is \$4.8, the dividend one year from now. The dividend of \$4 has just been paid and so anyone who buys FVG stock today will not receive it. Financial analysis is always about the future, not about the past; the past is only relevant to the extent to which it predicts the future. When you make any investment, real or financial, you receive the future cash flows of that investment and bear its future risks. This is captured by the following principle:

> ***The Forward-Looking Principle****: Cash flows and discount rates should reflect what is expected to happen in the future, not what has happened in the past.*

The above example illustrates one way to choose your forecast at horizon H: it's the date at which the firm is mature and it's reasonable to assume a constant dividend growth rate from then on. In fact, in the above example, you could also use a forecast horizon of $H = 1$. We would then have

$$P_0 = \frac{\text{DIV}_1}{1.19} + \frac{P_1}{1.19}$$

$$P_0 = \frac{\text{DIV}_1}{1.19} + \frac{\frac{\text{DIV}_2}{r-g}}{1.19}$$

$$P_0 = \frac{4.8}{1.19} + \frac{\frac{5.76}{0.19-0.05}}{1.19} = 38.61$$

Remember that the perpetual growth formula has *next* year's dividend in the numerator. Since dividends grow at a constant rate from date 2 onward, we can apply the perpetual growth formula at date 1. The terminal value of FVG at date 1 is the date 2 dividend, \$5.76, divided by $(r - g)$. Thus, FVG's stock price is the date 1 dividend plus the date 1 terminal value, both discounted by one year. This leads to the exact same answer.

Multiple Stages of Growth We can generalize the two-stage approach to as many stages as are needed. For example, FVG could grow at 20% for two years, then 10% for the next three, 15% for the next four, and finally a constant growth rate of 5% in year 10 and beyond. How many stages are "needed"? Recall that the growth rate is the plowback ratio multiplied by the return on equity, so growth changes when either of the two drivers change:

1. *Profitability.* This in turn depends on:
   - *Internal factors.* The rate of return on new projects is typically high for early-stage companies, which have an abundance of growth opportunities. As they mature, and new competitors enter, the return on equity typically declines. This is why, in the above example, FVG's growth rate starts off at 20% but falls to 5% in the long term.
   - *External factors.* The rate of return also depends on economic conditions. This is why, even though FVG's growth rate generally declines, it is higher in years 6–9 (15%) than in years 3–5 (10%): years 3–5 are forecast to be an economic downturn, while 6–9 will be an upswing.
2. *Reinvestment.* Similar to profitability, companies typically reinvest substantial amounts at early stages, which explains why many tech companies don't pay dividends. When they mature, they have fewer opportunities to reinvest at above the cost of capital. Such reinvestment opportunities will also change with economic conditions.

Here we have focused on finding the price of a stock given a cost of equity r and expectations about the growth rate of dividends. An alternative use of the formula is to turn it around and find the cost of equity r given the stock price and expected dividend growth rate. This approach is considered in the Appendix to this chapter.

Terminal Value: Comparables

You won't be able to estimate terminal value using the perpetual growth formula in two cases:

- You can't foresee any date H after which dividends will grow at a constant rate. The company is a long way off entering into a mature phase.
- You can forecast a date H after which dividends will grow at constant rate, but it's so far off that you're unable to reasonably forecast dividends until that date.

In this case, your choice of H will not be determined by when dividends will start to grow at a constant rate, but how far into the future you can reasonably forecast. You forecast dividends as far as you can, and then estimate terminal value at that point. Since you can't use the perpetual growth formula to estimate terminal value, you'll instead use valuation by comparables.

Let's look at an example. It's December 2024 and Copley Corporation's dividends per share for 2025–2028 are forecast to be \$0.77, \$0.84, \$0.91, and \$0.98, respectively. The most relevant value driver for Copley Corporation is its earnings, which are estimated to be \$1.50 per share in 2028. Its cost of equity is 12% and its nearest competitor, Leggett Limited, is currently trading at \$40 per share. Leggett Limited's earnings per share and P/E ratios are given as follows:

Year	Earnings per Share	P/E
2023A	\$3.75	10.67
2024E	\$4.00	10
2025E	\$4.30	9.30

Leggett has a December year end, so even though it's December 2024, its full year results won't be announced until January 2025, which is why the 2024 earnings are still estimates.

We can now use Equation 4.6 to calculate the fair price of Copley today:

$$P_0 = \frac{0.77}{1.12} + \frac{0.84}{1.12^2} + \frac{0.91}{1.12^3} + \frac{0.98}{1.12^4} + \frac{P_4}{1.12^4} \qquad \textbf{(4.7)}$$

We wish to estimate Copley's stock price in 2028. Its estimated earnings of $1.50 are also for 2028. We thus need to multiply $1.50 by an estimate of Copley's *current* P/E ratio in 2028, because the current P/E ratio compares a stock price to the earnings in that same year.

Leggett Limited's current P/E is 10. Since Leggett is a comparable company, and assuming that current P/E ratios will be stable between now and 2028, a reasonable estimate for Copley's current P/E in 2028 is 10. We thus have

$$P_4 = \$1.5 \times 10 = \$15$$

Plugging this into Equation 4.7 gives $P_0 = \$12.16$.

Notice the principle of "apples-to-apples."

- We had a forecast for Copley's earnings in 2028, and we wanted to estimate Copley's stock price in December 2028, which is why we needed an estimate of Copley's current P/E.
- If we instead had a forecast for Copley's earnings in 2029, the next full year *after* December 2028, we'd need an estimate of Copley's *forward* P/E. We could use Leggett's forward P/E of 9.3 as an estimate.
- If we instead had a forecast for Copley's earnings in 2027, the last full year *prior* to December 2028, we'd need an estimate of Copley's *historic* P/E. We could use Leggett's historic P/E of 10.67 as an estimate.

4-5 The Determinants of Price-Earnings Ratios

The most common multiple used in comparable companies analysis is the price-earnings ratio. In Table 4.2, we saw that this ratio differs significantly across companies: it was 38.7 for PepsiCo but only 6.4 for Citigroup. Why would someone buy PepsiCo and have to pay $38.70 for each dollar of earnings, when they could pay only $6.40 by buying Citigroup? Does this mean that PepsiCo is overpriced and Citigroup is underpriced?

Not necessarily. Let's return to our house price analogy. A property in Manhattan, New York City might cost $2,000 per square foot, compared to $200 in Bloomington, Indiana. This doesn't mean that the former is overpriced; it's because homeowners care not only about the size of their house, but also its location. Similarly, investors in a stock don't just care about its current earnings, but also its future growth. So you might think that companies with growing earnings will always have higher price-earnings ratios. This is on the right lines, but it turns out to be more complex than that. Let's see why.

No-Growth Company

Let's first consider Nogro, a company that does not grow at all. It doesn't plow back any earnings and simply produces a constant stream of dividends. Its stock would resemble the perpetual bond described in Chapter 2. For example, if the dividend is $10 per share and the cost of equity is 10%, the perpetual growth formula gives Nogro's price as

$$P_0 = \frac{\text{DIV}_1}{r - g} = \frac{10.00}{0.10 - 0} = \$100$$

Since earnings are the same as dividends, in this special case of a no-growth firm, we also have

$$P_0 = \frac{\text{EPS}_1}{r} = \frac{10.00}{0.10} = \$100$$

Growing Company, Rate of Return = Cost of Equity

Now consider Somegro, which is exactly the same as Nogro but has the opportunity to invest \$10 a share next year in a new project with an ROE of 10%. This would mean no dividend at $t = 1$. From $t = 2$ onward, the project will produce \$10 × 10% = \$1 per share, and so the dividend could be increased to \$11 per share from date 2.

What happens to the company's stock price? One share pays a dividend of \$11 per year from $t = 2$ onward. Applying the perpetual growth formula at date 1 gives

$$P_1 = \frac{\text{DIV}_2}{r} = \frac{\$11}{0.1} = \$110$$

and so

$$P_0 = \frac{P_1}{1 + r} = \frac{\$110}{1.1} = \$100$$

Somegro has exactly the same stock price as Nogro. This makes sense, because the project is zero NPV. To see this, Somegro invests \$10 at date 1 and receives \$1 every year from date 2 onward, which yields an NPV at date 1 of

$$\text{NPV}_1 = -10 + \frac{1}{0.10} = 0$$

The ROE on the project is exactly the same as the cost of equity, and so the project is zero NPV. As a result, Somegro's stock price is the same as Nogro's, and so it also has the same P/E of \$100/10 = 10. How can this be when they have different growth rates? Nogro's earnings per share never grow—they stay at \$10—but Somegro's earnings grow from \$10 to \$11 in year 2 and remain at this higher level forever.

Because growth is insufficient for a higher P/E. Growth is NPV positive only if the ROE exceeds the cost of equity. Only then will growth increase the stock price, and thus the P/E. We state this result in the following principle:

> ***The Growth Principle****: Growth only adds value to a company if the rate of return on reinvested capital exceeds the cost of capital.*

Looking at it another way, growth requires investment and thus a lower dividend today. Only if the increase in future earnings (and thus dividends) is high enough to compensate for the lower current dividend will the stock price—the present value of all dividends—increase. This will only be the case if the return on the project is sufficiently high.

Growing Company, General Case

Now that we've highlighted the importance of the project's return, Table 4.3 repeats our example for different assumptions about this rate.

As we can see, the P/E ratio of Somegro is higher than Nogro if and only if the rate of return on the project exceeds the cost of equity of 10%. Indeed, Somegro has a *lower* P/E ratio if the return is only 5%, as shown in the top row. Even though its earnings grow faster than Nogro's, this growth is insufficient to compensate for the lost dividend.

Project Rate of Return	Incremental Cash Flow (C)	Project NPV in Year 1[1]	Project's Impact on Share Price in Year 0[2]	Share Price in Year 0 (P_0)	$\frac{EPS_1}{P_0}$	r
0.05	$0.50	−$5.00	−$4.55	$ 95.45	9.545	0.10
0.10	1.00	0	0	100.00	10	0.10
0.15	1.50	+5.00	+4.55	104.55	10.455	0.10
0.20	2.00	+10.00	+9.09	109.09	10.909	0.10

TABLE 4.3 NPV of Reinvestment

Effect on stock price of investing an additional $10 in year 1 at different rates of return.

[1]Project costs $10.00 ($EPS_1$). NPV = −10 + C/r, where $r = 0.10$.
[2]NPV is calculated at year 1. To find the impact on P_0, discount for one year at $r = 0.10$.

Investors often refer to stocks with high P/E ratios as **growth stocks**, that reinvest their earnings in the hunt for future growth. They describe their low-P/E brethen as **income stocks**, who pay out most of their earnings as dividends to provide income for their investors. This distinction isn't entirely accurate. Even if a stock is reinvesting most of its earnings, it won't have a high P/E unless the rate of return on reinvested capital is sufficiently high. Conversely, a low-P/E stock might not be producing much of an income; instead, it could be foolishly reinvesting its earnings in negative-NPV projects.

The Present Value of Growth Opportunities

In general, we can think of stock price as the value of average earnings under a no-growth policy, plus **PVGO**, the **present value of growth opportunities**:

$$P_0 = \frac{\text{EPS}_1}{r} + \text{PVGO} \tag{4.8}$$

In turn, PVGO is given by[9]

$$\text{PVGO} = \frac{(y - r) \times k \times \text{EPS}_1}{r(r - g)} \tag{4.9}$$

where k is the **plowback ratio** and y is yield on reinvested capital, or **return on equity**. As with many formulas in this book, we won't derive it but will instead sense-check it.

- PVGO is positive if and only if $y > r$, that is, the return on equity exceeds the cost of equity. In particular, even if $g > 0$ (dividends are growing), PVGO could still be negative, as in the first row of Table 4.3.
- If $y > r$, and holding y and r constant, PVGO is increasing in k. The more a company can reinvest at a return on equity that exceeds its cost of equity, the more valuable its growth opportunities.

Using Equation 4.8, the forward P/E ratio is given by

$$\frac{P_0}{\text{EPS}_1} = \frac{1}{r} \Big/ \left(1 - \frac{\text{PVGO}}{P_0}\right) \tag{4.10}$$

[9]Later in the book we will interpret PVGO more generally as the NPV today of the firm's *real options* to expand in the future. See Chapter 23.

Equation 4.12 shows that a stock will have a high P/E for two reasons:

- *High PVGO*. In turn, Equation 4.9 shows that a high PVGO arises if $y > r$ and k is high, that is, the company can reinvest a high fraction of earnings at a return that exceeds the cost of equity.
- *Low cost of equity r*. If other investment opportunities are less attractive, the stock is more valuable. This is similar to why bond prices are higher when market interest rates are lower.

Growth stocks are therefore stocks in which PVGO is positive and accounts for a large fraction of the stock price. Investors in growth stocks are attracted by the capital gains they will get if expansion succeeds. **Income stocks** are stocks of companies where PVGO accounts for a small fraction of the stock price. Such companies may still grow, but are not able to invest at returns substantially above the cost of equity. The price of income stocks depends mostly on future earnings and dividends from existing assets.

Calculating the Present Value of Growth Opportunities

Let's calculate the present value of growth opportunities for that well-known growth stock, Establishment Electronics. Its current assets will generate earnings per share of \$8.33 in perpetuity. Next year it will pay a dividend of \$5; any reinvested capital earns a return of 25%. At a 15% cost of equity, what is the present value of its growth opportunities? We can calculate it in two ways.

Method 1: Direct The first method is to calculate PVGO directly by using Equation 4.9. We have the following inputs:

- y is the ROE of 25%
- k is the plowback ratio of $1 - 5/8.33 = 40\%$
- $g = yk$ is the growth rate of dividends, which is $25\% \times 40\% = 10\%$

This gives us

$$\text{PVGO} = \frac{(0.25 - 0.15) \times 0.4 \times 8.33}{0.15(0.15 - 0.1)} = \$44.44$$

Method 2: Indirect We can also calculate PVGO indirectly by using Equation 4.8. Establishment's stock price is given by the constant-growth DCF formula:

$$P_0 = \frac{\text{DIV}_1}{r - g} = \frac{5}{0.15 - 0.10} = \$100$$

If Establishment had a no-growth policy, and paid out all of its earnings as dividends, its price would be

$$\frac{\text{EPS}_1}{r} = \frac{8.33}{0.15} = \$55.56$$

Using Equation 4.8, we have

$$\text{PVGO} = P_0 - \frac{\text{EPS}_1}{r} = 100 - 55.56 = \$44.44$$

the same as above.

Explaining PVGO We've shown that investors must be valuing Establishment's growth opportunities at \$44.44. Let's see if we can explain that figure.

Each year Establishment plows back 40% of its earnings into new assets. In the first year, it invests \$3.33 at a permanent 25% return. Thus, the cash generated by this investment is $0.25 \times 3.33 = \$0.83$ per year starting at $t = 2$ and continuing forever in perpetuity. The net present value of the investment as of $t = 1$ is

$$\text{NPV}_1 = -3.33 + \frac{0.83}{0.15} = \$2.22$$

The investment in year 2 is 10% higher (remember $g = 0.10$) and so the cash it generates is also 10% higher. Thus, the net present value of the year 2 investment is

$$\text{NPV}_2 = -3.33 \times 1.10 + \frac{0.83 \times 1.10}{0.15} = \$2.44$$

NPV_2 is 10% higher than NPV_1, because all cash flows (both the initial investment and the future payoffs) are 10% higher. Similarly, the year 3 investment has a 10% higher net present value of $\$2.44 \times 1.10 = \$2.69$.

Establishment Electronics' growth opportunities thus comprise a series of tickets, one for each future year, each of which is 10% more valuable than the last. Does this sound familiar to you? It should: it is a growing perpetuity! The present value of the growth opportunities is therefore

$$\text{PVGO} = \frac{\text{NPV}_1}{r - g} = \frac{2.22}{0.15 - 0.10} = \$44.44$$

exactly as we calculated earlier. Establishment's stock price thus has two components:

$$\begin{aligned} \text{Share price} &= \text{present value of level stream of earnings} \\ &\quad + \text{present value of growth opportunities} \\ &= \frac{\text{EPS}_1}{r} + \text{PVGO} \\ &= \$55.56 + \$44.44 \\ &= \$100 \end{aligned}$$

Why is Establishment Electronics a growth stock? Not just because it is expanding at 10% per year. It is a growth stock because PVGO, the net present value of its future investments, accounts for 44% of the stock's price. The total price incorporates investors' expectations about the earnings power of the firm's current and future assets.

We admit that calculating PVGO is more difficult in practice than for Establishment Electronics. Companies are complex, and future investment and profitability are difficult to forecast. We may not know the rate of return on new investment, or the amount that a company can reinvest at that return. PVGO is nevertheless an important *practical* concept—the key to understanding the potential value of future investment and growth. Also, we can infer PVGO indirectly using Equation 4.8 even when we cannot calculate it directly as we did for Establishment.

EXAMPLE 4.1 • PVGO for Nike and Essential Utilities

Question 1: Table 4.1 reports statistics for Nike stock at the close of trading on one day in February 2024. The stock closed at \$103.77 per share. The forward P/E was 24.42. EPS forecast for 2025 were \$4.25. Assuming a cost of equity of 12%, is Nike a growth or income stock?

Answer 1: If Nike grew no further and paid all of its earnings out as dividends, its price would have been $4.25/0.12 = \$35.42$, about a third of its actual price. Thus Nike is a growth stock because PVGO is a large fraction of its stock price.

Question 2: Essential Utilities' closing price on one day in February 2024 was \$35, with forward EPS of \$2.12 and a forward P/E of 16.51. Assuming the same cost of equity of 8%, is Essential Utilities a growth or income stock?

Answer 2: If Essential Utilities paid out all of its earnings as dividends, its price would have been 2.12/0.08 = \$26.50. Since \$26.50/\$35 = 76%, Essential Utilities is an income stock because PVGO is a small fraction of its stock price.

4.3 Self-Test

Wilderness Matchmakers is growing rapidly, but intense competition restricts the rate of return on new investment to its cost of equity capital. How does the company's P/E ratio depend on its future rate of growth?

BEYOND THE PAGE

Valuing Alphabet

4-6 Valuation Based on Free Cash Flow

So far we have focused on price per share. However, it's often simpler or more convenient to value shareholders' equity, the aggregate of all outstanding shares. The DCF principle is the same, except that free cash flow takes the place of dividends per share:

PV(equity) = market capitalization = PV(expected future free cash flow)

Free cash flow (FCF) equals the post-tax cash flow generated by the company's operations. Importantly, this is after subtracting investment, just like the value of a share is the present value of future dividends rather than earnings. Free cash flow is the amount of cash available for payout to all shareholders, whereas dividends per share are paid to a single share. (Note that the company may choose not to pay out its free cash flow but retain some or all of it on the balance sheet, in which case it still belongs to shareholders.)

Here are three examples where this valuation approach would make sense.

1. The owners of a private business ask what it would be worth if it were publicly traded. The owners are less interested in price per share than in the potential total market value of their business.
2. A company has an opportunity to sell one of its manufacturing divisions. The division is wholly owned by the company and has no outside shareholders. The company wants to know what the division is worth overall.
3. A security analyst is trying to value a public company using DCF. The company is both paying dividends and repurchasing shares. The repurchases fluctuate year to year, which makes the number of remaining shares, and therefore earnings and dividends *per share,* hard to predict. So the analyst decides to forecast and value free cash flow. He knows that free cash flow is the *total* amount of cash that can be paid out to investors and doesn't want to worry about how payout is split between dividends and repurchases. (More on dividends vs. repurchases in Chapter 15.)

We now work through a numerical example.

Valuing the Concatenator Business

Concatenator Inc. is a privately owned business. A potential acquirer is trying to figure out how much to offer for it.

A	B	C	D	E	F	G	H	I	J	K
	1	2	3	4	5	6	7	8	9	10
1 Asset value, start of year	**10.00**	11.20	12.54	14.05	15.31	16.69	18.19	19.29	20.44	21.67
2 Return on equity	**0.12**	**0.12**	**0.12**	**0.12**	**0.12**	**0.12**	**0.12**	**0.12**	**0.12**	**0.12**
3 Earnings ((1) × (2))	1.20	1.34	1.51	1.69	1.84	2.00	2.18	2.31	2.45	2.60
4 Plowback ratio	**1.00**	**1.00**	**1.00**	**0.75**	**0.75**	**0.75**	**0.50**	**0.50**	**0.50**	**0.50**
5 Investment ((3) × (4))	1.20	1.34	1.51	1.26	1.38	1.50	1.09	1.16	1.23	1.30
6 Free cash flow (FCF) ((3) – (5))	0.00	0.00	0.00	0.42	0.46	0.50	1.09	1.16	1.23	1.30
7 Asset value, end of year ((1) + (5))	11.20	12.54	14.05	15.31	16.69	18.19	19.29	20.44	21.67	22.97
8 Asset growth rate		0.12	0.12	0.09	0.09	0.09	0.06	0.06	0.06	0.06
9 Earnings growth rate		0.12	0.12	0.12	0.09	0.09	0.09	0.06	0.06	0.06

TABLE 4.4 Cash-Flow Forecasts for Concatenator.
Inputs required for the table's calculations are in bold type. All figures in $ millions.

Notes:
1. Free cash flow equals earnings minus investment. For simplicity, we assume no depreciation and so the asset value rises by the amount of investment. Earnings are net of taxes.
2. Concatenator has no debt outstanding. Therefore asset value is equal to equity value, so the ROE is earnings divided by asset value and is equal to ROA.[10]

Table 4.4 gives a forecast of FCF for Concatenator. It forecasts earnings, investment, and assets, based on the following assumptions:

- Plowback ratio is 100% for the first three years, then falls to 75% in years 4-6 and 50% from year 7 onwards.
- Opportunity cost of capital is 10%

Free cash flow, the sixth line in Table 4.4, is equal to earnings less new investment.[11] Free cash flow is zero in years 1 to 3, because Concatenator is reinvesting all of its earnings during this period. Are the early zeros for free cash flow a bad sign? No: Free cash flow is zero because the business is growing rapidly, not because it is unprofitable. Rapid growth is good news, because the business is earning 12%, 2 percentage points over the 10% cost of capital.

Valuation Method

Concatenator's value equals the discounted value of its free cash flows out to a horizon date H, plus the discounted terminal value of the business at that date:

$$\mathrm{PV} = \underbrace{\frac{\mathrm{FCF}_1}{1+r} + \frac{\mathrm{FCF}_2}{(1+r)^2} + \ldots + \frac{\mathrm{FCF}_H}{(1+r)^H}}_{\text{PV(free cash flow)}} + \underbrace{\frac{\mathrm{PV}_H}{(1+r)^H}}_{\text{PV(terminal value)}} \qquad \textbf{(4.11)}$$

This equation is exactly the same as Equation 4.6, except that free cash flow takes the place of expected dividends.

[10] If Concatenator had debt, then we would use ROA. Chapter 18 explains this approach.
[11] We assume for simplicity that there is no depreciation and existing assets do not require maintenance or replacement.

Valuation horizons are often chosen arbitrarily. Sometimes the boss tells everybody to use 10 years because that's a round number. We will use year 6, because growth of the Concatenator business settles down to a long-run trend after year 7.

BEYOND THE PAGE
Try It! Table 4.4: Valuing the Concatenator business

Estimating Terminal Value

As discussed in Section 4-4, we can estimate terminal value using comparables or by fundamentals. We will start with the former, and use two multiples: P/E and P/B.

Terminal Value Based on P/E Ratios Suppose you can observe stock prices for good comparables—that is, for mature manufacturing companies whose scale, risk, and growth prospects today roughly match those projected for Concatenator in year 6.[12] Suppose further that these companies tend to sell at price–earnings ratios of about 11. Then you could reasonably guess that the price–earnings ratio of a mature, public concatenator company would likewise be 11. That implies

$$\text{PV(terminal value)} = \frac{11 \times 2.18}{1.1^6} = 13.5$$

The present value of free cash flow up to the horizon is $0.42/1.1^4 + 0.46/1.1^5 + 0.5/1.1^6 =$ \$0.9 million. Therefore,

$$\text{PV (business)} = 0.9 + 13.5 = \$14.4 \text{ million}$$

Terminal Value Based on P/B Ratios Suppose also that the market–book ratios of the sample of mature manufacturing companies tend to cluster around 1.5. If Concatenator's market-to-book ratio is 1.5 in year 6,

$$\text{PV(terminal value)} = \frac{1.5 \times 16.69}{1.1^6} = 14.1$$

$$\text{PV(business)} = 0.9 + 14.1 = \$15.0 \text{ million}$$

Terminal Value Based on DCF Now let us try the constant-growth DCF formula. This requires the following:

- Free cash flow in year 7, which is \$1.09 million from Table 4.4.
- A long-term growth rate of free cash flow. This is plowback ratio × ROE = 50% × 12% = 6%
- The discount rate, which is given as 10%.

Therefore,

$$\text{Terminal value at date 6} = \text{PV}_H = \frac{1.09}{0.10 - 0.06} = \$27.3 \text{ million}$$

$$\text{PV(terminal value)} = \frac{27.3}{1.1^6} = \$15.4 \text{ million}$$

The PV of the near-term free cash flows is \$0.9 million. Thus, the present value of the concatenator division is

$$\begin{aligned}\text{PV(business)} &= \text{PV(free cash flow)} + \text{PV(terminal value)}\\ &= 0.9 + 15.4\\ &= \$16.3 \text{ million}\end{aligned}$$

[12]We have not asked how the Concatenator business would be financed if it were a stand-alone business. Table 4.4 assumes 100% equity and zero debt. Therefore the comparables should also have little or no debt. If they do have debt, EBIT or EBITDA ratios would be better than P/E ratios. See the examples in Section 18-2.

Sensitivity Analysis Now, are we done? Well, the mechanics of this calculation are perfect. But doesn't it make you just a little nervous to find that 94% of the value of the business rests on the terminal value? Moreover, a little checking shows that terminal value can change dramatically in response to small changes in the assumed long-term growth rate.

Suppose the growth rate is 7% instead of 6%. That means that asset value has to grow by an extra 1% per year. The asset value at the start of year 7 is \$18.19 million. An extra 1% growth in earnings requires an extra investment of 0.01× \$18.19 = \$0.18 million in year 7, which reduces FCF_7 to \$1.09 – \$0.18 = \$0.91 million. Terminal value increases to PV_H = \$0.91/(0.10 – 0.07) = \$30.3 million in year 6 and to $\$30.3/1.1^6$ = \$17.1 million discounted to year zero. PV increases from \$16.3 million to \$0.9 + 17.1 = \$18.0 million.

When estimating terminal value, take care to heed the following warnings:

1. *Growth Requires Investment.* When you use the constant-growth DCF formula to calculate terminal value, always remember that faster growth requires increased investment, which reduces free cash flow. Indeed, when we considered the case in which Concatenator grows at 7% rather than 6%, we accounted for the increased investment of \$0.18 million required for faster growth.
2. *Growth Requires Returns to Exceed the Cost of Capital* The Growth Principle highlights how it is not sufficient for a company to be plowing back earnings for PVGO to be positive; the return on reinvested capital y needs to exceed the cost of equity r. For Concatenator, ROE is forecast to be 12% forever, which exceeds the discount rate of 10%. Thus, every dollar invested in period 7 and beyond generates positive NPV. This is why 7% instead of 6% growth increases the value of Concatenator by \$18.0 – 16.3 = \$1.7 million, even after accounting for the \$0.18 million investment required for faster growth.

   But is it realistic to assume that any business can keep on growing and making positive-NPV investments forever? Sooner or later your competitors will catch up with you. You may still be earning a superior return on past investments, but you will find that introductions of new products or attempts to expand profits from existing products trigger vigorous reactions from competitors who are just as smart and efficient as you are. When that time comes, the NPV of subsequent investments will average out to zero. After all, PVGO is positive only when investments can be expected to earn more than the cost of capital.
3. *Terminal Value Depends on Post-Horizon PVGO* Always check to see whether terminal value assumes that post-horizon PVGO is positive. You can check Warning 3 by changing the assumed long-term growth rate. If a higher growth rate increases terminal value—after you have taken care to respect Warning 1—then you are assuming positive post-horizon PVGO. From Warning 2, this means that you are assuming that the firm can earn more than the cost of capital in perpetuity. If this assumption is unrealistic, then adjust your forecasts accordingly.

There is an easy way to calculate terminal value if post-horizon PVGO is zero. Recall that PV depends on next period's earnings plus PVGO:

$$PV_t = \frac{\text{earnings}_{t+1}}{r} + \text{PVGO}$$

If PVGO = 0 at the horizon date H, then,

$$PV_H = \frac{\text{earnings}_{H+1}}{r}$$

In other words, when the competition catches up and the firm can only earn its cost of equity on new investment, the price–earnings ratio will equal $1/r$, because PVGO disappears and all earnings ends up as free cash flow.

If post-horizon PVGO is zero, we can calculate the terminal value at period 6 as the present value of a level stream of earnings starting in period 7 and continuing indefinitely. The resulting value for Concatenator is

$$
\begin{aligned}
\text{PV(terminal value)} &= \frac{1}{(1+r)^6}\left(\frac{\text{earnings in period 7}}{r}\right)\\
&= \frac{1}{(1.1)^6}\left(\frac{2.18}{0.10}\right)\\
&= \$12.3 \text{ million}\\
\text{PV(business)} &= 0.9 + 12.3 = \$13.2 \text{ million}
\end{aligned}
$$

A Value Range for Concatenator We now have four estimates of PV based on four different methods of estimating terminal value. There is no best method, although we like the last method of assuming PVGO = 0 at the horizon date, which forces managers to remember that sooner or later competition catches up.

Our calculated values for the concatenator business range from \$13.2 to \$16.3 million, a difference of about \$3 million. The width of the range may be disquieting, but it is not unusual. Discounted cash flow formulas only estimate market value, and the estimates change as forecasts and assumptions change. Managers cannot know market value for sure until an actual transaction takes place.

4.4 Self-Test

How do you check whether terminal value in a DCF model includes post-horizon PVGO? Why is this check important in practice?

KEY TAKEAWAYS

In this chapter, we have used present value (PV) formulas to explain stock prices.

- **Discounted cash flow (DCF) or dividend discount model** The value of a share equals the PV of the future dividends paid to the current and future owners of that share. Shares have no fixed maturity and may survive forever. The formula is

$$P_0 = \sum_{t=1}^{\infty} \frac{\text{DIV}_t}{(1+r)^t} \tag{4.3}$$

- **DCF model with terminal value** It is often convenient to calculate the PV of dividends out to some horizon date H and then add the PV of P_H, the forecast price on that date. In this case, the PV formula is

$$P_0 = \sum_{t=1}^{H} \frac{\text{DIV}_t}{(1+r)^t} + \frac{P_H}{(1+r)^H} \tag{4.6}$$

The forecast *terminal value* P_H stands in for the PV of dividends per share after date H. It can be calculated using fundamentals (expected future dividend growth) or comparables. Comparable companies are compared using multiples such as price-to-book or price-to-earnings. The price-to-earnings multiple can be trailing, historic, current, or forward; typically, forward P/Es are used.

- **Dividends and capital gains** These DCF formulas do not assume that investors purchase shares solely for dividends. The one-period PV formula values next period's dividend plus next period's price, which incorporates capital gains or losses:

$$P_0 = \frac{\text{DIV}_1 + P_1}{1+r} \tag{4.2}$$

The discount rate r (the *cost of equity*) is an opportunity cost of capital. It equals the expected rate of return on all stocks with the same risk as the stock being valued. The application of the one-period formula to repeated future dates leads to the long-term DCF formula, Equation 4.3.

- **Perpetual growth** If dividends are expected to grow forever at a constant rate g,

$$P_0 = \frac{\text{DIV}_1}{r - g} \tag{4.4}$$

where r must be greater than g. The growth rate g is the plowback ratio k multiplied by the rate of return on reinvested capital y.

- **Earnings and growth opportunities** Dividend discount models are most useful for mature firms that pay regular dividends. They are less useful for young, rapidly growing firms that pay out no dividends. The share price of such firms is better expressed as

$$P_0 = \frac{\text{EPS}_1}{r} + \text{PVGO} \tag{4.8}$$

The ratio EPS_1/r is the present value of earnings per share that the firm would generate if it did not grow. With zero growth, all of EPS could be paid out as dividends. PVGO is the NPV of the future investments that the firm will make in order to grow. A growth firm is one with large PVGO relative to the PV of its EPS with no growth. Rapid growth is insufficient for high PVGO: we also need the return on investment to exceed the cost of equity.

- **Valuation of a business based on free cash flow** Free cash flow (FCF) is the total amount of cash available for payout to investors after subtracting all operating expenses and investments necessary for growth. The PV formula is

$$\text{PV}_0 = \sum_{t=1}^{H} \frac{\text{FCF}_t}{(1+r)^t} + \frac{\text{PV}_H}{(1+r)^H}$$

where PV_H represents the PV of FCF for all future periods after date H. PV_H may be estimated using fundamentals (the perpetual-growth DCF formula) or comparables (assuming a future price-to-earnings or price-to-book ratio). PVs based on free cash flow are useful when the firm pays out cash by repurchases or by a combination of repurchases and dividends. They are also useful for valuing privately owned businesses or operating divisions of larger companies.

PROBLEM SETS

Select problems are available in McGraw-Hill's *Connect*. Please see the preface for more information.

1. **Stock markets (S4-1)** True or false?
   a. The sale of shares by a large investor usually takes place in the primary market.
   b. *Electronic communications network* refers to the automated ticker tape on the New York Stock Exchange.
2. **Stock quotes (S4-1)** Go to **finance.yahoo.com** or another Internet source and get trading quotes for IBM.
   a. What is the latest IBM stock price and market cap?
   b. What is IBM's dividend payment and dividend yield?
   c. What is IBM's trailing P/E ratio?
   d. Calculate IBM's forward P/E ratio using the EPS forecast by analysts for the next year.
   e. What is IBM's price-book (P/B) ratio?

3. **Valuation by comparables (S4-2)** Look up P/E and P/B ratios for Entergy (ticker symbol ETR), using Yahoo! Finance or another Internet source. Calculate the same ratios for the following potential comparables: American Electric Power (AEP), Eversource Energy (ES), and Southern Company (SO). Set out the ratios in the same format as Table 4.2. Are the ratios for these electric companies tightly grouped or scattered? If you didn't know Entergy's stock price, would the comparables give a good estimate?
4. **Dividend discount model (S4-3)** True or false?
   a. All stocks in an equivalent-risk class are priced to offer the same expected rate of return.
   b. The value of a share equals the PV of future dividends per share.
5. **Dividend discount model (S4-3)** Respond briefly to the following statement: "You say stock price equals the present value of future dividends? That's crazy! All the investors I know are looking for capital gains."
6. **Dividend discount model (S4-3)** Company X is expected to pay an end-of-year dividend of \$5 a share. After the dividend, its stock is expected to sell at \$110. If the cost of equity is 8%, what is the current stock price?
7. **Dividend discount model (S4-3)** Company Y does not plow back any earnings and is expected to produce a level dividend stream of \$5 a share. If the current stock price is \$40, what is the cost of equity?
8. **Terminal value (S4-4)** What is meant by the "terminal value" of a business? How can it be estimated?
9. **Constant-growth DCF model (S4-4)** Company Z's earnings and dividends per share are expected to grow indefinitely by 5% a year. If next year's dividend is \$10 and the cost of equity is 8%, what is the current stock price?
10. **Constant-growth DCF model (S4-4)** Consider three investors:
    a. Mr. Single invests for one year.
    b. Ms. Double invests for two years.
    c. Mrs. Triple invests for three years.

    Assume each invests in company Z (see Problem 9). Show that each expects to earn a rate of return of 8% per year.
11. **Constant-growth DCF model (S4-4)** Pharmecology just paid an annual dividend of \$1.35 per share. It's a mature company, but future EPS and dividends are expected to grow with inflation, which is forecasted at 2.75% per year.
    a. What is Pharmecology's current stock price? The nominal cost of capital is 9.5%.
    b. Redo part (a) using forecast real dividends and a real discount rate.
12. **Constant-growth DCF model (S4-4)**

    Here are forecasts for next year for two stocks:

	Stock A	Stock B
Return on equity	15%	10%
Earnings per share	\$2.00	\$1.50
Dividends per share	\$1.00	\$1.00

    a. What are the dividend payout ratios for each firm?
    b. What are the expected sustainable dividend growth rates for each stock?
    c. If investors require a return of 15% on each stock, what are their values?

13. **Constant-growth DCF model (S4-4)** Look up General Mills (GIS), Kellogg (KLG), and Campbell Soup (CPB).

    a. What are the current P/E and P/B ratios for these food companies? What are the dividend and dividend yield for each company?

    b. What are the growth rates of EPS and dividends for each company over the last five years? What EPS growth rates are forecast by analysts? Do these growth rates appear to be on a steady trend that could be projected for the long run?

    c. Would you be confident in applying the constant-growth DCF model to measure these companies' costs of equity? Why or why not?

14. **Two-stage DCF model (S4-4)** Company Z-prime is like Z in Problem 9 in all respects except one: Its growth will stop after year 4. In year 5 and afterward, it will pay out all earnings as dividends. What is Z-prime's stock price? Assume next year's EPS is $15.

15. **Two-stage DCF model (S4-4)** Consider the following three stocks:

    a. Stock A is expected to provide a dividend of $10 a share forever.

    b. Stock B is expected to pay a dividend of $5 next year. Thereafter, dividend growth is expected to be 4% a year forever.

    c. Stock C is expected to pay a dividend of $5 next year. Thereafter, dividend growth is expected to be 20% a year for five years (years 2 through 6, as C recovers from a severe recession) and zero thereafter.

    If the cost of equity for each stock is 10%, which stock is the most valuable? What if the capitalization rate is 7%?

16. **Two-stage DCF model (S4-4)** Company Q's current return on equity (ROE) is 14%. It pays out one half of earnings as dividends (payout ratio = 0.5). Current book value per share is $50. Book value per share will grow as Q reinvests earnings. Assume that the ROE and payout ratio stay constant for the next four years. After that, competition forces ROE down to 11.5% and the payout ratio increases to 0.8. The cost of equity is 11.5%.

    a. What are Q's EPS and dividends next year? How will EPS and dividends grow in years 2, 3, 4, 5, and subsequent years?

    b. What is Q's stock worth per share? How does that value depend on the payout ratio and growth rate after year 4?

17. **Two-stage DCF model (S4-4)** Compost Science Inc. (CSI) is in the business of converting Boston's sewage sludge into fertilizer. The business is not in itself very profitable. However, to induce CSI to remain in business, the Metropolitan District Commission (MDC) has agreed to pay whatever amount is necessary to yield CSI a 10% book return on equity. At the end of the year, CSI is expected to pay a $4 dividend. It has been reinvesting 40% of earnings and growing at 4% a year.

    a. Suppose CSI continues on this growth trend. What is the expected long-run rate of return from purchasing the stock at $100? What part of the $100 price is attributable to the present value of growth opportunities?

    b. Now the MDC announces a plan for CSI to treat Cambridge sewage. CSI's plant will, therefore, be expanded gradually over five years. This means that CSI will have to reinvest 80% of its earnings for five years. Starting in year 6, however, it will again be able to pay out 60% of earnings. What will be CSI's stock price once this announcement is made and its consequences for CSI are known?

18. **Growth opportunities (S4-5)** If company Z (see Problem 9) were to distribute all its earnings, it could maintain a level dividend stream of $15 a share. How much is the market actually paying per share for growth opportunities?

19. **Growth opportunities (S4-5)** Look up Intel (INTC), Oracle (ORCL), and HP (HPQ) on finance.yahoo.com or another Internet source. Rank the companies' forward P/E ratios from

highest to lowest. What are the possible reasons for the different ratios? Which of these companies appears to have the most valuable growth opportunities?

20. **Growth opportunities (S4-5)** Alpha Corp's earnings and dividends are growing at 15% per year. Beta Corp's earnings and dividends are growing at 8% per year. The companies' assets, earnings, and dividends per share are now (at date 0) exactly the same. Yet PVGO accounts for a greater fraction of Beta Corp's stock price. How is this possible? (*Hint:* There is more than one possible explanation.)

21. **Growth opportunities (S4-5)** Suppose the horizon date is set at a time when the firm will run out of positive-NPV investment opportunities. How would you calculate the terminal value? (*Hint:* What is the P/E ratio when PVGO = 0?)

22. **Free cash flow (S4-6)** What do financial managers mean by "free cash flow"? How is free cash flow calculated? Briefly explain.

23. **Valuing a business (S4-6)** Permian Partners (PP) produces from aging oil fields in west Texas. Production is currently 1.8 million barrels per year, but is declining at 7% per year for the foreseeable future. Costs of production, transportation, and administration add up to $25 per barrel. The current oil price is $65 per barrel. PP has 7 million shares outstanding. The cost of equity is 9%. All of PP's net income is distributed as dividends. For simplicity, assume that the company will stay in business forever and that costs per barrel are constant at $25. Also, ignore taxes.

    a. What is the value of one PP share? Assume that oil prices are expected to fall to $60 per barrel next year, and to $55 and $50 per barrel in the two years following. After that decrease, assume a long-term trend of oil-price increases at 5% per year.

    b. What is PP's E/P ratio, and why is it not equal to the 9% cost of capital?

24. **Valuing a business (S4-6)** Construct a new version of Table 4.4, assuming that competition drives down profitability (on existing assets as well as new investment) to 11.5% in year 6, 11% in year 7, 10.5% in year 8, and 8% in year 9 and all later years. What is Concatenator's PV?

25. **Valuing a business (S4-6)** Mexican Motors' market cap is 200 billion pesos. Next year's free cash flow is 8.5 billion pesos. Security analysts are forecasting that free cash flow will grow by 7.5% per year for the next five years.

    a. Assume that the 7.5% growth rate is expected to continue forever. What rate of return are investors expecting?

    b. Mexican Motors has generally earned about 12% on book equity (ROE = 12%) and reinvested 50% of earnings. The remaining 50% of earnings has gone to free cash flow. Suppose the company maintains the same ROE and investment rate for the long run. What is the implication for the growth rate of earnings and free cash flow? For the cost of equity? Should you revise your answer to part (a) of this question?

26. **Valuing a business (S4-6)** Phoenix Corp. faltered during the COVID-19 pandemic but is recovering. Free cash flow has grown rapidly. Forecasts made in 2024 are as follows:

($ millions)	2025	2026	2027	2028	2029
Net income	1.0	2.0	3.2	3.7	4.0
Investment	1.0	1.0	1.2	1.4	1.4
Free cash flow	0	1.0	2.0	2.3	2.6

Phoenix's recovery will be complete by 2029, and there will be no further growth in net income or free cash flow.

a. Calculate the PV of free cash flow, assuming a cost of equity of 9%.

b. Assume that Phoenix has 12 million shares outstanding. What is the price per share?

c. Confirm that the expected rate of return on Phoenix stock is exactly 9% in each of the years from 2025 to 2029.

CHALLENGE

30. **Constant-growth DCF formula (S4-4)** The constant-growth DCF formula

$$P_0 = \frac{\text{DIV}_1}{r - g}$$

is sometimes written as

$$P_0 = \frac{y(1 - k)\text{BVPS}}{r - yk}$$

where BVPS is book equity value per share, k is the plowback ratio, and y is the return on equity, the ratio of earnings per share to BVPS. Use this equation to show how the price-to-book ratio varies as ROE changes. What is price-to-book when $y = r$?

31. **DCF valuation (S4-4)** Portfolio managers are frequently paid a proportion of the funds under management. Suppose you manage a \$100 million equity portfolio offering a dividend yield (DIV_1/P_0) of 5%. Dividends and portfolio value are expected to grow at a constant rate. Your annual fee for managing this portfolio is 0.5% of portfolio value and is calculated at the end of each year. Assuming that you will continue to manage the portfolio from now to eternity, what is the PV of the management contract? How would the contract value change if you invested in stocks with a 4% yield?

32. **Valuing a business (S4-6)** Construct a new version of Table 4.4, assuming that Concatenator grows at 20%, 12%, and 6%, instead of 12%, 9%, and 6%. You will get negative early free cash flows.
    a. Recalculate the PV of free cash flow. What does your revised PV say about Concatenator's PVGO?
    b. Suppose Concatenator has no other resources. Thus, it will have to issue stock to cover the negative free cash flows. Does the need to issue shares change your valuation? Explain. (*Hint:* Suppose first that Concatenator's existing shareholders buy all of the newly issued shares. What is the value of the company to these shareholders? Now suppose instead that all the shares are issued to new shareholders, so that existing shareholders don't have to contribute any cash. Does the value of the company to the existing shareholders change, assuming that the new shares are sold at a fair price?)

SOLUTIONS TO SELF-TEST QUESTIONS

4.1 a. Investors expect Thor's stock price to increase by 11%. The 11% capital gain makes up for a dividend of zero.
b. Steamtool's higher risk drives down its stock price, increasing the expected rate of return. Steamtool's higher return comes as a combination of dividends and capital gains.

4.2 Yes. Mr. Devon could use Equation 4.6 with the horizon date set at $H = 2$. Aunt Cleo's trust could use the same equation with $H = 10$.

4.3 It doesn't. Future investment for growth contributes no net present value when expected rates of return just equal the cost of capital. In this case, the rate of growth does not affect price P or the P/E ratio.

4.4 Increase the assumed post-horizon growth rate, taking care to increase investment when the growth rate is higher. If higher growth generates higher present value, then your model must be generating positive PVGO post-horizon.

The major stock exchanges have wonderful websites. Start with the NYSE (**nyse.com**) and Nasdaq (**nasdaq.com**). Make sure you know how trading takes place on these exchanges.

FINANCE ON THE WEB

MINI-CASE

Reeby Sports

Ten years ago, in 2014, Precious Reeby founded a small mail-order company selling high-quality sports equipment. Since those early days, Reeby Sports has grown steadily and been consistently profitable. The company has issued 2 million shares, all of which are owned by Precious Reeby and her five children.

For some months, Precious has been wondering whether the time has come to take the company public. This would allow her to cash in on part of her investment and would make it easier for the firm to raise capital should it wish to expand in the future.

But how much are the shares worth? Precious' first instinct is to look at the most recent year-end balance sheet, which shows that the book value of the equity was $26.34 million, or $13.17 per share. A share price of $13.17 would put the stock on a P/E ratio of 6.6. That is quite a bit lower than the 13.1 P/E ratio of Reeby's larger rival, Molly Sports.

Precious suspects that book value is not necessarily a good guide to a share's market value. She thinks of her son Terrell, who works in an investment bank. He would undoubtedly know what the shares are worth. She decides to phone him after he finishes work that evening at 11 o'clock or before he starts the next day at 9 a.m.

Before phoning, Precious jots down some basic data on the company's profitability. After recovering from its early losses, the company has earned a return that is higher than its estimated 10% cost of equity. Precious is fairly confident that the company could continue to grow fairly steadily for the next six to eight years. In fact, she feels that the company's growth has been somewhat held back in the last few years by the demands from two of the children for the company to make large dividend payments. Perhaps, if the company went public, it could hold back on dividends and plow more money back into the business.

There are some clouds on the horizon. Competition is increasing, and only that morning, Molly Sports announced plans to form an online division. Precious is worried that beyond the next six or so years, it might become difficult to find worthwhile investment opportunities.

Precious realizes that Terrell will need to know much more about the prospects for the business before he can put a final figure on the value of Reeby Sports, but she hopes that the information is sufficient for him to give a preliminary indication of the value of the shares.

	2025	2026	2027	2028	2029	2030	2031	2032	2033	2034E
Earnings per share ($)	−2.10	−0.70	0.23	0.81	1.10	1.30	1.52	1.64	2.00	2.03
Dividend ($)	0.00	0.00	0.00	0.20	0.20	0.30	0.30	0.60	0.60	0.80
Book value per share ($)	9.80	7.70	7.00	7.61	8.51	9.51	10.73	11.77	13.17	14.40
ROE (%)	−27.10	−7.1	3.0	11.6	14.5	15.3	16.0	15.3	17.0	15.4

QUESTIONS

1. Help Precious forecast dividend payments for Reeby Sports and estimate the value of the stock. Of course, you will have to make some simplifying assumptions. For example, you could assume an ROE of 15% at least for the first six years, and a dividend payout ratio of 30%. Those assumptions will drive growth in assets, earnings, and dividends. What happens if ROE falls to 10% after six years?
2. How much of your estimate of the value of Reeby's stock comes from the present value of growth opportunities?

APPENDIX

Using Stock Prices to Estimate the Cost of Equity

Our growing perpetuity formula (Equation 4.4) explained P_0 in terms of next year's expected dividend DIV_1, the projected growth trend *g,* and the expected rate of return on other securities of equivalent risk *r*. The formula can also be turned around to obtain an estimate of *r* from DIV_1, P_0, and g*:*

$$r = \frac{\text{DIV}_1}{P_0} + g \tag{4.12}$$

The expected return equals the **dividend yield** (DIV_1/P_0) plus the expected rate of growth in dividends (*g*).

Using the Constant-Growth DCF Model to Set Water, Gas, and Electricity Prices

In the United States, the prices charged by local water, electric, and gas utilities are regulated by state commissions. The regulators try to keep consumer prices down but are supposed to allow the utilities to earn a fair rate of return. But what is fair? It is usually interpreted as *r,* the cost of equity. In other words, the fair rate of return on equity for a public utility ought to be its cost of equity—that is, the rate offered by securities that have the same risk as the utility's stock.[13]

Small variations in estimates of this return can have large effects on the prices charged to customers and on the regulated firm's profits. So both the firms' managers and regulators work hard to estimate the cost of equity. They've noticed that most utilities are mature, stable companies that pay regular dividends. Such companies could be tailor-made for application of the constant-growth DCF formula.

Suppose you wished to estimate the cost of equity for American States Water, a water distribution company. American's stock (ticker symbol AWR) was selling for \$76.32 per share at mid-October 2020. Dividend payments for the next year were expected to be \$1.34 a share. Thus, it was a simple matter to calculate the first half of the constant-growth DCF formula:

$$\text{Dividend yield } = \frac{\text{DIV}_1}{P_0} = \frac{1.34}{76.32} = 0.018 \textit{ or } 1.8\%$$

The hard part is estimating *g,* the expected rate of dividend growth. One option is to consult the views of security analysts who study the prospects for each company. Analysts are rarely prepared to stick their necks out by forecasting dividends far out in the future, but they often forecast growth rates of earnings and dividends over the next five years, and these estimates may provide an indication of the expected long-run growth path. In the case of American, analysts in 2020 were forecasting an annual growth of 5.6%.[14] This, together with the dividend yield, gives an estimate of the cost of equity:

$$r = \frac{\text{DIV}_1}{P_0} + g = 0.018 + 0.056 = 0.074 \textit{ or } 7.4\%$$

[13] This is the accepted interpretation of the U.S. Supreme Court's directive in 1944 that "The returns to the equity owner [of a regulated business] should be commensurate with returns on investments in other enterprises having corresponding risks." *Federal Power Commission v. Hope Natural Gas Company,* 302 U.S. 591 at 603.

[14] The growth rate was based on the average earnings growth forecasted by analysts as compiled by Bloomberg. In this calculation, we are therefore assuming that earnings and dividends are forecasted to grow forever at the same rate *g*. We showed how to relax this assumption earlier in the chapter in Section 4-4.

An alternative approach is to use the formula $g = y \times k$ that we introduced earlier. The payout ratio for American was about 60%, implying a plowback ratio of 40%. American's ratio of earnings to equity was about 13%. If American earns 13% on equity and reinvests 40% of earnings, then equity will increase by $0.40 \times 0.13 = 0.052$, or 5.2%. Earnings and dividends per share will also increase by 5.2%. We can thus arrive at a second estimate of the cost of equity:

$$r = \frac{\text{DIV}_1}{P_0} + g = 0.018 + 0.052 = 0.07 \ or \ 7\%$$

Although these estimates of American's cost of equity seem reasonable, there are obvious dangers in analyzing any single firm's stock with the constant-growth DCF formula. First, the underlying assumption of regular future growth is at best an approximation. Second, even if it is an acceptable approximation, errors can creep into the estimate of g. Third, American's shares could be over- or underpriced. As a result, any estimate of r for a single stock is "noisy" and subject to error. Good practice does not put too much weight on single-company estimates of the cost of equity. It collects samples of similar companies, estimates r for each, and takes an average. The average gives a more reliable benchmark for decision making.

The next-to-last column of Table 4.5 gives constant-growth DCF cost-of-equity estimates for American and five other water companies. These are all stable, mature companies for which the constant-growth DCF formula *ought* to work. But notice the variation in the cost-of-equity estimates. Some of the variation may reflect differences in the risk, but most is just temporary noise. The outlandishly high growth rates for California Water and SJW Group reflected recovery from the COVID-19 pandemic. These growth rates could not be maintained in perpetuity, as the constant-growth DCF formula assumes.

	Stock Price	Dividend[1]	Dividend Yield	Long-Term Growth Rate[1]	DCF Cost of Equity[2]	Multistage DCF Cost of Equity[3]
American States Water	$ 76.32	$1.34	1.8%	5.6%	7.4%	6.1%
American Water Works	151.43	2.20	1.5%	7.3%	8.7%	6.0%
California Water	44.69	0.85	1.9%	17.7%	19.6%	8.7%
Middlesex Water	63.74	1.04	1.6%	3.3%	4.9%	5.7%
SJW Corp.	61.37	1.28	2.1%	14.8%	16.9%	8.3%
York Water Co.	43.43	0.75	1.7%	5.1%	6.8%	6.0%
				Average	10.7%	6.8%

TABLE 4.5 Water Company Cost of Equity Estimates

Data as of October 2020. The long-term growth rate is based on security analysts' forecasts. In the multistage DCF model, growth after five years is assumed to adjust gradually to the estimated long-term growth rate of gross domestic product (GDP).

[1]Dividend and analysts' long-term growth-rate forecasts in October 2020.
[2]Sum of dividend yield and long-term growth rate. This column contains some small rounding differences.
[3]Long-term growth rate of GDP was forecast at 4.1% by Blue-Chip Economic Indicators.

Source: The Brattle Group, Inc.

The last column of Table 4.5 shows results from a multistage DCF model, where the constant growth formula is applied only at a horizon date H, as in Section 4-4. These results seem much more stable and reasonable than the constant-growth estimates.

Constant-Growth DCF Formulas—Summary

The constant-growth DCF formula is an extremely useful guide, but no more than that. Naive trust in the formula has led many financial analysts to incorrect conclusions.

We have stressed the difficulty of estimating r by analysis of one stock only. Try to use a large sample of equivalent-risk securities. Even that may not work, but at least it gives the analyst a fighting chance because the inevitable errors in estimating r for a single security tend to balance out across a broad sample.

Also, resist the temptation to apply the formula to firms having high current rates of growth. Such growth can rarely be sustained indefinitely, but the constant-growth DCF formula assumes it can. This erroneous assumption leads to an overestimate of r. Instead, we need to use the two-stage approach of Section 4-4.

CHAPTER 18

Financing and Valuation

In Chapters 4 and 6, we showed how to value a capital investment project by a three-step procedure:

1. Forecast post-tax cash flows, assuming all-equity financing.
2. Assess the project's risk and use it to estimate the opportunity cost of capital, the required return with all-equity financing.
3. Calculate NPV, using the opportunity cost of capital as the discount rate.

This procedure works in an MM world. If capital structure is irrelevant for the value of a project, we can simply assume all-equity financing. But if capital structure does matter, we need to include the value contributed by financing decisions. There are two ways to do this:

1. *Adjust the cost of capital.* To take account of the interest tax shields, companies usually adjust the cost of capital downward. They do this by calculating a post-tax weighted-average cost of capital (WACC).
2. *Adjust the present value.* The *adjusted present value* (APV) method calculates value in two steps. You start with a base-case net present value assuming all-equity financing. Then you calculate the present values added or subtracted by financing. You add this to the all-equity NPV to get the complete project value.

Section 18-1: The Post-Tax Weighted-Average Cost of Capital
We explain how to calculate the post-tax weighted-average cost of capital (WACC) and show how to use it to value an investment project.

Section 18-2: Valuing Businesses
We work through a realistic example to show how WACC can also be used to value an entire business.

Section 18-3: Using WACC in Practice
This section covers some practical problems. It offers helpful suggestions about estimating inputs and adjusting the WACC when business risk or financial risk changes.

Section 18-4: Adjusted Present Value
The adjusted present value (APV) method starts by valuing a project or business as if it were all-equity financed and then separately adds in the extra value from financing. We show how to use APV to value the same business we valued in Section 18-2 using a post-tax WACC. The two valuation approaches lead to the same result when financing matters only because of taxes and the firm maintains a constant debt ratio.

Section 18-5: Your Questions Answered
This is a question-and-answer section designed to clarify points that can often be confusing.

18-1 The Post-Tax Weighted-Average Cost of Capital

We first addressed problems of valuation and capital budgeting in Chapters 4 and 6. In those early chapters, we ignored financing decisions, viewing them as separate from investment decisions. If the investment project was positive NPV, we concluded that the firm should go ahead, without asking how the project would be financed. We were really assuming a Modigliani–Miller (MM) world in which all financing decisions are irrelevant. In such a world, firms can analyze investments as if they are all-equity-financed; the actual financing plan is a value-irrelevant detail to be worked out later.

Under MM assumptions, decisions to spend money can be separated from decisions to raise money. Now we reconsider the capital budgeting decision when investment and financing decisions interact and cannot be separated.

One reason why financing and investment decisions interact is taxes. Debt interest generates **tax shields** because it is a tax-deductible expense. The tax shields are accounted for in the **post-tax weighted-average cost of capital** or **WACC**:

$$\text{WACC} = (1 - t_c)\, r_D \frac{D}{V} + r_E \frac{E}{V} \tag{18.1}$$

The WACC is the correct discount rate whenever the firm adjusts its amount of debt to maintain a constant debt ratio over time.[1] Notice that the WACC formula uses the *post-tax* cost of debt $(1 - t_c)r_D$. The firm pays out r_D of interest to bondholders, but the tax deductibility of interest means that this interest only costs it $(1 - t_c)r_D$. As a result, unlike in Chapter 16, WACC is no longer the same as r_A, the expected return on a company's assets. That is given by the weighted average of the returns to the company's debt and equity investors:

$$r_A = r_D \frac{D}{V} + r_E \frac{E}{V}$$

While the returns to investors are r_D and r_E, the costs to the firm are $(1 - t_c)r_D$ and r_E. WACC is always less than r_A due to the tax deductibility of interest.

[1]We can prove this statement as follows. Denote expected after-tax cash flows (assuming all-equity financing) as $C_1, C_2, \ldots, C_T$. With all-equity financing, these flows would be discounted at the opportunity cost of capital r. But we need to value the cash flows for a firm that is financed partly with debt.

Start with value in the next to last period: $V_{T-1} = D_{T-1} + E_{T-1}$. The total cash payoff to debt and equity investors is the cash flow plus the interest tax shield. The expected total return to debt and equity investors is

$$\text{Expected cash payoff in } T = C_T + t_c r_D D_T - 1 \tag{1}$$

$$= V_{T-1}\left(1 + r_D \frac{D_{T-1}}{V_{T-1}} + r_E \frac{E_{T-1}}{V_{T-1}}\right) \tag{2}$$

Assume the debt ratio is constant at $L = D/V$. Equate (1) and (2) and solve for V_{T-1}:

$$V_{T-1} = \frac{C_T}{1 + (1 - t_c)r_D L + r_E(1 - L)} = \frac{C_T}{1 + \text{WACC}}$$

The logic repeats for V_{T-2}. Note that the next period's payoff includes V_{T-1}: Expected cash payoff in $T - 1 = C_{T-1} + t_c r_D D_{T-2} + V_{T-1}$

$$= V_{T-2}\left(1 + r_D \frac{D_{T-2}}{V_{T-2}} + r_E \frac{E_{T-2}}{V_{T-2}}\right)$$

$$V_{T-2} = \frac{C_{T-1} + V_{T-1}}{1 + (1 - t_c)r_D L + r_E(1 - L)} = \frac{C_{T-1} + V_{T-1}}{1 + \text{WACC}} = \frac{C_{T-1}}{1 + \text{WACC}} + \frac{C_T}{(1 + \text{WACC})^2}$$

We can continue all the way back to date 0:

$$V_0 = \sum_{t=1}^{T} \frac{C_t}{(1 + \text{WACC})^t}$$

Notice too that all the variables in the WACC formula refer to the firm as a whole. The Discount Rate Principle of Section 2-1 highlighted how the discount rate for a project depends on the risk of that project, not the risk of the company undertaking it. As a result, the WACC formula gives the right discount rate only for projects that are carbon copies of the firm. It is incorrect for projects that are safer or riskier than the firm as a whole, or that would be financed with a different capital structure from the firm. It is incorrect for projects whose acceptance would lead to an increase or decrease in the firm's target debt ratio. Many firms start with a single, companywide WACC but adjust it when business and financial risks change.[2] We show how to make these adjustments later in this chapter.

The WACC is based on the firm's *current* characteristics, but managers use it to discount *future* cash flows. That's fine as long as the firm's business and financial risks are expected to remain constant, but when they are expected to change, we suggest using the long-term leverage ratio and business risk. The Forward-Looking Principle of Section 4-4 highlighted how the discount rate should reflect what is expected to happen in the future, not what has happened in the past.

The next example shows how to apply the WACC formula.

EXAMPLE 18.1 • Calculating Sangria's WACC

Question: Sangria is a U.S.-based company whose products aim to promote happy, low-stress lifestyles. Its book- and market-value balance sheets are

Sangria Corporation (Book Values, $ millions)

Asset value	$1,000	$ 500	Debt
		500	Equity
	$1,000	$1,000	

Sangria Corporation (Market Values, $ millions)

Asset value	$1,250	$ 500	Debt
		750	Equity
	$1,250	$1,250	

We calculated the market value of equity on Sangria's balance sheet by multiplying its current stock price ($7.50) by 100 million, the number of its outstanding shares. The company's future prospects are good, so the stock is trading above book value ($7.50 vs. $5.00 per share). However, interest rates have been stable since the firm's debt was issued, so the book and market values of debt are, in this case, equal.

Sangria's cost of debt (the interest rate on new borrowing)[3] is 6%. Its cost of equity is 12.5%. Sangria is consistently profitable and pays taxes at the marginal rate of 21%. What is its WACC?

Answer: The Market Value Principle of Section 4-1 highlighted how only market values are relevant, not book values. Shareholders' stake in the firm is worth $750 million. Even if they bought their shares for less, they could sell them for $750 million today and invest this $750 million elsewhere. Thus, the firm needs to give a 12.5% return on the market value of $750 million, not the book value of $500 million.

[2]See also "Perfect Pitch and the Cost of Capital" in Section 9-3.

[3]Always use an up-to-date interest rate (yield to maturity), not the interest rate when the firm's debt was first issued and not the coupon rate on the debt's book value.

The other inputs are summarized here:

Cost of debt (r_D)	0.06
Cost of equity (r_E)	0.125
Marginal tax rate (t_c)	0.21
Debt ratio (D/V)	500/1,250 = 0.4
Equity ratio (E/V)	750/1,250 = 0.6

The company's post-tax WACC is

$$\begin{aligned} \text{WACC} &= (1 - 0.21) \times 0.06 \times 0.4 + 0.125 \times 0.6 \\ &= 0.094, \text{ or } 9.4\% \end{aligned}$$

18.1 Self-Test

Serial Music has a long-standing policy of financing with 50% debt and 50% equity at book values. The average interest rate on existing debt is 4.5%, but new debt can be issued at 3.5%. Its stock is trading at two times book value per share. The cost of equity is 9.5%. The corporate tax rate is 21%. What is Serial's post-tax WACC?

Now let's see how Sangria would *use* its post-tax WACC.

EXAMPLE 18.2 • Using Sangria's WACC to value a project

Question 1: Sangria's enologists have proposed investing \$12.5 million in the construction of a perpetual crushing machine to make wine, which generates a perpetual stream of earnings and cash flow of \$1.487 million per year pre-tax. The project is average risk for the company, so we can use the company's WACC. The post-tax cash flow is \$1.487 × (1 − 21%) = \$1.175 million. What is the NPV of the machine?

Answer 1: The crusher generates a perpetual post-tax cash flow of C = \$1.175 million, so NPV is

$$\text{NPV} = -12.5 + \frac{1.175}{0.094} = 0$$

NPV = 0 means a barely acceptable investment. The annual cash flow of \$1.175 million per year amounts to a 9.4% rate of return on investment (1.175/12.5 = 0.094), exactly equal to Sangria's WACC.

Question 2: If project NPV is exactly zero, the return to equity investors must exactly equal the cost of equity, 12.5%. Confirm that Sangria shareholders can indeed look forward to a 12.5% return on their investment in the perpetual crusher project.

Answer 2: Suppose Sangria sets up this project as a mini-firm. Its market-value balance sheet looks like this:

Perpetual Crusher (Market Values, \$ millions)

Asset value	\$12.5	\$ 5.0	Debt
		7.5	Equity
	\$12.5	\$12.5	

Calculate the expected dollar return to shareholders:

$$\begin{aligned}\text{Post-tax interest} &= (1 - t_c)r_D D \\ &= (1 - 0.21) \times 0.06 \times 5 = 0.237 \\ \text{Expected equity income} &= C - (1 - t_c)r_D D \\ &= 1.175 - 0.237 = 0.938\end{aligned}$$

The project's earnings are level and perpetual, so the expected rate of return on equity is equal to the expected equity income divided by the equity value:

$$\begin{aligned}\text{Expected equity return} = r_E &= \frac{\text{expected equity income}}{\text{equity value}} \\ &= \frac{0.938}{7.5} = 0.125, \text{ or } 12.5\%\end{aligned}$$

The expected return on equity equals the cost of equity, so it makes sense that the project's NPV is zero.

Review of Assumptions

It is appropriate to discount the perpetual crusher's cash flows at Sangria's WACC only if

- The project's business risks are the same as those of Sangria's other assets and remain so for the life of the project.
- The project supports the same fraction of debt to value as in Sangria's overall capital structure and does so for the life of the project.

You can see the importance of these two assumptions: If the perpetual crusher had greater business risk than Sangria's other assets, or if the acceptance of the project would lead to a permanent increase in Sangria's debt ratio, then Sangria's shareholders would not be content with a 12.5% expected return on their equity investment in the project.

But users of WACC need not worry about small or temporary fluctuations in debt ratios. Even if the project does not maintain the same debt ratio at all times in the future, the errors in using WACC are typically small, so discounting by WACC is approximately correct. Nor should they be misled by the immediate source of financing. Suppose that Sangria decides to borrow \$12.5 million to get a quick start on construction of the crusher. This does not necessarily change Sangria's long-term financing targets. The crusher's debt *capacity* is only \$5 million. If Sangria decides for convenience to borrow \$12.5 million for the crusher, then sooner or later it will have to borrow \$12.5 – \$5 = \$7.5 million *less* for other projects. The project only generates \$5 million of tax shields for Sangria as a whole, and so \$5 million enters into the WACC formula. We highlight this point in the following principle:

> ***The Project Leverage Principle***: *The leverage used when calculating the discount rate for a project should depend on the project's debt capacity, not how the project is initially financed.*

Common Mistakes in Using the WACC Formula

The weighted-average formula is very useful, but it also dangerous. It tempts people to make conceptual errors. For example, Rodrigo, a manager who is campaigning for a pet project, might look at the formula

$$\text{WACC} = (1 - t_c)r_D \frac{D}{V} + r_E \frac{E}{V} \tag{18.1}$$

and think, "Aha! My firm has a good credit rating. It could borrow, say, 90% of the project's cost if it likes. That means $D/V = 0.9$ and $E/V = 0.1$. My firm's borrowing rate r_D is 8%, and the required return on equity, r_E, is 15%. The tax rate is now 21%. Therefore,

$$\text{WACC} = (1 - 0.21)(0.08)(0.9) + 0.15(0.1) = 0.072$$

or 7.2%. When I discount at that rate, my project looks great."

Rodrigo is wrong on several counts:

1. The WACC formula works only for projects that are carbon copies of the firm. The pet project may have more business risk than the firm.
2. The Project Leverage Principle highlights how the discount rate for a project does not depend on its immediate source of funds, but its overall contribution to the firm's borrowing power. A dollar invested in Rodrigo's pet project won't increase the firm's debt capacity by $0.90. If the firm borrows 90% of the project's cost, it is really borrowing in part against its *existing* assets. Any advantage from financing the new project with more debt than normal should be attributed to the old assets, not to the new ones.
3. Even if the firm were willing and able to lever up to 90% debt, its cost of capital would not decline to 7.2%, as Rodrigo's naive calculation predicts. You cannot increase the debt ratio without creating financial risk for shareholders and thereby increasing r_E, the expected rate of return they demand from the firm's equity. Going to 90% debt would certainly increase the borrowing rate, too.

18.2 Self-Test

Look back to your answer to Self-Test 18.1. Serial has accumulated $40 million of surplus cash, which is earning a 1.5% post-tax interest rate. It can use the cash to pay for a new synthesizer line. The line has the same business risk as Serial's other assets. Should the future cash flows from the new line be discounted at the 1.5% interest rate or at Serial's post-tax WACC? Explain briefly.

18-2 Valuing Businesses

On most workdays, the financial manager concentrates on valuing projects, arranging financing, and helping to run the firm more effectively. The valuation of the business as a whole is left to investors and financial markets. But on some days, the financial manager has to take a stand on what an entire business is worth. When this happens, a big decision is typically in the offing. For example:

- If firm A is about to make a takeover offer for firm B, then A's financial managers have to decide how much the combined business A + B is worth under A's management. This task is particularly difficult if B is a private company with no observable share price.
- If firm C is considering the sale of one of its divisions, it has to decide what the division is worth in order to negotiate with potential buyers.
- When a firm goes public, it must evaluate how much the firm is worth in order to set the issue price.
- If a mutual fund owns shares in a company that is not traded, then the fund's directors are obliged to estimate a fair value for those shares. If the directors do a sloppy job of coming up with a value, they are liable to find themselves in court.

In addition, thousands of analysts in stockbrokers' offices and investment firms spend every workday burrowing away in the hope of finding undervalued firms. Many of these analysts use the valuation tools we are about to cover.

In Chapter 4, we took a first pass at valuing free cash flows from an entire business. We assumed then that the business was financed solely by equity. Now we will show how WACC can be used to value a company that is financed by a mixture of debt and equity. You just treat the company as if it were one big project. You forecast the company's free cash flows (the hardest part of the exercise) and discount back to present value. But be sure to remember three important points:

1. *Discount unlevered cash flows.* Do not deduct interest when estimating free cash flows. Calculate taxes as if the company were all-equity-financed. The value of interest tax shields is reflected in the post-tax WACC formula.
2. *Estimate terminal value.* Unlike most projects, companies are potentially immortal. But that does not mean that you need to forecast every year's cash flow from here to eternity. Financial managers usually forecast to a horizon date and then add a **terminal value**, as in the two-stage method for valuing stocks of Section 4-4. The terminal value is the present value at the horizon date of all subsequent cash flows. Estimating the terminal value requires careful attention because it often accounts for the majority of the company's value.
3. *Subtract debt.* Discounting at WACC values the company's assets. If the goal is to value the company's equity, don't forget to subtract the value of the company's outstanding debt.

Here is an example.

Valuing Rio Corporation

Sangria is tempted to acquire the Rio Corporation, which is also in the business of promoting relaxed, happy lifestyles. Rio has developed a special weight-loss program called the Brazil Diet, based on barbecues, red wine, and sunshine. The firm guarantees that within three months you will have a body that will allow you to fit right in at Ipanema or Copacabana Beach in Rio de Janeiro. But before you head for the beach, you've got the job of working out how much Sangria should pay for Rio.

Rio is privately held, so Sangria has no stock price to rely on. Rio is in the same line of business as Sangria, so we will assume that it has the same business risk as Sangria, and, like Sangria, its debt capacity is 40% of firm value. Therefore, we can use Sangria's WACC.

Your first task is to forecast Rio's **free cash flow** (FCF). Free cash flow is the amount of cash that the firm can pay out to investors after making all investments necessary for growth (see Section 4-6), and assuming all-equity financing. Discounting the free cash flows at the post-tax WACC gives the total value of Rio (debt plus equity). To find the value of its equity, you will need to subtract the 40% of the firm that can be financed with debt.

We will forecast each year's free cash flow out to a **horizon date** (H) and predict the business's value at that horizon (PV_H). The cash flows and terminal value are then discounted back to the present:

$$PV = \underbrace{\frac{FCF_1}{1 + WACC} + \frac{FCF_2}{(1 + WACC)^2} + \cdots + \frac{FCF_H}{(1 + WACC)^H}}_{\text{PV (free cash flow)}} + \underbrace{\frac{PV_H}{(1 + WACC)^H}}_{\text{PV (terminal value)}}$$

Free cash flow can be negative for rapidly growing firms, even if the firms are profitable, because investment exceeds cash flow from operations. Free cash flow turns positive as growth slows down and the payoffs from prior investments start to roll in.

Table 18.1 sets out the information that you need to forecast Rio's free cash flows. We start by projecting sales. In the year just ended, Rio had sales of $83.6 million. In recent years, sales have grown between 5% and 8% a year. You forecast that sales will grow about 7% a year for the next three years. Growth will then slow to 4% for years 4 to 6 and to 3% starting in year 7.

The other components of cash flow in Table 18.1 are driven by these sales forecasts:

- *Costs.* Costs are forecast at 74% of sales in the first year, with a gradual increase to 76.5% of sales in later years, reflecting increased marketing costs as Rio's competitors gradually catch up.
- *Fixed assets.* Increasing sales are likely to require further investment in fixed assets. Rio's net fixed assets are currently about \$0.79 for each dollar of sales. Unless Rio has surplus capacity or can squeeze more output from its existing plant and equipment, its investment in fixed assets will need to grow along with sales. Therefore, we assume that every dollar of sales growth requires an increase of \$0.79 in net fixed assets.
- *Working capital.* We also assume that working capital grows in proportion to sales.

BEYOND THE PAGE

Try it! Rio's spreadsheet

Rio's free cash flow is calculated in Table 18.1 as operating cash flow minus investment, the change in the stock of gross fixed assets and working capital from the previous year. Operating cash flow is profit after tax, plus depreciation (see Equation 6.6). For example, in year 1:

$$\begin{aligned}\text{Free cash flow} &= \text{profit after tax} + \text{depreciation} - \text{investment} \\ &\quad\ \text{in fixed assets} - \text{investment in working capital} \\ &= 10.6 + 9.9 - (109.6 - 95.0) - (11.6 - 11.1) \\ &= \$5.3 \text{ million}\end{aligned}$$

Operating cash flow can also be calculated starting with earnings before interest, taxes, depreciation, and amortization (EBITDA) and then deducting tax (see Equation 6.4). Here EBITDA = \$23.3 million for year 1 in Table 18.1.

$$\begin{aligned}\text{Free cash flow} &= \text{EBITDA} - \text{tax} - \text{investment in fixed} \\ &\quad\ \text{assets and working capital} \\ &= 23.3 - 2.8 - (109.6 - 95.0) - (11.6 - 11.1) \\ &= \$5.3 \text{ million}\end{aligned}$$

Estimating Terminal Value

We will forecast cash flows for each of the first six years. After that, Rio's sales are expected to settle down to stable, long-term growth starting in year 7. To find the present value of the cash flows in years 1 to 6, we discount at the 9.4% WACC:

$$\text{PV} = \frac{5.3}{1.094} + \frac{5.2}{1.094^2} + \frac{5.5}{1.094^3} + \frac{8.0}{1.094^4} + \frac{8.3}{1.094^5} + \frac{8.2}{1.094^6} = \$29 \text{ million}$$

Now we need to find the value of the cash flows from year 7 onward. In Chapter 4, we looked at several ways to estimate terminal value. Here we will use the constant-growth DCF formula. This requires a forecast of the free cash flow for year 7, which we have worked out in the final column of Table 18.1, assuming a long-run growth rate of 3% per year.[4] The free cash flow is \$8.5 million, so

$$\text{PV}_H = \frac{\text{FCF}_{H+1}}{\text{WACC} - g} = \frac{8.5}{0.094 - 0.03} = \$132.7 \text{ million}$$

$$\text{PV at year 0} = \frac{1}{1.094^6} \times 132.7 = \$77.4 \text{ million}$$

[4]Notice that expected free cash flow increases by about 4% from year 6 to year 7 because the transition from 4% to 3% sales growth reduces required investment. But sales, investment, and free cash flow will all increase at 3% once the company settles into stable growth. Recall that the first cash flow in the constant-growth DCF formula occurs in the next year, year 7 in this case. Growth progresses at a steady-state 3% from year 7 onward. Therefore, it's OK to use the 3% growth rate in the terminal-value formula.

		Latest							
		Year	Forecast						
		0	1	2	3	4	5	6	7
1	Sales	83.6	89.5	95.8	102.5	106.6	110.8	115.2	118.7
2	Cost of goods sold	63.1	66.2	71.3	76.3	79.9	83.1	87.0	90.8
3	EBITDA (1 – 2)	20.5	23.3	24.4	26.1	26.6	27.7	28.2	27.9
4	Depreciation	3.3	9.9	10.6	11.3	11.8	12.3	12.7	13.1
5	Profit before tax (EBIT) (3 – 4)	17.2	13.4	13.8	14.8	14.9	15.4	15.5	14.8
6	Tax	3.6	2.8	2.9	3.1	3.1	3.2	3.3	3.1
7	Profit after tax (5 – 6)	13.6	10.6	10.9	11.7	11.7	12.2	12.2	11.7
8	Investment in fixed assets (change in gross fixed assets)	11.0	14.6	15.5	16.6	15.0	15.6	16.2	15.9
9	Investment in working capital	1.0	0.5	0.8	0.9	0.5	0.6	0.6	0.4
10	Free cash flow (7 + 4 – 8 – 9)	4.9	5.3	5.2	5.5	8.0	8.3	8.2	8.5
	PV free cash flow, years 1–6	29.0							
	PV terminal value	77.4			(Terminal value in year 6)			132.7	
	PV of company	106.4							
	Assumptions:								
	Sales growth, %	6.7	7.0	7.0	7.0	4.0	4.0	4.0	3.0
	Costs (percent of sales)	75.5	74.0	74.5	74.5	75.0	75.0	75.5	76.5
	Working capital (percent of sales)	13.3	13.0	13.0	13.0	13.0	13.0	13.0	13.0
	Net fixed assets (percent of sales)	79.2	79.0	79.0	79.0	79.0	79.0	79.0	79.0
	Depreciation (percent of net fixed assets)	5.0	14.0	14.0	14.0	14.0	14.0	14.0	14.0
	Tax rate, %	21.0							
	WACC, %	9.4							
	Long-term growth forecast, %	3.0							
	Fixed assets and working capital								
	Gross fixed assets	95.0	109.6	125.1	141.8	156.8	172.4	188.6	204.5
	Less accumulated depreciation	29.0	38.9	49.5	60.8	72.6	84.9	97.6	110.7
	Net fixed assets	66.0	70.7	75.6	80.9	84.2	87.5	91.0	93.8
	Net working capital	11.1	11.6	12.4	13.3	13.9	14.4	15.0	15.4

TABLE 18.1 Free-Cash-Flow Projections and Company Value for Rio Corporation ($ millions)

We now have all we need to value Rio:

$$\begin{aligned}\text{PV(company)} &= \text{PV(cash flow years 1 to 6)} + \text{PV(terminal value)} \\ &= \$29.0 + 77.4 = \$106.4 \text{ million}\end{aligned}$$

This is the total value of Rio. To find the value of the equity, we simply subtract the 40% of firm value that will be financed with debt:

$$\text{Value of debt} = 0.40 \times 106.4 = \$42.6 \text{ million}$$
$$\text{Value of equity} = \$106.4 - 42.6 = \$63.8 \text{ million}$$

If Rio has 1.5 million shares outstanding, its value per share is

$$\text{Value per share} = 63.8/1.5 = \$42.53$$

Thus, Sangria could afford to pay up to \$42.53 per share for Rio.

You now have an estimate of the value of Rio Corporation. But how confident can you be in this figure? Notice that only about a quarter of Rio's value comes from cash flows in the first six years. The rest comes from the terminal value. Moreover, this terminal value can change in response to minor changes in assumptions. For example, if the long-run growth rate is 4% rather than 3%, firm value increases from \$106.4 million to \$110.5 million.

Thus, faster growth increases Rio's terminal value and thus total value. At this point, we must check the three warnings from the similar Concatenator valuation example that we considered in Chapter 4:

1. Did we account for the extra investment required to support the faster long-run growth? Yes. Growth at 4% instead of 3% increases year 7's investment in fixed assets from \$15.9 to \$16.9 million and investment in working capital from \$0.4 to \$0.6 million. (To confirm this, go to the Rio Spreadsheet Beyond the Page and change the long-run growth rate.)
2. Is the ROA going forward greater than the WACC? Yes, the ROA is 11.7% and thus greater than the 9.4% WACC.[5] This explains why Rio's growth opportunities are valuable, consistent with the Growth Principle of Section 4-5.
3. Did we include post-horizon PVGO in terminal value? Yes, because we showed that the increased investment starting in year 7 increased NPV, so Rio has valuable growth opportunities.

Valuation by Comparables

Most of this chapter covers discounted cash flow (DCF) valuation models. What about valuation by comparables? In Chapter 4 we described how financial managers use price-earnings (P/E) or price-to-book value (P/B) ratios to help value shares. You take the average P/E or P/B ratio for a sample of comparable companies, and then multiply this average ratio by the earnings or book value of the stock you want to value.

When we are valuing shares, the numerator of the above multiples (P/E and P/B) is the price of a share. Thus, to be apples-to-apples, the denominator of the multiple is the value to the owner of one share: earnings per share or book value per share. These are values to shareholders only, not to the whole enterprise (equity and debt). Earnings per share are after deducting interest; book value per share is after deducting debt.

Here, we are not valuing shares, but businesses. The numerator of the multiple is thus **enterprise value**, $V = D + E$, the value of the whole business, both debt and equity. Thus, to be apples-to-apples, the denominator of the multiple must be cash flows to the entire business, both debtholders and equityholders, and thus be *before* subtracting interest. Examples of such ratios are

- EV/EBIT: enterprise value divided by earnings before interest and taxes.
- EV/EBITDA: enterprise value divided by earnings before interest, taxes, depreciation, and amortization.
- EV/Sales: enterprise value divided by sales. This is typically only used for startup companies where sales are the main value driver; profitability net of costs is less of a concern. For more mature businesses, costs are important and so EV/EBIT or EV/EBITDA is more appropriate.

[5]Rio's net assets at date 0 are the net fixed assets of 66.0 and the net working capital of 11.1 to give a total of 77.1. The ROA is the discount rate which leads to the discounted free cash flow equaling net assets. In other words, it solves the following equation:

$$77.1 = \frac{5.3}{1 + ROA} + \frac{5.2}{(1 + ROA)^2} + \frac{5.5}{(1 + ROA)^3} + \frac{8.0}{(1 + ROA)^4} + \frac{8.3}{(1 + ROA)^5} + \frac{8.2}{(1 + ROA)^6} + \frac{8.5}{(1 + ROA)^6(ROA - 0.03)}$$

EBIT, EBITDA, and sales are cash flows that go to the whole business and thus can be used to pay both debtholders and equityholders. Thus, an EV/Sales ratio makes sense but P/Sales would not.

We can use comparables in two ways:

1. *Estimate terminal value by comparables.* Suppose that lifestyle companies similar to Rio are trading at a forward EV/EBITDA ratio of 4.8. Then Sangria's manager could estimate Rio's terminal value as 4.8 × 27.9 = \$133.9 million in year 6 and \$78.1 million discounted to year 0. This would suggest that Rio is currently worth \$29.0 + 78.1 = \$107.1 million, slightly higher than our initial DCF estimate. This valuation uses the two-stage method of Section 4-4: It values early free cash flows using DCF and terminal value using comparables.
2. *Estimate company value by comparables.* This method bypasses DCF entirely and values the entire business using comparables in a single stage, similar to Section 4-2. For example, Rio could be valued by multiplying the forward EV/EBITDA ratio by EBITDA in year 1, which is \$23.3 million. This gives a value of 4.8 × 23.3 = \$111.8 million, a bit higher still than the DCF and two-stage approaches.

WACC versus the Flow-to-Equity Method

When valuing Rio, we forecast the cash flows assuming all-equity financing and we used the WACC to discount these cash flows. The WACC formula picked up the value of the interest tax shields. Then, to find the equity value, we subtracted the value of debt from the total value of the firm.

An alternative is to value equity directly, rather than via subtraction. To do so, we discount cash flows to equity after interest and taxes at the cost of equity capital. This is called the **flow-to-equity method** and is given in Equation 18.2 (in the case where cash flows are a level perpetuity):

$$E = \frac{C - (1 - t_c)r_D D}{r_E} \qquad \textbf{(18.2)}$$

where C is the post-tax operating cash flow.

If the company's debt ratio is constant over time, the flow-to-equity method should give the same answer as discounting total cash flows at the WACC and then subtracting the value of the debt, as in Equation 18.3:

$$E = \frac{C}{\text{WACC}} - D \qquad \textbf{(18.3)}$$

If, instead of post-tax cash flows C, we are given pre-tax cash flows X, Equations 18.2 and 18.3 become

$$E = \frac{(1 - t_c)(X - r_D D)}{r_E}$$

$$E = \frac{(1 - t_c)X}{\text{WACC}} - D$$

It sounds straightforward, but in practice, it can be tricky to do it right. The problem arises because each year's interest payment depends on the amount of debt at the start of the year, and this depends in turn on Rio's value at the start of the year (remember Rio's debt is assumed to be a constant proportion of value). So you seem to have a catch-22 situation in which you first need to know Rio's value each year before you can calculate cash flows to equity. Fortunately, a simple formula allows you to solve simultaneously for the company's

BEYOND THE PAGE

Try it! Cash-flow-to-equity method

value and the cash flow in each year. We won't get into that here, but if you would like to see how the flow-to-equity method can be used to value Rio, click on the nearby Beyond the Page feature to access the worked example.

18-3 Using WACC in Practice

Handling Complications

Sangria had just one asset and two sources of financing. A real company's market-value balance sheet has many more entries, for example:[6]

Current assets, including cash, inventory, and accounts receivable	Current liabilities, including accounts payable and short-term debt
Property, plant, and equipment	Long-term debt (*D*)
	Preferred stock (*P*)
Growth opportunities (PVGO)	Equity (*E*)
Total assets	Total liabilities plus equity

Several questions immediately arise:

How does the formula change when there are more than two sources of financing? Easy: There is one cost for each element. The weight for each element is proportional to its market value. For example, if the capital structure includes both preferred and common shares, then:

$$\text{WACC} = (1 - t_c)\,r_D\frac{D}{V} + r_P\frac{P}{V} + r_E\frac{E}{V}$$

where r_P is investors' expected rate of return on the preferred equity, P is the amount of preferred equity outstanding, and $V = D + P + E$.

What about cash and short-term debt? Many companies consider only long-term financing when calculating WACC. They leave out the cost of short-term debt. This is incorrect. The lenders who hold short-term debt are investors who can claim their share of operating earnings. A company that ignores this claim will misstate investors' required return. Thus, the total amount of debt should include both short-term and long-term debt.

Debt should also be net of cash, because it can typically be used to pay down debt at any time. Thus, the D in the WACC formula should be net debt. In sum:

$$\text{Net debt} = \text{short-term debt} + \text{long-term debt} - \text{cash}$$

What about other current liabilities? Non-debt current liabilities are usually subtracted from current assets. The difference is entered as **net working capital** on the left-hand side of the balance sheet.

[6]This balance sheet is for exposition and should not be confused with a real company's books. It includes the value of growth opportunities, which accountants do not recognize, though investors do. It excludes certain accounting entries, for example, deferred taxes.

Deferred taxes arise when a company uses faster depreciation for tax purposes than it uses in reports to investors. That means the company reports more in taxes than it pays. The difference is accumulated as a liability for deferred taxes. In a sense, there is a liability because the Internal Revenue Service "catches up," collecting extra taxes as assets age. But this is irrelevant in capital investment analysis, which focuses on actual post-tax cash flows and uses accelerated tax depreciation.

Deferred taxes should not be regarded as a source of financing or an element of the weighted-average cost of capital formula. The liability for deferred taxes is not a security held by investors. It is a balance sheet entry created for accounting purposes.

Deferred taxes can be important in regulated industries, however. Regulators take deferred taxes into account in calculating allowed rates of return and the time patterns of revenues and consumer prices.

Net working capital = current assets − current liabilities	Long-term debt (*D*)
Property, plant, and equipment	Preferred equity (*P*)
Growth opportunities	Equity (*E*)
Total assets	Total capitalization (*V*)

Forecasts of project cash flows must treat increases in net working capital as a cash outflow and decreases as an inflow. This is standard practice, which we followed in Section 6-1. We also did so when we estimated the future investments that Rio would need to make in working capital.

How are the costs of financing calculated? You can often use the CAPM to get an estimate of r_E, the expected rate of return demanded by shareholders. The borrowing rate r_D and the debt and equity ratios D/V and E/V can be directly observed or estimated without too much trouble.[7] Estimating the value and required return for preferred shares is likewise usually fairly straightforward.

Estimating the required return on other security types can be tricky. Convertible debt, where the investors' return comes partly from an option to exchange the debt for the company's stock, is one example. We leave convertibles to Chapter 25.

Junk debt, where the risk of default is high, is likewise difficult. The higher the odds of default, the lower the market price of the debt, and the higher is the **promised** rate of interest. But the cost of debt r_D is an **expected**, not a promised one. For example, early in the COVID-19 crisis, the long-term debt issues of the retail chain Macy's were selling at deep discounts to **par value**. Its 7% coupon bonds maturing in February 2028 were priced at 54% of par value, offering a **promised yield** of 18.5%. But the bond's **expected yield** was far less than 18.5%, as investors would have suffered a negative return if Macy's defaulted. Including 18.5% as a "cost of debt" in a calculation of WACC would, therefore, vastly overstate Macy's true cost of capital. Section 24-1 describes the differences between promised and expected yields in more detail.

This is bad news: There is no easy way of estimating the true expected rate of return on most junk debt issues. The good news is that for most debt, the odds of default are small. That means the promised and expected rates of return are close, and the promised rate can be used as an approximation in calculating the weighted-average cost of capital.

Should a diversified company use a single, companywide WACC? It depends on whether the divisions of the diversified company differ significantly in business risk. If they do differ significantly, the higher-risk divisions should be assigned higher WACCs, the lower-risk divisions lower WACCs.

Here is an extreme example. Kansas City Southern used to be a portfolio of:

1. The Kansas City Southern Railroad, with operations running from the U.S. Midwest south to Texas and Mexico.
2. Stillwell Financial, an investment-management company that included the Janus mutual funds.

[7]Most corporate debt is not actively traded, so its market value cannot be observed directly. But you can usually value a nontraded debt security by looking to securities that *are* traded and that have approximately the same default risk and maturity. See Chapter 24.

For healthy firms the market value of debt is usually not too far from book value, so many managers and analysts use book value for *D* in the weighted-average cost of capital formula. However, be sure to use market, not book, values for *E*.

It's hard to think of two more dissimilar businesses. Kansas City Southern's overall WACC was not right for either of them. The company would have been well-advised to set a separate, higher railroad WACC and a lower investment-management WACC. Railroads are riskier than established investment-management operations.

A more recent example: Large electric utilities typically operate in both regulated and unregulated markets:

1. Their unregulated, "merchant" generating plants sell electricity at market prices, which can fluctuate wildly.
2. They also operate traditional regulated businesses, which include transmission, local distribution, and also some regulated generation. The regulation stabilizes prices and (in normal times, at least) provides a "floor" on profitability.

Applying a single WACC for both businesses would *underestimate* the required return for the unregulated merchant business and lead to overinvestment. Conversely, it would result in an *overestimate* of the required return and, therefore, inadequate investment in regulated activities. Thus, such companies typically set two WACCs: a lower one for the safer regulated and a higher one for the riskier merchant business.

Use of an industry WACC for a particular company's investments assumes that the company and industry have approximately the same business risk.[8] Industry WACCs also have to be adjusted (by the three-step procedure given below) if industry-average debt ratios differ from the target debt ratio for the project to be valued.

What tax rate should I use? Taxes are complicated. Companies can often reduce average tax rates by taking advantage of special provisions in the tax code, such as R&D credits. But the WACC formula calls for the *marginal* tax rate—that is, the cash taxes paid as a percentage of each dollar of additional income generated by a capital-investment project.

The examples in this chapter use 21%, the current (2024) U.S. corporate rate. In practice, U.S. companies add three or four percentage points to cover state taxation. Thus, a company operating nationwide—and paying income taxes in most states—might use a 24% or 25% rate to calculate WACC.

What if the company can't use all its interest tax shields? So far, we have assumed that the company is consistently profitable and will pay taxes at the full statutory rate. Thus, the company can still use a new project's depreciation tax shields even if a project endures a period of startup losses. The project's depreciation expense shields some of the company's overall taxable income. Interest tax shields on debt supported by the new project can be captured in the same way.

Sometimes interest tax shields from new debt cannot be captured immediately because (1) the company is suffering losses overall or (2) its total interest payments exceed 30% of EBIT.[9] Should the company change its WACC if it finds itself in one or both of these unfortunate states?

Probably not, if the losses or constraint are temporary. Tax losses and nondeductible interest can be carried forward and used to shield future income. (The tax rate in the WACC formula could be reduced to account for the delay in using tax shields.) But if the wait to use interest tax shields from additional borrowing is long enough, it may be best to use the APV method, which we explain in the next section.

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[8]Use industry WACCs cautiously. The operations and business risks of the major U.S. and Canadian railroads are similar, so railroad industry WACCs can make sense. But a look into other industry groupings, for example, Miscellaneous Consumer Goods, will reveal wide differences in business risks. Levi and Welch argue against using industry-average betas to predict betas for individual companies in Y. Levi and I. Welch, "Best Practice for Cost-of-Capital Estimates," *Journal of Financial and Quantitative Analysis* 52 (2017), pp. 427–463.

[9]See the summary of the U.S. Tax Cuts and Jobs Act in Chapter 6.

Adjusting WACC When Debt Ratios and Business Risks Differ

The WACC formula assumes that the project or business to be valued will be financed in the same debt–equity proportions as the company as a whole. What if that is not true? For example, what if Sangria's perpetual crusher project supports only 20% debt versus 40% for Sangria overall?

Moving from 40% to 20% debt could change all the inputs to the WACC formula:[10]

- The financing weights clearly change.
- The cost of equity r_E is less because financial risk is reduced: MM's Proposition 2.
- The cost of debt may also be lower.

Take a look at Figure 18.1, which plots Sangria's costs of capital as a function of its debt-to-equity ratio, and compare it to Figure 16.3, which is the equivalent diagram in a world without taxes.

- r_E: As in Figure 16.3, the cost of equity increases with the debt-to-equity ratio due to financial risk. In the case of debt rebalanced (where the company targets a debt-to-value ratio, rather than a dollar amount of debt), the formula for how r_E increases with leverage is exactly the same as in the no-tax world of Figure 16.3: $r_E = r_A + (r_A - r_D)D/E$.
- r_D: As in Figure 16.3, we assume this is independent of leverage. However, the main insights will not change if the cost of debt increases with leverage, as in Figure 16.4.
- r_A: The asset cost of capital is the return that investors would require if the company were all-equity-financed. As in Figure 16.3, it is flat and independent of leverage, because it depends on business risk only. As Figure 16.4 showed, r_A remains flat even if the cost of debt increases with leverage.
- WACC: This is the cost of capital to a levered firm. In Figure 16.3, this was exactly the same as r_A, the cost of capital to an unlevered firm. No matter how much leverage a firm takes, in a perfect market its cost of capital is the same as if it were unlevered. Even though debt is cheaper than equity, using more debt causes r_E to rise. With taxes, WACC is lower than r_A due to interest tax shields. The more debt a firm takes on, the lower its WACC because the greater the tax shields.

Figure 18.1 shows the shape of the relationship between leverage and WACC, but initially we have numbers only for Sangria's current 40% debt ratio. We want to recalculate WACC at a 20% ratio. We do so in three steps:

BEYOND THE PAGE

Try it! Figure 18.1: Sangria's WACC

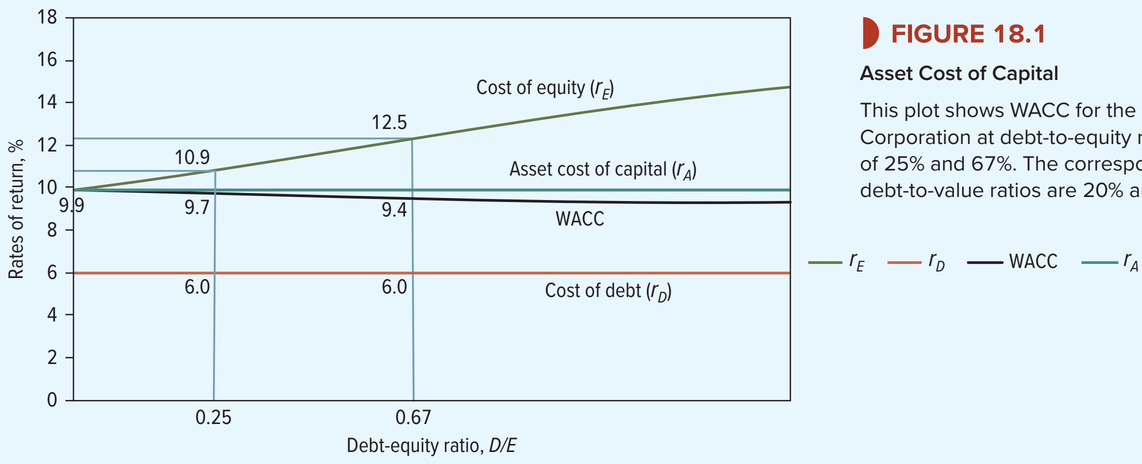

FIGURE 18.1

Asset Cost of Capital

This plot shows WACC for the Sangria Corporation at debt-to-equity ratios of 25% and 67%. The corresponding debt-to-value ratios are 20% and 40%.

[10]Even the tax rate could change. For example, Sangria might have enough taxable income to cover interest payments at 20% debt but not at 40% debt. In that case, the effective marginal tax rate would be higher at 20% than 40% debt.

Step 1 *Unlever the cost of equity* r_E *to obtain the asset cost of capital* r_A. This step involves *unlevering* the cost of equity. In the case of "debt rebalanced," the unlevering formula is given by[11]

$$\text{Asset cost of capital} = r_A = r_D D/V + r_E E/V$$

Notice that this is exactly the same as in the no-tax world of Modigliani and Miller's Proposition 1 (see Section 16-1).

Step 2 *Estimate the cost of debt* r_D *at the new debt ratio, and relever the asset cost of capital* r_A *to obtain the cost of equity* r_E *at the new debt ratio.* In the case of debt rebalanced, this is given by

$$r_E = r_A + (r_A - r_D)D/E$$

This formula is Modigliani and Miller's Proposition 2 (see Section 16-3). It calls for *D/E,* the ratio of debt to *equity,* not debt to value.

Step 3 *Recalculate WACC at the new debt ratio:*

$$\text{WACC} = (1 - t_c)r_D D/V + r_E E/V$$

Let's run the numbers for Sangria at $D/V = 0.20$.

Step 1 Sangria's current debt ratio is $D/V = 0.4$. So the asset cost of capital is

$$r_A = 0.06(0.4) + 0.125(0.6) = 0.099, \text{ or } 9.9\%$$

[11]Here's why the formulas work with continuous rebalancing. Think of a market-value balance sheet with assets and interest tax shields on the left and debt and equity on the right, with $D + E = \text{PV(assets)} + \text{PV(tax shield)}$. The total risk (beta) of the firm's debt and equity equals the blended risk of PV(assets) and PV(tax shield):

$$\beta_D \frac{D}{V} + \beta_E \frac{E}{V} = \alpha\beta_A + (1 - \alpha)\beta_{\text{tax shield}} \qquad (1)$$

where α is the proportion of the total firm value from its assets and $1 - \alpha$ is the proportion from interest tax shields. If the firm readjusts its capital structure to keep *D/V* constant, then the beta of the tax shield must be the same as the beta of the assets. With rebalancing, an $x\%$ change in firm value V changes debt D by $x\%$. So the interest tax shield $t_c\, r_D\, D$ will change by $x\%$ as well. Thus the risk of the tax shield must be the same as the risk of the firm as a whole:

$$\beta_{\text{tax shield}} = \beta_A = \beta_D \frac{D}{V} + \beta_E \frac{E}{V} \qquad (2)$$

This is our unlevering formula expressed in terms of beta. Since expected returns depend on beta:

$$r_A = r_D \frac{D}{V} + r_E \frac{E}{V} \qquad (3)$$

Rearrange formulas (2) and (3) to get the relevering formulas for β_E and r_E. (Notice that the tax rate t_c has dropped out.)

$$\beta_E = \beta_A + (\beta_A - \beta_E)D/E$$
$$r_E = r_A + (r_A - r_D)D/E$$

All this assumes continuous rebalancing. Suppose instead that the firm rebalances once a year, so that the next year's interest tax shield, which depends on this year's debt, is known. Then you can use a formula developed by Miles and Ezzell:

$$r_{\text{Miles-Ezzell}} = r_A - (D/V)\, r_D t_c \left(\frac{1 + r_A}{1 + r_D}\right)$$

See J. Miles and J. Ezzell, "The Weighted Average Cost of Capital, Perfect Capital Markets, and Project Life: A Clarification," *Journal of Financial and Quantitative Analysis* 15 (September 1980), pp. 719–730.

Step 2 We will assume that the cost of debt stays at 6% when the debt ratio falls to 20%. A debt-to-value ratio of 0.2 is a debt-to-equity ratio of 0.2/0.8 = 0.25. Thus,

$$r_E = r_A + (r_A - r_D)D/E = 0.099 + (0.099 - 0.06)(0.25) = 0.109, \text{ or } 10.9\%$$

Step 3 Recalculate WACC.

$$\text{WACC} = (1 - 0.21)(0.06)(0.2) + 0.109(0.8) = 0.097, \text{ or } 9.7\%$$

BEYOND THE PAGE
WACC and changing debt ratios

Figure 18.1 enters these numbers on the plot of WACC versus the debt-to-equity ratio.

Unlevering and Relevering Betas

Our three-step procedure for finding WACCs at different debt ratios unlevers and then relevers the cost of equity. Some financial managers find it convenient to unlever and then relever the equity beta. Given the equity beta at the new debt ratio, the cost of equity can be calculated from the CAPM. We can then compute the WACC at the new debt ratio.

Suppose Sangria's debt and equity betas at a 40% leverage ratio are $\beta_D = 0.135$ and $\beta_E = 1.07$.[12] If the risk-free rate is 5%, and the market risk premium is 7%, then Sangria's cost of equity is

$$r_E = r_f + (r_m - r_f)\beta_E = 0.05 + (0.07)1.07 = 0.125, \text{ or } 12.5\%$$

This matches the cost of equity in our example at a 40/60 debt-to-equity ratio.

To find Sangria's weighted-average cost of capital at a 20% debt ratio, we can follow a three-step procedure that matches the procedure that we used earlier.

Step 1 *Unlever the equity beta β_E to obtain the asset beta β_A.* This is the unlevered beta: what the equity beta would be if the company had business risk only and no financial risk. The formula for unlevering beta was given in Section 16-3 in the case of no taxes. When there are taxes, the formula remains the same in the case of "debt rebalanced":

$$\beta_A = \beta_D D/V + \beta_E E/V$$

This equation says that the beta of a firm's assets (β_A) is equal to the beta of a portfolio of all of the firm's outstanding debt and equity. An investor who bought such a portfolio would own all the assets and bear only business risk. For Sangria,

$$\beta_A = \beta_D D/V + \beta_E E/V = 0.135 \times 0.4 + 1.07 \times 0.6 = 0.696$$

Step 2 *Estimate the debt beta β_D at the new debt ratio and relever the asset beta β_A to obtain the equity beta β_E at the new debt ratio.* In the case of debt rebalanced, the formula for relevering beta resembles MM's Proposition 2, except that betas are substituted for rates of return:

$$\beta_E = \beta_A + (\beta_A - \beta_D)D/E$$

[12]Debt betas are generally small, and many managers simplify and assume $\beta_D = 0$. However, this is incorrect in general. For example, junk-debt betas can be well above zero.

This is the same as Equation 9.6 in the absence of taxes. If the beta of Sangria's debt stays at 0.135 at the new debt-to-equity ratio of 0.2/0.8 = 0.25, then

$$\beta_E = \beta_A + (\beta_A - \beta_D)D/E = 0.696 + (0.696 - 0.135)0.25 = 0.836$$

Step 3 *Recalculate the cost of equity and the WACC at the new debt ratio:*

$$r_E = r_f + (r_m - r_f)\beta_E = 0.05 + 0.07(0.836) = 0.109, \text{ or } 10.9\%$$

$$\text{WACC} = (1 - 0.21)(0.06)(0.2) + 0.8(0.109) = 0.097, \text{ or } 9.7\%$$

exactly the same as we calculated earlier.

Unlevering and Relevering with Debt Fixed

We have shown that, with taxes and in the case of debt rebalanced, the formulas for r_A, r_E, β_A, and β_E were exactly the same as in a no-tax world. However, if the firm pursues a "debt fixed" policy, where it fixes the dollar amount of debt, the relevering formulas now contain a tax term:

$$r_E = r_A + (1 - t_c)(r_A - r_D)D/E$$

$$\beta_E = \beta_A + (1 - t_c)(\beta_A - \beta_D)D/E$$

Practitioners often use the above formulas without thinking, reasoning "in the real world we have taxes, so we must include a tax term." That's incorrect because, even if there are taxes, there's no tax term in the relevering formulas if the firm pursues a "debt rebalanced" policy. Only under debt fixed is there a tax term.

In addition, practitioners sometimes use the following formulas without thinking:

$$r_E = r_A\,(1 + (1 - t_c)\,D/E)$$

$$\beta_E = \beta_A\,(1 + (1 - t_c)\,D/E)$$

These formulas assume both that the company pursues a "debt fixed" policy, and that the debt beta is zero. The latter is true for safe firms, but won't be true in general.

The unlevering formulas are also different under "debt fixed" and given as follows:

$$r_A = (1 - t_c)\,r_D\,\frac{D}{V - t_c D} + r_E\,\frac{E}{V - t_c D} \tag{18.4}$$

$$\beta_A = (1 - t_c)\beta_D\,\frac{D}{V - t_c D} + \beta_E\,\frac{E}{V - t_c D} \tag{18.5}$$

WACC with Bankruptcy and Agency Costs

The WACC curve in Figure 18.1 considers only tax shields. As a result, it is always downward-sloping and so WACC is minimized with 99.99% debt; this implies that firm value is maximized with 99.99% debt. However, as we saw in Figure 17.5, when there are bankruptcy and agency costs, firm value is first increasing with leverage (due to the tax shields) and then decreasing (due to bankruptcy costs and the agency costs of debt). As a result, the WACC curve is first decreasing and then increasing, as shown in Figure 18.2. The leverage ratio that maximizes firm value (point C in Figure 17.5) is also the leverage ratio that minimizes the WACC (point C in Figure 18.2).

A common interview question for finance jobs is "draw the WACC curve and explain why it looks like that." You'd draw the U-shaped curve in Figure 18.2 and offer the following explanation:

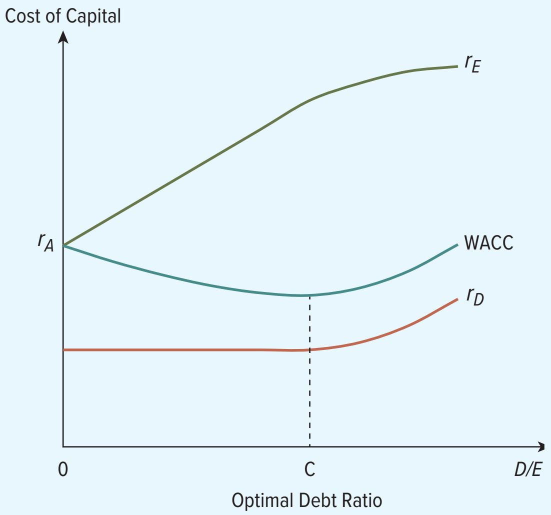

FIGURE 18.2

The WACC Curve with Bankruptcy and Agency Costs

When there are bankruptcy and agency costs, the WACC is first decreasing with leverage and then increasing.

1. The WACC curve is first decreasing in leverage because debt is cheaper than equity. Actually, you wouldn't—as we showed in Chapter 16, this alone doesn't lead to WACC decreasing. You'd say: The WACC curve is first decreasing in leverage because interest is tax deductible.
2. It then starts to flatten out as the company exhausts its interest tax shields (due to hitting the 30% EBIT limit in the United States, or interest equals its entire profits in countries with no limit).
3. WACC then starts to increase because debtholders are worried about bankruptcy costs and the agency costs of debt. They demand a higher cost of debt to compensate for this, which causes WACC to rise.

Part 3 is nuanced. Figure 16.4 showed that, even if there were no bankruptcy or agency costs, the cost of debt rises with leverage. Even if, in Section 17-4, the debtholders of the Devonshire Toy company could recover the full €30 in bankruptcy, they still make a loss because they invested €50, and will demand a higher cost of debt to compensate. A cost of debt rising with leverage is not enough to cause WACC to increase: Figure 16.4 shows that the cost of equity rises less rapidly, offsetting the rising cost of debt and leading to WACC remaining constant.

But in the presence of bankruptcy and agency costs, the cost of debt rises *even faster* than it would have otherwise. Because debtholders know that they'll recover less than €30 in a bankruptcy, they'll demand an even higher cost of debt than in Figure 16.4, and this leads to WACC rising rather than remaining constant.

Calculating Divisional WACCs

Diversified companies are typically mixes of high- and low-risk divisions. We have noted how discounting at a single, company-average WACC biases investment toward high-risk divisions and away from low-risk divisions. The bias can be canceled out by setting different WACCs for different divisions.

Suppose the financial manager singles out a high-risk division. First she estimates the asset beta β_A for the division and uses the CAPM to set the asset cost of capital for that division. Then she sets the target debt ratio for the division, levers up the cost of equity, determines the post-tax cost of debt, and calculates the divisional WACC.

FINANCE IN PRACTICE

Drawing the WACC Curve

❯ Figure 18.2 suggests that companies should try to find the "sweet spot" leverage ratio that minimizes the WACC, since this also maximizes firm value. To do this, they might ask their investment bank advisors to help them draw the WACC curve. How would an investment banking analyst do so in practice?

1. Model the company's cash flows under different debt-to-equity ratios. You'd then calculate various credit ratios under these different financing scenarios. Credit ratios are numbers that debtholders will look at to assess the creditworthiness of a company and thus the cost of debt to ask for. As discussed in Section 29-4, examples include the interest coverage ratio (EBIT divided by interest).
2. Take these credit ratios to their credit ratings advisory department. This is an internal department, often staffed by professionals who used to work at rating agencies such as Standard & Poor's and Moody's, which advises clients on what their credit rating would likely be in different situations. This department in turn would give the credit rating for these different debt-to-equity ratios.
3. Take these credit ratings to the debt capital markets department and ask them what costs of debt investors are currently demanding for companies in the same industry with those credit ratings. Those costs of debt will take into account bankruptcy and agency costs. Note that these costs of debt are *promised* returns. To get to *expected* returns, you would need to subtract the probability of default multiplied by the expected loss given default (see Section 24-1). This gives you the cost of debt for all the different debt-to-equity ratios that you've modeled.
4. Calculate the cost of equity under the different debt-to-equity ratios, using either $r_E = r_A + (r_A - r_D)D/E$ (debt rebalanced) or $r_E = r_A + (1 - t_c)(r_A - r_D)D/E$ (debt fixed) and then plugging the costs of debt and equity into the WACC formula.

Even though companies often do draw the WACC curve in practice, they rarely fine-tune their leverage ratio to be exactly at the sweet spot, the lowest point on the WACC curve. Adjusting leverage by issuing new debt or equity is itself costly, both due to issuance costs and (in the case of equity) the fall in the stock price as discussed in Section 17-7. However, knowing the general shape of the WACC curve is useful to ensure that their leverage doesn't drift too far away from the optimum.

EXAMPLE 18.3 ● Calculating a divisional WACC

Suppose that a company is composed of a high-risk division H with $\beta_A = 1.25$ and a low-risk division L with $\beta_A = 0.75$. Suppose the risk-free interest rate is 3% and the market risk premium is 6%. Then the asset cost of capital for H is

$$r_A = r_f + \beta_A(r_m - r_f) = 0.03 + 1.25 \times 0.06 = 0.105, \text{ or } 10.5\%$$

The financial manager decides on a prudent target debt-to-value ratio of 1/3 (and thus a debt-to-equity ratio of 0.5) and a cost of debt of $r_D = 4\%$. So the cost of equity is

$$r_E = r_A + (r_A - r_D)D/E = 0.105 + (0.105 - 0.04)0.5 = 0.1375, \text{ or } 13.75\%$$

Suppose the corporate tax rate is 25%. The divisional post-tax WACC is

$$\text{WACC} = (1 - t_c)r_D D/V + r_E E/V = (1 - 0.25)(0.04)(1/3) + 0.1375(2/3) = 0.102, \text{ or } 10.2\%.$$

18.3 Self-Test

Calculate the post-tax WACC for the lower-risk division L with $\beta_A = 0.75$. In this case, the target debt-to-value ratio is 0.5. The cost of debt and tax rate remain at 4% and 25%.

The Assumption of a Constant Debt Ratio in the Post-Tax WACC

The post-tax WACC is calculated using the company's existing, overall debt ratio. If the company then uses that WACC to discount future cash flows, it is assuming that the debt ratio does not change in the future. That assumption requires the company to rebalance its capital structure to maintain the same market-value debt ratio in the future. In Section 17-1, we referred to this case as "debt rebalanced."

But real companies do not rebalance capital structure in this obsessive and compulsive way. Does their lax rebalancing behavior undermine practical use of the post-tax WACC?

The answer is normally "No." The typical financial manager does not care much if her debt ratio drifts up or down in a range of reasonable financial leverage. She acts as if a plot of WACC against the debt ratio is flat (constant) over this range. This seems to be incorrect, because Figure 18.1 showed that the WACC always declines with leverage. However, Figure 18.2 illustrated that, when there are bankruptcy and agency costs, and that the tax benefits of debt are exhausted after a point, WACC is reasonably flat around the optimal debt ratio: It doesn't change even if the debt ratio moves slightly away from the optimum. If so, the financial manager is wise to focus on operating and investment decisions, relying on the flat WACC, rather than on fine-tuning the debt ratio.

There is another way to interpret the constant debt ratio in the WACC formula. Assume that a project's **debt capacity** is a constant fraction of project value. ("Capacity" does not mean the maximum amount that could be borrowed, but the amount that the financial manager would optimally choose to borrow.) As the project's value evolves, its debt capacity is assumed to change proportionally, keeping the ratio of debt capacity to value constant. Discounting at WACC with a fixed debt ratio then gives the project credit for interest tax shields that would be generated by the project's fluctuating future value and debt capacity, even if the firm does not regularly rebalance capital structure to keep its overall debt ratio constant. The project's debt capacity can be a constant ratio even if the firm's actual debt ratio is not constant.

The Modigliani–Miller Formula

In the introduction to this chapter we explained that we can incorporate the tax benefits of leverage by either adjusting the cost of capital or adjusting the present value. So far, we have focused on the former, and adjusted the cost of capital by using the post-tax WACC. To estimate the post-tax WACC for different leverage ratios, Step 1 involved unlevering the cost of equity to calculate the asset cost of capital. In the case of debt rebalanced, the formula was simple:

$$r_A = r_D D/V + r_E E/V$$

But in the case of debt fixed, Equation 18.4 showed that the formula is significantly more complex:

$$r_A = (1 - t_c)\, r_D \frac{D}{V - t_c D} + r_E \frac{E}{V - t_c D}$$

Fortunately, in the case of debt fixed, there is another way to calculate the adjusted cost of capital. Modigliani and Miller considered a company or project generating a level, perpetual stream of cash flows financed with fixed, perpetual debt. There is then a simple relationship between the adjusted cost of capital (r_{MM}) and the asset cost of capital (r_A):[13]

$$r_{\mathrm{MM}} = r_A(1 - t_c D/V) \tag{18.6}$$

[13]The formula first appeared in F. Modigliani and M. H. Miller, "Corporate Income Taxes and the Cost of Capital: A Correction," *American Economic Review* 53 (1963), pp. 433–443. It is explained more fully in M. H. Miller and F. Modigliani: "Some Estimates of the Cost of Capital to the Electric Utility Industry, 1954–1957," *American Economic Review* 56 (1966), pp. 333–391. Given perpetual fixed debt,

$$V = \frac{C}{r} + t_c D$$

$$V = \frac{C}{r(1 - t_c D/V)} = \frac{C}{r_{\mathrm{MM}}}$$

How do we use Equation 18.6? The earlier three-step method involved the following:

1. Unlever the cost of equity r_E to obtain the asset cost of capital r_A.
2. Estimate the cost of debt r_D at the new debt ratio, and relever the asset cost of capital r_A to obtain the cost of equity r_E at the new debt ratio.
3. Recalculate the WACC at the new debt ratio.

We needed to unlever and relever the cost of equity, and then plug the relevered r_E into the WACC formula. Equation 18.6 allows us to unlever the WACC directly, allowing us to recalculate the WACC under a different debt level in just two steps:

1. Unlever the WACC to obtain the asset cost of capital r_A.
2. Relever the asset cost of capital r_A at the new debt ratio to obtain the new WACC.

Let's now use Equation 18.6. Take the numbers for Sangria in Example 18.1, and instead assume that these numbers are for the case in which Sangria fixes the amount of debt at $500 million. It now wishes to find how the WACC could change if it increased the amount of debt to $750 million. We proceed in two steps:

1. Unlever the WACC of 9.4% using Equation 18.6. We have $9.4\% = r_A(1 - 0.21 \times 500/1{,}250)$ which yields $r_A = 10.3\%$.
2. Relever the r_A at the new debt level of $750 million. We have $r_{MM} = 10.3\%(1 - 0.21 \times 750/1{,}250) = 9.0\%$.

This passes a sense check: The WACC is lower because debt, and thus tax shields, are higher.

MM's formula is sometimes still used in practice, but it is exact only in the special case where there is a level, perpetual stream of cash flows and fixed, perpetual debt ("debt fixed"). However, the formula is not a bad approximation for projects that are not perpetual as long as debt is issued in a fixed amount.[14]

Summing Up

We plead guilty. We acknowledge that we've thrown a lot of formulas at you in this section. This isn't to be deliberately brutal, but because the correct formulas depend on whether the firm has a debt fixed or debt rebalanced policy. In repentance, we sum up everything in the table below.

Item	Debt Fixed	Debt Rebalanced
$V_L =$	$V_U + t_c D$	$V_U + \dfrac{t_c r_D D}{r_A}$
$r_A =$	$\dfrac{D}{V - t_c D}(1 - t_c)\, r_D + \dfrac{E}{V - t_c D}\, r_E$	$\dfrac{D}{V} r_D + \dfrac{E}{V} r_E$
$\beta_A =$	$\dfrac{D}{V - t_c D}(1 - t_c)\, \beta_D + \dfrac{E}{V - t_c D}\, \beta_E$	$\dfrac{D}{V} \beta_D + \dfrac{E}{V} \beta_E$
$r_E =$	$r_A + \left(\dfrac{D}{E}\right)(1 - t_c)(r_A - r_D)$	$r_A + \left(\dfrac{D}{E}\right)(r_A - r_D)$
$\beta_E =$	$\beta_A + \left(\dfrac{D}{E}\right)(1 - t_c)(\beta_A - \beta_D)$	$\beta_A + \left(\dfrac{D}{E}\right)(\beta_A - \beta_D)$
WACC =	$(1 - t_c)\left(\dfrac{D}{V}\right) r_D + \left(\dfrac{E}{V}\right) r_E$	$(1 - t_c)\left(\dfrac{D}{V}\right) r_D + \left(\dfrac{E}{V}\right) r_E$
	or $r_A\left(1 - t_c \dfrac{D}{V}\right)$	only

[14]See S. C. Myers, "Interactions of Corporate Financing and Investment Decisions—Implications for Capital Budgeting," *Journal of Finance* 29 (1974), pp. 1–25.

18-4 Adjusted Present Value

We now turn to an alternative way to take account of financing decisions. This is to calculate an **adjusted present value** or **APV**.[15] The idea behind APV is to divide and conquer. Instead of capturing the effects of financing by adjusting the discount rate, APV makes a series of present value calculations:

1. We first calculate a base-case NPV: the value of the project or firm if it were all-equity-financed. The discount rate for the base-case value is the unlevered cost of capital r_A, not the WACC which takes into account the tax shields from debt.
2. If the project is not all-equity-financed, we calculate the value of each side effect that results from using debt financing. These may include
   - The interest tax shields on debt (a plus)
   - Issuance costs (a minus)
   - Expected bankruptcy costs and agency costs of debt (a minus)
   - Subsidized financing from a supplier or government (a plus)
3. Finally, all the present values are added together to estimate the project's value:

$$\text{APV} = \text{base-case NPV} + \text{sum of PVs of financing side effects}$$

The APV and WACC methods can be easily compared if taxes are the only market imperfection. Consider a project that yields a perpetual post-tax cash flow of C. If it were all-equity financed, its value would be given by

$$V_U = \frac{C}{r_A}$$

If the project is partially debt financed, there are two ways of incorporating the tax benefits of debt. The WACC method discounts by the WACC, which incorporates tax shields, rather than the r_A which assumes all-equity financing:

$$V_L = \frac{C}{\text{WACC}}$$

The APV method does not change the discount rate, but adds tax shields as a separate term:

$$V_L = \frac{C}{r_A} + \frac{t_c r_D D}{r_D \text{ (debt fixed) or } r_A \text{ (debt rebalanced)}}$$

It may seem that the WACC method is simpler as this gives us the project's value in one go, rather than requiring two steps. However, APV has three potential advantages over WACC:

1. It gives the financial manager an explicit view of the factors that are adding or subtracting value. For example, it could be that the APV is overall positive, but this is entirely due to the tax shields of debt: The base-case NPV is negative, so the project itself destroys value. That should prompt the manager to look around to see if there is a project with similar debt capacity but a positive base-case NPV.
2. APV is flexible. It allows for debt management policies that are neither debt fixed nor debt rebalanced. In particular, it allows debt to be paid down according to a predetermined schedule, as we'll consider shortly.
3. Another source of flexibility is that APV allows for market imperfections other than taxes.

[15]The adjusted-present-value rule was developed in S. C. Myers, "Interactions of Corporate Financing and Investment Decisions—Implications for Capital Budgeting," *Journal of Finance* 29 (1974), pp. 1–25.

APV for the Perpetual Crusher

APV is easiest to understand with an example. Let's apply it to Sangria's perpetual crusher project. We start by showing that APV is equivalent to discounting at WACC if we make the same assumptions about debt policy and taxes.

Step 1 The first step is to calculate base-case NPV. This is the project's NPV with all-equity financing. To find it, we discount the post-tax project cash flows of $1.175 million by the unlevered cost of capital. In Section 18-3, we calculated r_A by unlevering Sangria's cost of equity under "debt rebalanced" to obtain $r_A = 9.9\%$. After subtracting the $12.5 million outlay, we have

$$\text{Base-case NPV} = -12.5 + \frac{1.175}{0.099} = -\$0.63 \text{ million}$$

Thus, the project would not be worthwhile with all-equity financing.

Step 2 Fortunately, the project supports debt of $5 million. At a 6% borrowing rate ($r_D = 0.06$) and a 21% tax rate ($t_c = 0.21$), annual tax shields are $0.21 \times 0.06 \times 5 = 0.063$, or $63,000.

What are those tax shields worth? If Sangria always sticks to a 40% debt ratio for the perpetual crusher, then we are in the "debt rebalanced" case and discount the tax shields at $r_A = 9.9\%$:

$$\text{PV(interest tax shields, debt rebalanced)} = \frac{63{,}000}{0.099} = \$0.63 \text{ million}$$

Step 3 APV is the sum of base-case value and PV(interest tax shields):

$$\text{APV} = -0.63 \text{ million} + 0.63 \text{ million} = 0$$

This is exactly the same as we obtained by one-step discounting with WACC. The perpetual crusher is a break-even project by either valuation method.[16]

COMMON CONFUSION: THE COST OF EQUITY VERSUS THE COST OF CAPITAL ASSUMING ALL-EQUITY FINANCING

Step 1 of the APV method uses a discount rate that assumes all-equity financing. Some people see the word *equity* and think that this refers to r_E, the cost of equity.

This is not the case. r_E is the cost of equity for a levered firm, one with debt financing. It includes not only business risk (the only source of risk for an all-equity firm) but also financial risk, as shown by the calculation $r_E = r_A + (r_A - r_D)D/E$.

The cost of capital for an all-equity firm is r_A, which depends on business risk only. We need to unlever the cost of equity r_E to get the unlevered cost of capital r_A.

[16]Calculating the present value of the tax shields is straightforward when the project is a perpetuity. When it is not, the expected value of the project changes as time passes and so does the expected tax shield. With a finite project and debt that is a constant proportion of project value, we would need to calculate the expected project value at each future date before calculating the present value of the tax shields. Therefore, whenever the debt ratio is constant, managers use WACC to account for the interest tax shield, and they save APV for times when debt is repaid on a fixed schedule.

Debt Fixed Suppose that, instead of rebalancing debt to a constant 40% of the project's value, Sangria instead holds the dollar amount of debt fixed at \$5 million. We are then in the "debt fixed" case and discount the tax shields at the cost of debt, $r_D = 6\%$:

$$\text{PV(tax shields, debt fixed)} = \frac{63{,}000}{0.06} = \$1.05 \text{ million}$$

$$\text{APV} = -0.63 + 1.05 = \$0.42 \text{ million}$$

Now the project is more attractive. With fixed debt, the interest tax shields are safe and therefore worth more. (Whether the fixed debt is safer for Sangria is another matter. If the perpetual crusher project fails, the \$5 million of fixed debt may end up as a burden on Sangria's other assets.)

18.4 Self-Test

True or false? Explain briefly.

a. In an APV calculation, you need to use the WACC to discount any tax shields if debt is constantly rebalanced.
b. The APV is higher when debt is fixed than when it is a constant proportion of value.

Other Financing Side Effects

Suppose Sangria has to finance the perpetual crusher by issuing debt and equity. It issues \$7.5 million of equity with issue costs of 7% (\$0.53 million) and \$5 million of debt with issue costs of 2% (\$0.10 million). Assume the debt is fixed once issued, so that interest tax shields are worth \$1.05 million. Now we can recalculate APV, taking care to subtract the issue costs:

$$\text{APV} = -0.63 + 1.05 - 0.53 - 0.10 = -\$0.21 \text{ million, or } -\ \$210{,}000$$

The issue costs would result in a negative APV. If the issue costs were immediately tax-deductible, the APV would still be negative:

$$\text{APV} = -0.63 + 1.05 - 0.53(1 - 0.21) - 0.10(1 - 0.21) = -\$0.08 \text{ million}$$

Sometimes there are favorable financing side effects that have nothing to do with taxes. For example, suppose that a potential manufacturer of crusher machinery offers to sweeten the deal by leasing it to Sangria on favorable terms. Then, to calculate APV, you would need to add in the NPV of the lease. Or suppose that a local government offers to lend Sangria \$5 million at a very low interest rate if the crusher is built and operated locally. The NPV of the subsidized loan could be added in to APV. (We cover leases in Chapter 26 and covered subsidized loans in Section 12-1.)

APV for Entire Businesses

APV can also be used to value entire businesses. Let's take another look at the valuation of Rio. In Table 18.1, we assumed a constant 40% debt ratio (debt rebalanced) and discounted free cash flow at Sangria's WACC. We've also seen how to handle a fixed dollar amount of debt. However, some cases involve neither fixed nor rebalanced debt, but **amortized debt** where debt is paid down according to a predetermined schedule. As mentioned earlier, one of the advantages of APV is that it is flexible and can deal with such cases. Table 18.2 runs the same analysis as in Table 18.1, but with debt being repaid according to an amortization schedule.

	Latest							
	Year	Forecast						
	0	1	2	3	4	5	6	7
Free cash flow	4.9	5.3	5.2	5.5	8.0	8.3	8.2	8.5
PV free cash flow, years 1–6	28.5							
PV terminal value	75.3			(Terminal value in year 6)			132.7	
Base-case PV of company	103.8							
Debt	62.0	60.0	60.0	58.0	56.0	54.0	53.0	
Interest		3.72	3.60	3.60	3.48	3.36	3.24	
Interest tax shield		0.78	0.76	0.76	0.73	0.71	0.68	
PV interest tax shields	3.6							
APV	107.5							
Tax rate, %	21.0							
Asset cost of capital, %	9.9							
WACC, % (to discount terminal value to year 6)	9.4							
Long-term growth forecast, %	3.0							
Cost of debt, % (years 1–6)	6.0							

TABLE 18.2 APV Valuation of Rio Corporation ($ millions)

We'll suppose that Sangria has decided to make an offer for Rio. If successful, it plans to finance the purchase with $62 million of debt. It intends to pay down the debt to $53 million in year 6. Recall Rio's terminal value of $132.7 million, which is calculated in Table 18.1 and shown again in Table 18.2. The debt ratio at the horizon date is therefore projected at 53/132.7= 0.40, or 40%. Thus, Sangria plans to take Rio back to a normal 40% debt ratio at the horizon date.[17] But Rio will be carrying a heavier debt load before the horizon. For example, the $62 million of initial debt is about 56% of company value as calculated in Table 18.1.

BEYOND THE PAGE

Try it! Rio's spreadsheet

Let's see how Rio's APV is affected by this more aggressive borrowing schedule. Table 18.2 shows projections of free cash flows from Table 18.1.[18] Now we need Rio's base-case value. This is its value with all-equity financing, so we discount these flows at the asset cost of capital (9.9%), not at WACC. We calculate the base-case value for Rio in two stages:

- We discount the cash flows in years 1–6 by 9.9% and add them up to give a PV of $28.5 million.
- We discount the terminal value in year 6 of $132.7 million by 9.9% to give a PV of $\$132.7/1.099^6 = \$75.3$ million. Note this is lower than the $77.4 million in Table 18.1 which was discounted using the WACC of 9.4%.

The resulting base-case value for Rio is $28.5 + 75.3 = $103.8 million.

[17]Therefore, we still calculate the terminal value in year 6 by discounting subsequent free cash flows at WACC. The terminal value in year 6 is discounted back to year 0 at the asset cost of capital, however.

[18]Many of the assumptions and calculations in Table 18.1 have been hidden in Table 18.2. The hidden rows can be recalled in the Beyond the Page spreadsheets for Tables 18.1 and 18.2.

Table 18.2 also projects debt levels, interest payments, and interest tax shields. Since the debt levels are predetermined by the debt paydown schedule, they are fixed. The tax shields are safe and thus should be discounted at the 6% cost of debt. The resulting PV of interest tax shields is

$$\text{PV (tax shields)} = \frac{0.78}{1.06} + \frac{0.76}{1.06^2} + \frac{0.76}{1.06^3} + \frac{0.73}{1.06^4} + \frac{0.71}{1.06^5} + \frac{0.68}{1.06^6} = \$3.6 \text{ million}$$

Thus,

$$\begin{aligned} \text{APV} &= \text{base-case NPV} + \text{PV(interest tax shields)} \\ &= \$103.8 + 3.6 = \$107.5 \text{ million} \end{aligned}$$

an increase of \$1.0 million from the NPV in Table 18.1. The increase can be traced to the higher early debt levels and to the assumption that the debt levels and interest tax shields are fixed and safe.[19]

Now a difference of \$1.0 million is not a big deal, considering all the lurking risks and pitfalls in forecasting Rio's free cash flows. But you can see the advantage of the flexibility that APV provides. The APV spreadsheet allows you to explore the implications of different financing strategies without locking into a fixed debt ratio or having to calculate a new WACC for every scenario.

APV and Limits on Interest Deductions

The United States now limits the amount of interest that can be deducted for tax to 30% of each year's EBIT. The European Union restricts interest deductibility to 30% of EBITDA.

Most companies will not be affected by these limits. But what about the few that are affected? How should a financial manager take limits on interest-expense deductions into account?

Suppose the 30% constraint is and will be binding. Assume the firm is profitable and paying taxes. Then the future interest tax shields generated by a new investment project are proportional to its future EBIT. The financial manager should forecast EBIT and the associated tax shields and discount at a rate depending on the risk of EBIT. In most cases, the risk of EBIT will be similar to the risk of the project's overall cash flows, in which case the interest tax shields can be discounted at the asset cost of capital r_A. The APV formula is the same as before:

$$\text{APV} = \text{base-case NPV} + \text{PV(interest tax shields)}$$

but PV(interest tax shields) now depends on the project's forecast EBIT.

Those projects that generate plenty of EBIT are especially valuable to tax-paying firms that are subject to the 30% constraint. The EBIT of the project can relax the constraint for the firm as a whole, thus unlocking interest tax shields on the firm's existing debt.

The APV of an entire business or company subject to the 30% constraint should also include the PV of interest tax shields generated by its expected future EBIT. If the 30% limit on interest deductions is temporary—in one or two low-profit years, for example—then the unused tax shields are not lost but can be carried forward indefinitely and may, therefore, be merely delayed. The financial manager could assign the tax shields to future years, discount to PV and include them in APV.

[19] But will Rio really *support* debt at the levels shown in Table 18.2? If not, then the debt must be partly supported by Sangria's other assets, and only part of the \$3.6 million in PV(interest tax shields) can be attributed to Rio itself.

APV for International Investments

APV is most useful when financing side effects are numerous and important. This is frequently the case for large international investments, which may have custom-tailored **project financing** and special contracts with suppliers, customers, and governments. Here are a few examples of financing side effects resulting from the financing of a project.

We explain project finance in more detail in Chapter 25, but for now we highlight a few key features:

- *High leverage.* Project finance typically involves very high debt ratios to start, with most or all of a project's early cash flows committed to debt service. Equity investors have to wait. Since the debt ratio will not be constant, you have to turn to APV.
- *Subsidized financing.* Project financing may include debt available at favorable interest rates. Most governments subsidize exports by making special financing packages available, and manufacturers of industrial equipment may stand ready to lend money to help close a sale. Suppose, for example, that your project requires construction of an on-site electricity generating plant. You solicit bids from suppliers in various countries. Don't be surprised if the competing suppliers sweeten their bids with offers of low interest rate project loans or if they offer to lease the plant on favorable terms. You should then calculate the NPVs of these loans or leases and include them in your project analysis.
- *Favorable contracts.* Sometimes international projects are supported by contracts with suppliers or customers. Suppose a manufacturer wants to line up a reliable supply of a crucial raw material—powdered magnoosium, say. The manufacturer could subsidize a new magnoosium smelter by agreeing to buy 75% of production and guaranteeing a minimum purchase price. The guarantee is clearly a valuable addition to the smelter's APV: If the world price of powdered magnoosium falls below the minimum, the project doesn't suffer. You would calculate the value of this guarantee (by the methods explained in Chapters 21 to 23) and add it to APV.
- *Restrictions.* Sometimes local governments impose costs or restrictions on investment or disinvestment. For example, Chile, in an attempt to slow down a flood of short-term capital inflows in the 1990s, required investors to "park" part of their incoming money in non-interest-bearing accounts for a period of two years. An investor in Chile during this period could have calculated the cost of this requirement and subtracted it from APV.[20]

18-5 Your Questions Answered

Question 1: All these cost of capital formulas—which ones do financial managers actually use?

Answer 1: The post-tax WACC, nearly all of the time. WACC is estimated for the company, or sometimes for an industry. Industry WACCs can be useful when data are available for firms with similar assets, operations, and business risks.

Of course, conglomerate companies, with divisions operating in two or more industries facing different business risks, should not use a single WACC. Such firms should try to estimate a different WACC for each operating division.

Question 2: But WACC is the correct discount rate only for "average" projects. What if the project's financing differs from the company's or industry's?

[20] Such capital controls have been described as financial roach motels: Money can get in, but it can't get out.

Answer 2: Remember, investment projects are usually not separately financed. Even when they are, the Project Leverage Principle highlights how you should focus on the project's contribution to the firm's overall debt capacity, not on its immediate financing. (Suppose it's convenient to raise all the money for a particular project with a bank loan. That doesn't mean the project itself supports 100% debt financing. The company is borrowing against its existing assets as well as the project.)

But if the project's debt capacity is materially different from the company's existing assets, or if the company's overall debt policy changes, WACC should be adjusted. The adjustment can be done by the three-step procedure explained in Section 18-3.

Question 3: Could we do one more numerical example?

Answer 3: Sure. Suppose that WACC has been estimated as follows at a 30% debt ratio:

$$\begin{aligned}\text{WACC} &= (1 - t_c) r_D \frac{D}{V} + r_E \frac{E}{V} \\ &= (1 - 0.21)(0.09)(0.3) + 0.15(0.7) = 0.126, \text{ or } 12.6\%\end{aligned}$$

What is the correct discount rate at a 50% debt ratio?

Step 1. Calculate the asset cost of capital.

$$\begin{aligned}r_A &= r_D D/V + r_E E/V \\ &= 0.09(0.3) + 0.15(0.7) = 0.132, \text{ or } 13.2\%\end{aligned}$$

Step 2. Calculate the new costs of debt and equity. The cost of debt will be higher at 50% debt than 30%. Say it is $r_D = 0.095$. The new cost of equity is

$$\begin{aligned}r_E &= r_A + (r_A - r_D)D/E \\ &= 0.132 + (0.132 - 0.095)50/50 \\ &= 0.169, \text{ or } 16.9\%\end{aligned}$$

Step 3. Recalculate WACC.

$$\begin{aligned}\text{WACC} &= (1 - t_c) r_D D/V + r_E E/V \\ &= (1 - 0.21)(0.095)(0.5) + 0.169(0.5) = 0.122, \text{ or } 12.2\%\end{aligned}$$

Question 4: How do I use the capital asset pricing model (CAPM) to calculate the post-tax weighted-average cost of capital?

Answer 4: First plug the equity beta into the CAPM formula to calculate r_E, the expected return to equity. Then use this figure, along with the post-tax cost of debt and the debt-to-value and equity-to-value ratios, in the WACC formula.

Of course, the CAPM is not the only way to estimate the cost of equity. For example, you might be able to use the dividend discount model (see the Appendix to Chapter 4).

Question 5: But suppose I do use the CAPM? What if I have to recalculate the equity beta for a different debt ratio?

Answer 5: You first unlever the equity beta to find the asset beta. In the case of debt rebalanced, the asset beta is a weighted average of the debt and equity betas:

$$\beta_A = \beta_D D/V + \beta_E E/V$$

Then you plug in the new debt ratio and new debt beta under this debt ratio to relever the asset beta and calculate the new equity beta:

$$\beta_E = \beta_A + (\beta_A - \beta_D)D/E$$

You then plug this into the CAPM to obtain r_E. Notice that, if you only needed the asset cost of capital r_A, you could calculate β_A from the first equation and then plug it into the CAPM to obtain r_A.

Question 6: I think I understand how to adjust for differences in debt capacity or debt policy. How about differences in business risk?

Answer 6: If business risk is different, then r_A, the opportunity cost of capital, is different.

Figuring out the right r_A for an unusually safe or risky project is never easy. Sometimes the financial manager can use estimates of risk and expected return for companies similar to the project. Suppose, for example, that a traditional pharmaceutical company is considering a major commitment to biotech research. The financial manager could pick a sample of biotech companies, estimate their average beta and cost of capital, and use these estimates as benchmarks for the biotech investment.

But in many cases, it's difficult to find a good sample of matching companies for an unusually safe or risky project. Then the financial manager has to adjust the opportunity cost of capital by judgment. Section 9-3 may be helpful in such cases.

Question 7: When do I need adjusted present value (APV)?

Answer 7: The WACC formula picks up only one financing side effect: the value of interest tax shields on debt supported by a project. If there are other side effects—subsidized financing tied to a project, for example—you should use APV.

You can also use APV to break out the value of interest tax shields:

$$\text{APV} = \text{base-case NPV} + \text{PV(tax shield)}$$

Suppose, for example, that you are analyzing a company just after a leveraged buyout. The company has a very high initial debt level but plans to pay down the debt as rapidly as possible. APV could be used to obtain an accurate valuation.

Question 8: When should personal taxes be incorporated into the analysis?

Answer 8: Always use t_c, the marginal corporate tax rate, when calculating WACC as a weighted average of the costs of debt and equity. The discount rate is adjusted *only* for corporate taxes. This is because personal taxes are already incorporated into r_D and r_E, the pre-tax return required by debtholders and equityholders. If personal taxes are high, debtholders and equityholders will demand a higher r_D and r_E. As a result, personal taxes don't enter into the WACC formula.

In principle, APV can be adjusted for personal taxes by replacing the marginal corporate rate t_c with an effective tax rate that combines corporate and personal taxes and reflects the net tax advantage per dollar of interest paid by the firm. We provided simple calculations of this advantage in Section 17-2. The effective tax rate is almost surely less than t_c, but it is very difficult to pin down the numerical difference. Therefore, in practice t_c is almost always used as an approximation.

Question 9: Are taxes really that important? Do financial managers really fine-tune the debt ratio to minimize WACC?

Answer 9: As we saw in Chapter 17, financing decisions reflect many forces beyond taxes, including bankruptcy and agency costs. There may not be a sharply defined optimal capital structure. Therefore, most financial managers don't fine-tune their companies' debt ratios, and they don't rebalance financing to keep debt ratios strictly constant. In effect, they assume that a plot of WACC for different debt ratios is "flat" over a reasonable range of moderate leverage.

KEY TAKEAWAYS

In Chapters 4 and 6, when we first looked at valuing businesses and investment projects, we ignored any effects of financing on value. In effect, we assumed all-equity financing. In this chapter, we show how to take financing, including interest tax shields, into account.

- **Post-tax weighted-average cost of capital** When estimating present value, managers can adjust for interest rate tax shields by discounting at the post-tax weighted-average cost of capital (WACC):

$$\text{WACC} = (1 - t_c)r_D D/V + r_E\, E/V$$

Here $(1 - t_c)r_D$ is the *post-tax* cost of debt and r_E the cost of equity.

When calculating WACC, remember to use market values for the debt and equity ratios D/V and E/V.

- **WACC versus the asset cost of capital** Do not confuse WACC with the asset cost of capital, the cost of capital assuming all equity financing:

$$\text{Asset cost of capital} = r_A = r_D D/V + r_E E/V$$

The asset cost of capital is investors' opportunity cost of capital for an all-equity-financed investment in the firm.

- **Cost of equity** The cost of equity r_E depends on the debt-to-equity ratio due to financial risk:

$$r_E = r_A + (r_A - r_D)D/E$$

This formula can be used to lever or unlever the cost of equity when there is continuous rebalancing.

- **Betas** The formulas for the asset and equity betas under "debt rebalanced" are

$$\beta_A = \beta_D D/V + \beta_E E/V$$

$$\beta_E = \beta_A + (\beta_A - \beta_D)D/E$$

- **WACC assumptions** Discounting at WACC works for projects that are carbon copies of the existing firm—projects with the same business risk that will be financed to maintain the firm's current leverage ratio. But firms can use WACC as a benchmark rate, which is then adjusted for differences in business risk or financing. We gave a three-step procedure for adjusting WACC for different debt ratios:

1. Unlever the cost of equity r_E to obtain the asset cost of capital r_A.

2. Estimate the cost of debt r_D at the new debt ratio, and relever the asset cost of capital r_A to obtain the cost of equity r_E at the new debt ratio.

3. Recalculate WACC at the new debt ratio.

All of this chapter's examples reflect assumptions about the amount of debt supported by a project or business. Remember not to confuse "supported by" with the immediate source of funds for investment.

- **Adjusted present value (APV)** APV provides an alternative way to incorporate financing side effects. First calculate a base NPV assuming all-equity financing, and then add the present value of interest tax shields or other financing side effects.

$$\text{APV} = \text{all-equity NPV} + \text{PV(financing side effects)}$$

Common financing side effects include interest tax shields, bankruptcy costs, agency costs, issue costs, and subsidized financing.

- **Valuation based on free cash flow (FCF)** Free cash flow is the amount of cash that can be paid out to all investors, debt as well as equity, after deducting cash needed for new investment or increases in working capital but not interest. The WACC formula accounts for interest tax shields by using the post-tax cost of debt. APV adds PV(interest tax shields) to base-case value.
- **Business valuation** Businesses are usually valued in two steps. First, free cash flow is forecast out to a horizon date assuming all-equity financing and is discounted back to present value using WACC. Second, a terminal value is calculated and also discounted back. Be particularly careful to avoid unrealistically high terminal values. By the time the horizon arrives, competitors will have had several years to catch up. Also, when you are done valuing the business, don't forget to subtract its debt to get the value of the firm's equity.

FURTHER READING

The Harvard Business Review has published a popular account of APV:

T. A. Luehrman, "Using APV: A Better Tool for Valuing Operations," *Harvard Business Review* 75 (1997), pp. 145–154.

There have been dozens of articles on the weighted-average cost of capital and other issues discussed in this chapter. Here are three:

J. Miles and R. Ezzell, "The Weighted Average Cost of Capital, Perfect Capital Markets, and Project Life: A Clarification," *Journal of Financial and Quantitative Analysis* 15 (1980), pp. 719–730.

R. S. Ruback, "Capital Cash Flows: A Simple Approach to Valuing Risky Cash Flows," *Financial Management* 31 (2002), pp. 85–103.

R. A. Taggart Jr., "Consistent Valuation and Cost of Capital Expressions with Corporate and Personal Taxes," *Financial Management* 20 (1991), pp. 8–20.

PROBLEM SETS

Select problems are available in McGraw Hill's *Connect.* Please see the preface for more information.

1. **WACC (S18-1)** True or false? Use of the WACC formula assumes
   a. A project supports a fixed amount of debt over the project's economic life.
   b. The ratio of the debt supported by a project to project value is constant over the project's economic life.
   c. The firm rebalances debt each period, keeping the debt-to-value ratio constant.
2. **WACC (S18-1)** The WACC formula seems to imply that debt is "cheaper" than equity—that is, that a firm with more debt could use a lower discount rate. Does this make sense? Explain briefly.
3. **WACC (S18-1)** Calculate the weighted-average cost of capital (WACC) for Federated Junkyards of America, using the following information:
   - Debt: \$75,000,000 book value outstanding. The debt is trading at 90% of book value. The yield to maturity is 9%.

- Equity: 2,500,000 shares selling at $42 per share. Assume the expected rate of return on Federated's stock is 18%.
- Taxes: Federated's marginal tax rate is $t_c = 0.21$.

4. **WACC (S18-1)** Whispering Pines Inc. is all-equity-financed. The expected rate of return on the company's shares is 12%.
   a. What is the opportunity cost of capital for an average-risk Whispering Pines investment?
   b. Suppose the company issues debt, repurchases shares, and moves to a 30% debt-to-value ratio ($D/V = 0.30$). What will be the company's WACC at the new capital structure? The borrowing rate is 7.5% and the tax rate is 21%.

5. **WACC (S18-1)** Table 18.3 shows a *book* balance sheet for the Wishing Well motel chain. The company's long-term debt is secured by its real estate assets, but it also uses short-term bank loans as a permanent source of financing. It pays 10% interest on the bank debt and 9% interest on the secured debt. Wishing Well has 10 million shares of stock outstanding, trading at $90 per share. The expected return on Wishing Well's equity is 18%. Calculate Wishing Well's WACC. Assume that the book and market values of Wishing Well's debt are the same. The marginal tax rate is 21%.

Cash and marketable securities	100	Bank loan	280
Accounts receivable	200	Accounts payable	120
Inventory	50	Current liabilities	400
Current assets	350		
Real estate	2,100	Long-term debt	1,800
Other assets	150	Equity	400
Total	2,600	Total	2,600

TABLE 18.3 Book Balance Sheet for Wishing Well Inc. ($ millions)

6. **WACC (S18-1)** Table 18.4 shows a simplified balance sheet for the Dutch manufacturer Rensselaer Felt. The debt has just been refinanced at an interest rate of 6% (short term) and 8% (long term). The expected rate of return on the company's shares is 15%. There are 7.46 million shares outstanding, and the shares are trading at €46. The tax rate is 25%. Calculate Rensselaer Felt's WACC.

Cash and marketable securities	1,500	Short-term debt	75,600
Accounts receivable	120,000	Accounts payable	62,000
Inventory	125,000	Current liabilities	137,600
Current assets	246,500		
Property, plant, and equipment	212,000	Long-term debt	208,600
Deferred taxes	45,000		
Other assets	89,000	Shareholders' equity	246,300
Total	592,500	Total	592,500

TABLE 18.4 Simplified Book Balance Sheet for Rensselaer Felt (€ thousands)

7. **WACC (S18-1)** Nevada Hydro is 40% debt-financed and has a weighted-average cost of capital of 10.2%:

$$\text{WACC} = (1 - t_c)r_D D/V + r_E E/V = (1 - 0.21)(0.085)(0.40) + 0.125(0.60) = 0.102$$

Goldensacks Company is advising Nevada Hydro to issue \$75 million of preferred stock at a dividend yield of 9%. The proceeds would be used to repurchase and retire equity. The preferred issue would account for 10% of the pre-issue market value of the firm.

Goldensacks argues that these transactions would reduce Nevada Hydro's WACC to 9.84%:

$$\begin{aligned}\text{WACC} &= (1 - 0.21)(0.085)(0.40) + 0.09(0.10) + 0.125(0.50)\\ &= 0.0984, \text{ or } 9.84\%\end{aligned}$$

Do you agree with this calculation? Explain.

8. **Forecasting cash flow (S18-1)** Suppose Wishing Well (see Problem 5) is evaluating a new motel and resort on a romantic site in Madison County, Wisconsin. Explain how you would forecast the post-tax cash flows for this project. (*Hints:* How would you treat taxes? Interest expense? Changes in working capital?)

9. **Company valuation (S18-2)** Chiara Company's management has made the projections shown in Table 18.5. Use this table as a starting point to value the company as a whole. The WACC for Chiara is 12%, and the forecast long-run growth rate after year 5 is 4%. The company, which is located in South Africa, has ZAR5 million of debt and 865,000 shares outstanding. What is the value per share?

	Historical			Forecast				
Year	−2	−1	0	1	2	3	4	5
1. Sales	35,348	39,357	40,123	36,351	30,155	28,345	29,982	30,450
2. Cost of goods sold	17,834	18,564	22,879	21,678	17,560	16,459	15,631	14,987
3. Other costs	6,968	7,645	8,025	6,797	5,078	4,678	4,987	5,134
4. EBITDA (1 – 2 – 3)	10,546	13,148	9,219	7,876	7,517	7,208	9,364	10,329
5. Depreciation	5,671	5,745	5,678	5,890	5,670	5,908	6,107	5,908
6. EBIT (Pretax profit) (4 – 5)	4,875	7,403	3,541	1,986	1,847	1,300	3,257	4,421
7. Tax at 28%	1,365	2,073	991	556	517	364	912	1,238
8. Profit after tax (6 – 7)	3,510	5,330	2,550	1,430	1,330	936	2,345	3,183
9. Change in working capital	325	566	784	−54	−342	−245	127	235
10. Investment (change in gross fixed assets)	5,235	6,467	6,547	7,345	5,398	5,470	6,420	6,598

TABLE 18.5 Cash-Flow Projections for Chiara (ZAR thousands)

10. **Flow-to-equity valuation (S18-2)** What is meant by the flow-to-equity valuation method? What discount rate is used in this method? What assumptions are necessary for this method to give an accurate valuation?

11. **Using WACC in Practice (S18-3)** Suppose Federated Junkyards (see Problem 3) decides to move to a more conservative debt policy. A year later, its debt ratio is down to 15% (D/V = 0.15). The pre-tax cost of debt has dropped to 8.6%. Recalculate Federated's WACC under these new assumptions. The company's business risk, opportunity cost of capital, and tax rate have not changed. Use the three-step procedure explained in Section 18-3.

12. **Using WACC in Practice (S18-3)** See Problem 6. How will Rensselaer's WACC and cost of equity change if it issues €50 million in new equity and uses the proceeds to retire long-term debt? Assume the company's borrowing rates are unchanged. Use the three-step procedure from Section 18-3.

13. **APV (S18-4)** True or false? The APV method
    a. Starts with a base-case value for the project.
    b. Calculates the base-case value by discounting project cash flows, forecast assuming all-equity financing, at the WACC for the project.
    c. Is especially useful when debt is to be paid down on a fixed schedule.

14. **APV (S18-4)** A project costs $1 million and has a base-case NPV of exactly zero (NPV = 0). What is the project's APV in the following cases?
    a. If the firm invests, it has to raise $500,000 by a stock issue. Issue costs are 15% of *net* proceeds.
    b. If the firm invests, there are no issue costs, but its debt capacity increases by $500,000. The present value of interest tax shields on this debt is $76,000.
15. **APV (S18-4)** Consider a project lasting one year only. The initial outlay is $1,000, and the expected inflow is $1,200. The opportunity cost of capital is $r_A = 0.20$. The borrowing rate is $r_D = 0.10$, and the tax shield per dollar of interest is $t_c = 0.21$.
    a. What is the project's base-case NPV?
    b. What is its APV if the firm borrows 30% of the project's required investment?
16. **APV (S18-4)** To finance the Madison County project (see Problem 8), Wishing Well needs to arrange an additional $80 million of long-term debt and make a $20 million equity issue. Underwriting fees, spreads, and other costs of this financing will total $4 million. How would you take this into account in valuing the proposed investment?
17. **APV (S18-4)** Digital Organics (DO) has the opportunity to invest $1 million now ($t = 0$) and expects post-tax returns of $600,000 in $t = 1$ and $700,000 in $t = 2$. The project will last for two years only. The appropriate cost of capital is 12% with all-equity financing, the borrowing rate is 8%, and DO will borrow $300,000 against the project. This debt must be repaid in two equal installments of $150,000 each. Assume debt tax shields have a net value of $0.30 per dollar of interest paid. Calculate the project's APV using the procedure followed in Table 18.2.
18. **APV (S18-4)** Consider another perpetual project like the crusher described in Section 18-1. Its initial investment is $1,000,000, and the expected cash inflow is $95,000 a year in perpetuity. The opportunity cost of capital with all-equity financing is 10%, and the project allows the firm to borrow at 7%. The tax rate is 21%.

    Use APV to calculate the project's value.
    a. Assume first that the project will be partly financed with $400,000 of debt and that the debt amount is to be fixed and perpetual.
    b. Then assume that the initial borrowing will be increased or reduced in proportion to changes in the market value of this project.

    Explain the difference between your answers to (a) and (b).
19. **Opportunity cost of capital (S18-1)** Suppose the project described in Problem 18 is to be undertaken by a university. Funds for the project will be withdrawn from the university's endowment, which is invested in a widely diversified portfolio of stocks and bonds. However, the university can also borrow at 7%. The university is tax exempt. The university treasurer proposes to finance the project by issuing $400,000 of perpetual bonds at 7% and by selling $600,000 worth of shares from the endowment. The expected return on the shares is 10%. He therefore proposes to evaluate the project by discounting at a weighted-average cost of capital, calculated as

$$\begin{aligned} r &= r_D\, D/V + r_E\, E/V \\ &= 0.07(400{,}000/1{,}000{,}000) + 0.10\,(600{,}000/1{,}000{,}000) \\ &= 0.088, \text{ or } 8.8\% \end{aligned}$$

    What's right or wrong with the treasurer's approach? Should the university invest? Should it borrow? Would the project's value to the university change if the treasurer financed the project entirely by selling shares from the endowment?
20. **APV (S18-4)** Consider a project to produce solar water heaters. It requires a $10 million investment and offers a level post-tax cash flow of $1.75 million per year for 10 years. The opportunity cost of capital is 12%, which reflects the project's business risk.

a. Suppose the project is financed with \$5 million of debt and \$5 million of equity. The interest rate is 8% and the marginal tax rate is 21%. An equal amount of the debt will be repaid in each year of the project's life. Calculate APV.

b. How does APV change if the firm incurs issue costs of \$400,000 to raise the \$5 million of required equity?

21. **APV and debt capacity (S18-4)** Suppose KCS Corp. buys out Patagonia Trucking, a privately owned business, for \$50 million. KCS has only \$5 million cash in hand, so it arranges a \$45 million bank loan. A normal debt-to-value ratio for a trucking company would be 50% at most, but the bank is satisfied with KCS's credit rating. Suppose you were valuing Patagonia by APV in the same format as Table 18.2. How much debt would you include? Explain briefly.

22. **APV and issue costs (S18-4)** The Bunsen Chemical Company is currently at its target debt ratio of 40%. It is contemplating a \$1 million expansion of its existing business. This expansion is expected to produce a cash inflow of \$130,000 a year in perpetuity. The company is uncertain whether to undertake this expansion and how to finance it. The two options are a \$1 million issue of equity or a \$1 million issue of 20-year debt. The flotation costs of a stock issue would be around 5% of the amount raised, and the flotation costs of a debt issue would be around 1½%. Bunsen's financial manager, Polly Ethylene, estimates that the required return on the company's equity is 14%, but she argues that the flotation costs increase the cost of new equity to 19%. On this basis, the project does not appear viable. On the other hand, she points out that the company can raise new debt on a 7% yield, which would make the cost of new debt 8½%. She therefore recommends that Bunsen should go ahead with the project and finance it with an issue of long-term debt. Is Ms. Ethylene right? How would you evaluate the project?

23. **APV and limits on interest tax shields (S18-4)** Take another look at the APV calculation for the perpetual crusher project in Section 18-4. This time assume that the company investing in the project has hit the 30% constraint on interest deductions as a percentage of EBIT. How does the constraint change the project's APV? Notice that the crusher's pre-tax cash flow of \$1.487 million a year is also its EBIT. The project is perpetual, so there is no depreciation or amortization. Assume for simplicity that the constraint is permanently binding, but that the firm will continue to pay tax at the 21% statutory rate.

BEYOND THE PAGE

Try it! Rio's spreadsheet

24. **WACC and APV (S18-1–S18.4)** Take another look at the valuations of Rio in Tables 18.1 and 18.2. Now use the live spreadsheets in Connect to show how Rio's value depends on

a. The forecast long-term growth rate.

b. The required amounts of investment in fixed assets and working capital.

c. The cost of capital. (*Note:* You can vary the cost of capital in Table 18.1.)

d. Profitability—that is, cost of goods sold as a percentage of sales.

e. The assumed amount of debt financing.

CHALLENGE PROBLEMS

25. **Miles-Ezzell formula (S18-1)** In footnote 11, we referred to the Miles–Ezzell discount rate formula, which assumes that debt is not rebalanced continuously, but at one-year intervals. Derive this formula. Then use it to unlever Sangria's WACC and calculate Sangria's asset cost of capital. Your answer will be slightly different from the asset cost of capital that we calculated in Section 18-3. Can you explain why?

26. **Terminal value (S18-2)** Modify Table 18.1 on the assumption that competition eliminates any opportunities to earn more than WACC on new investment after year 7 (PVGO = 0). How does the valuation of Rio change?

27. **WACC and rebalancing (S18-4)** The WACC formula assumes that debt is rebalanced to maintain a constant debt ratio *D/V*. Rebalancing ties the level of future interest tax shields to the future value of the company. This makes the tax shields risky. Does that mean that fixed debt levels (no rebalancing) are better for shareholders?

SOLUTIONS TO SELF-TEST QUESTIONS

18.1 Serial Music's debt-to-equity ratio is 1-to-1 in book-value terms. Since Serial's market equity is twice its book equity, its debt-to-equity ratio is 1-to-2 in market-value terms. Thus, its debt-to-value ratio is 1/3. We have WACC = (1 – 0.21)(3.5) × 1/3 + 9.5 × 2/3 = 7.26%.

18.2 It should use WACC. The immediate source of funds is irrelevant. What matters is what shareholders could expect to earn by investing in the capital markets in securities with similar risk to the proposed synthesizer.

18.3 $r_A = r_f + \beta_A(r_m - r_f) = 0.03 + 0.75 \times 0.06 = 0.075$, or 7.5%

So

$r_E = r_A + (r_A - r_D)D/E = 0.075 + (0.075 - 0.04)1.0 = 0.11$, or 11.0%

$\text{WACC} = (1 - t_c)r_D D/V + r_E E/V = (1 - 0.25)(0.04)(1/2) + 0.11(1/2) = 0.07$, or 7.0%

18.4 a. False. The tax shields should be discounted at the asset cost of capital. To use the post-tax WACC would be double-counting.

b. True. Fixed interest tax shields should be discounted at the cost of debt. Take care, however, to check that the debt-financed assets actually support the fixed debt schedule so that the debt does not poach on the debt capacity of other assets.

FINANCE ON THE WEB

Table 18.6 is a simplified book balance sheet for Nike in February 2021. Here is some further information:[21]

Number of outstanding shares (N)	1.27 billion
Price per share (P)	$138
Beta	0.86
Treasury bill rate	0.1%
20-year Treasury bond rate	2.1%
Cost of debt (r_D)	3.5%
Marginal tax rate	21%

Current assets	$24,700	Current liabilities	$ 8,894
Net property, plant, and equipment	8,107	Long-term debt	12,376
Investments and other assets	3,378	Other liabilities	2,984
		Shareholders' equity	11,931
Total	$36,185	Total	$36,185

TABLE 18.6 Simplified Book Balance Sheet for Nike, February 2021 ($ millions)

1. Calculate Nike's WACC. Use the capital asset pricing model and the additional information given above. Make additional assumptions and approximations as necessary.
2. What is Nike's asset cost of capital?
3. Finally, go to **finance.yahoo.com** and update your answers to parts (1) and (2). (You may also find it necessary to check Nike's most recent financial statements.)

[21]Long-term debt includes lease obligations. Property, plant, and equipment includes the value in use of the corresponding leased assets.