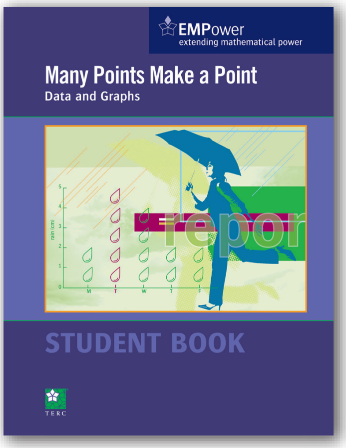
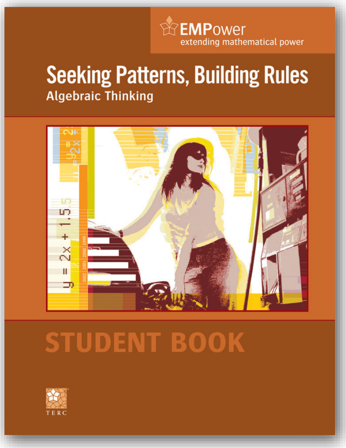
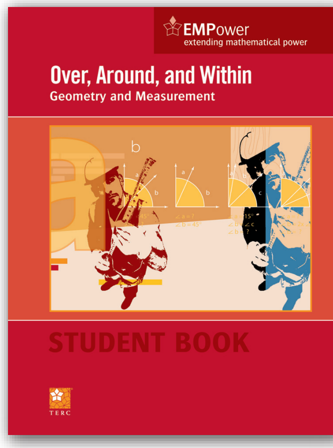
PROGRAM SAMPLER

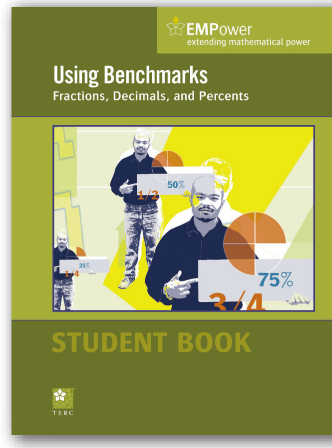
EMPower

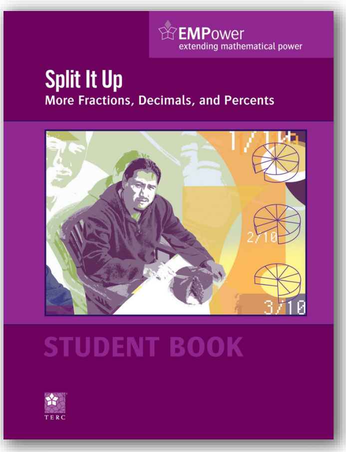
extending mathematical power

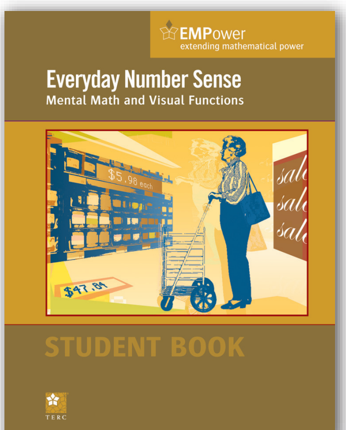


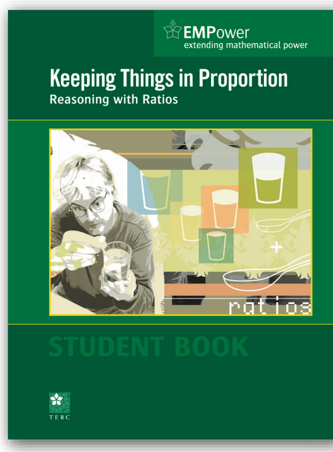


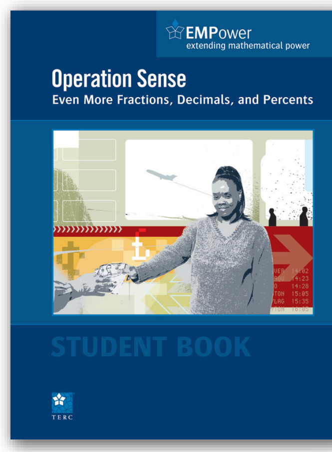












A Program Sampler for Adult Basic Education

This **Program Sampler** will empower **YOU** to:

- **Preview** all eight EMPower titles for Adult Basic Education.
- **Copy and use** the three full activities in this Program Sampler for your classes.
- **Make an adoption or purchasing decision** regarding the EMPower series.

Dear Colleague:

Developing a more numerate population is an essential mission of adult education. When adults take the courageous step to return to the classroom, they deserve the highest quality experience possible, one that respects their styles, intuitions, and experiences and one that acknowledges the roles they play as community members, workers, and parents.

With this in mind, we have looked to leaders in mathematics education. The National Council of Teachers of Mathematics (NCTM) *Principles and Standards for School Mathematics* are based on the principle that "our students deserve and need the best mathematics education possible, one that enables them to fulfill personal ambitions and career goals in an ever-changing world." The *EMPower* units are designed to extend that promise to adults who return to study math as well as to their teachers.

The *EMPower* curriculum changes "business as usual" in four critical aspects: content, sequence, pedagogy, and teacher support.

- **The content is different.** What is "basic" has been rethought through the lens of the mathematics that adults need to be able to do in contemporary society, a culture which depends on flexible, fluent, and accurate command of numerical, statistical, algebraical, and geometrical understandings. These four conceptual strands have been the focus of the NCTM standards and much research in learning styles.
- **The sequence is different**. Algebraical, statistical, and geometrical ideas are developed along with, sometimes before, numerical ones. In this way, the developmental progression has a solid footing in all four strands.
- **The pedagogy is different.** Classrooms are learning communities, where participants share strategies and results of mathematical investigations.
- **The teaching experience is different.** Their role is supported by many elements of the Teacher Book, an essential component of the *EMPower* program.

In spirit and in practice, EMPower authors/staff, as well as the Publisher, join with teachers who wish to offer a curriculum that is mathematically rich, and personally relevant to adult learners. We, along with the authoring team, welcome teachers with varying levels of math background to join us in helping to change the face of basic math teaching to a more active and empowering one for adult learners and teachers. Adult learners deserve the best!

Sincerely,

EMPower Project Co-directors

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TERC is a not-for-profit education research and development organization dedicated to improving mathematics, science, and technology teaching and learning.

http://empower.terc.edu

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Contents

This Sampler provides you with an overview of EMPower, and gives you Student and Teacher materials for one lesson from three of the eight units. You will also find an in-depth description of each unit, as well as a correlation grid detailing the mathematical concepts covered in each unit and lesson.

Contents

Introduction to the EMPower Curriculum

Background

*Extending Mathematical Power (*EMPower*)* integrates recent mathematics education reform into the field of education for adults and out-of-school youth. EMPower was designed especially for those students who return for a second chance at education by enrolling in remedial and adult basic education programs, high school equivalency programs, and developmental programs at community colleges. However, the curriculum is appropriate for a variety of other settings as well, such as high schools, workplaces, and parent and paraprofessional education programs. EMPower builds interest and competency in mathematical problem solving and communication.

Over the course of four years (2000–2004), a collaboration of teachers and researchers with expertise in adult numeracy education and K–12 mathematics reform developed and piloted eight contextualized curriculum units. These units are organized around four central topics: number and operation sense; patterns, functions, and relations; geometry and measurement; and data and graphs. The EMPower program serves as a model for a cohesive mathematics curriculum that offers content consistent with the *Principles and Standards for School Mathematics* (NCTM, 2000), as well as frameworks that are adult-focused, such as the *Equipped for the Future Content Standards* (Stein, 2000), the *Massachusetts ABE Curriculum Frameworks for Mathematics and Numeracy* (Massachusetts Department of Education, 2001), and the *Adult Numeracy Network's Framework for Adult Numeracy Standards* (Curry, Schmitt, & Waldron, 1995). The curriculum fosters a pedagogy of learning for understanding; it embeds teacher support and is transformative, yet realistic, for multilevel classrooms.

EMPower challenges students and teachers to consistently extend their ideas of what it means to do math. The curriculum focuses on mathematical reasoning, communication, and problem solving with a variety of approaches and strategies, not just rote memorization and symbol manipulation. The program fosters a learning community in which students are encouraged to expand their understanding of mathematics through open-ended investigations, working collaboratively, sharing ideas, and discovering multiple ways for solving problems. The goal of EMPower is to help people build experience managing the mathematical demands of various life situations, such as finances and commerce, interpretation of news stories, and leisure activities, and to connect those experiences to mathematical principles.

A Focus on Mathematical Content

The EMPower curriculum supports students' and teachers' growth by directing attention to significant mathematical understandings.

EMPower emphasizes:

- Data analysis, geometry and measurement, algebra, and number and operations at all student levels.
- Reliance on benchmark numbers—such as powers and multiples of 10, common fractions, and their decimal and percent equivalents—for making mental calculations.
- Early use of calculators to support computation.
- Development of reasoning on proportion and parts of quantities before consideration of formal operations with rational numbers.
- Making decisions about data where students generate, as well as interpret, graphical representations.
- Geometry and measurement based on opportunities to see and touch in developing an understanding of spatial relationships and formulas.
- Leading with patterns and relationships in contextual situations and the representations of these situations with diagrams, tables, graphs, verbal rules, and symbolic notation to develop algebraic competence.

A Focus on Pedagogy

Mathematics is meaningful within a social context. While mathematical truths are universal, the meaning and relevance of numbers changes according to the setting and culture. Therefore, the EMPower pedagogy is focused on sets of connected activities that require communication and discourse.

EMPower asks students to

- Work collaboratively with others on open-ended investigations;
- Share strategies orally and in writing; and
- Justify answers in multiple ways.

Key features of curriculum activities provide

- Clear mathematical goals
- Contexts that are engaging and useful for young people and adults
- Opportunities to strengthen mathematical language and communication skills
- Various ways of entering and solving problems
- Puzzles that draw students into problems and motivate them to seek a solution.

Changing the Culture

The authors have created this curriculum to follow the National Council of Teachers of Mathematics (NCTM) Principles and Standards; however, every teacher who uses EMPower faces the challenge of transforming the prevailing culture of his or her math classroom. EMPower pilot teachers offer some ideas for facilitating this transition:

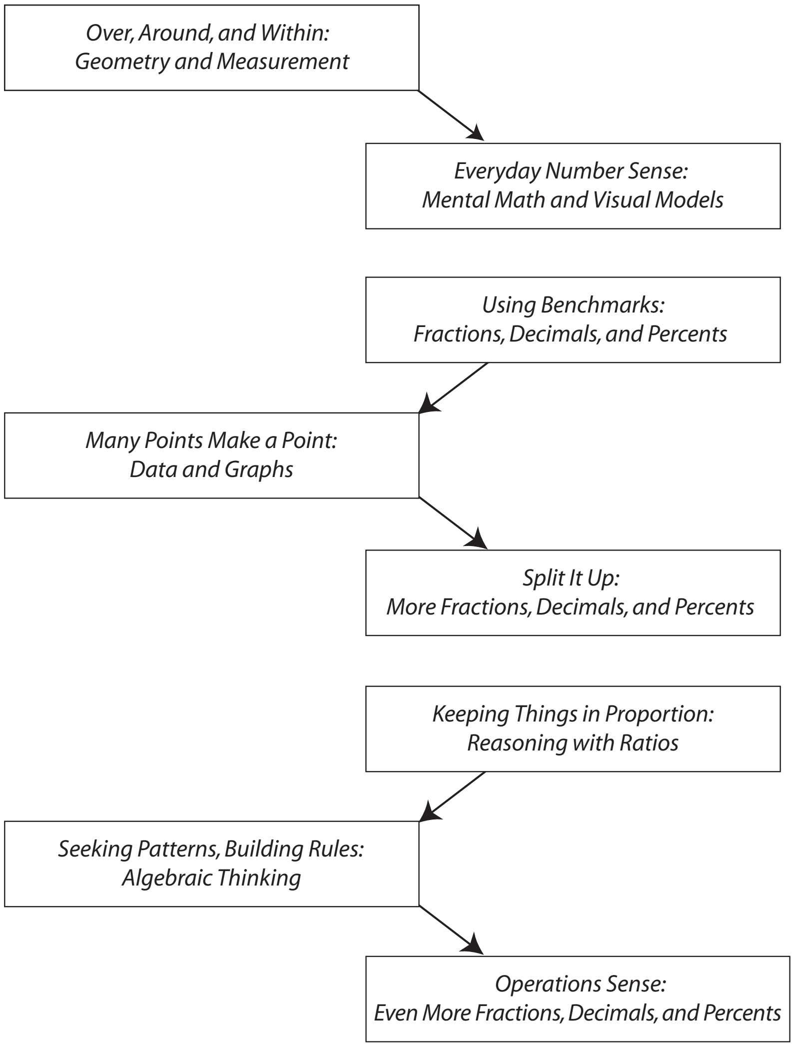
- Set the stage. As a class, set ground rules. Explicitly state that this is a space for everyone to learn. As one teacher said, "We are in this together. Share, even if you do not think you are right. Whatever you add will be helpful. It lets us see how you are looking at things."
- Group your students. Match students whose learning styles and background knowledge complement each other. Ask questions, such as, "How did it go to work together?" "How did everyone contribute?"
- Allow wait time. Studies have shown that teachers often wait less than three seconds before asking another question. Students need more time to think.
- Sit down. Watch students before interrupting to help them. Listen for logic and evidence of understanding. Follow the thread of students' thinking to uncover unconventional approaches. During discussions with the whole group, hand over the chalk.
- Review written work. Look beyond right and wrong answers to learn everything you can about what a student knows. Determine what seems solid and easy, as well as patterns in errors. If students are scattered, suggest ways they can organize their work; this is likely to lead to more efficient problem solving and clearer communication.
- Question. Hearing the right answer is not necessarily a cue to move on. Question students at this point too. Specific questions are included in the lesson facilitation.

Unit Sequences and Connections

The sequence in which the EMPower units can be used effectively with your class will depend on the backgrounds and interests of students. The units are not numbered so teachers can order them according to their class needs; however, the authors suggest specific unit arrangements that will support students' progression through certain concepts.

The authors do not recommend sequencing the units according to the traditional basic math model that begins with whole numbers, follows with fractions, decimals, and percents; data and graphs; algebra; and then geometry. Instead, the authors suggest you integrate the five units that focus on numbers with the units on geometry, data, and algebra. The authors found this integration of topics helped to motivate the adult students in their pilot classes.

Although the units were not specifically designed to build on one another, there are clear connections between some of the units in the series. *Over, Around, and Within: Geometry and Measurement* provides a nice introduction to the program because it focuses on small whole numbers. *Everyday Number Sense: Mental Math and Visual Models* could follow to further develop whole number mental math skills and visual models. *Using Benchmarks: Fractions, Decimals, and Percents* provides the necessary groundwork with fractions, decimals, and percents to describe approximate relationships between data sets in *Many Points Make a Point: Data and Graphs*. And *Split It Up: More Fractions, Decimals, and Percents* continues to expand students' repertoire of familiar fractions, decimals, and percents. *Seeking Patterns, Building Rules: Algebraic Thinking* builds upon the tools and relationships used in *Keeping Things in Proportion: Reasoning with Ratios*; finally, *Operations Sense: Even More Fractions, Decimals, and Percents* introduces more complex fractions and operations in geometric, graphic, and algebraic contexts. The following diagram demonstrates this integrated sequence:



Unit Descriptions

Over, Around, and Within: Geometry and Measurement
Students explore the features and measures of basic shapes. Perimeter and area of two-dimensional shapes and volume of rectangular solids provide the focus.

Everyday Number Sense: Mental Math and Visual Models
Students solve problems and compute with whole numbers using mental math strategies with benchmarks of 1, 10, 100, and 1,000. Number lines, arrays, and diagrams support their conceptual understanding of number relationships and the four operations.

Using Benchmarks: Fractions, Decimals, and Percents
Students use the fractions $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, and $\frac{1}{10}$; the decimals 0.1, 0.5, 0.25, and 0.75; and the percents 50%, 25%, 75%, 100%, and the multiples of 10% as benchmarks to describe and compare all part-whole relationships.

Many Points Make a Point: Data and Graphs
Students collect, organize, and represent data using frequency, bar, and circle graphs. They use line graphs to describe change over time. They use benchmark fractions and the three measures of central tendency—mode, median, and mean—to describe sets of data.

Split It Up: More Fractions, Decimals, and Percents
Building on their command of common benchmark fractions, students add thirds, eighths, hundredths and their decimal and percent equivalents to their repertoire of part-whole relationships.

Keeping Things in Proportion: Reasoning with Ratios
Students use various tools—objects, diagrams, tables, graphs, and equations—to understand proportional and nonproportional relationships.

Seeking Patterns, Building Rules: Algebraic Thinking
Students use a variety of representational tools—diagrams, words, tables, graphs, and equations—to understand linear patterns and functions. They connect the rate of change with the slope of a line and compare linear with nonlinear relationships. They also gain facility with and comprehension of basic algebraic notation.

Operation Sense: Even More Fractions, Decimals, and Percents
Students extend their understanding of the four operations with whole numbers as they puzzle over questions such as "How is it possible that two fractions multiplied might yield a smaller amount than either fraction?" and "What does it mean to divide one-half by six?"

Frequently Asked Questions

Q: *I have classes that are widely multilevel. Can this work?*

A: Many teachers see a wide range of levels within the group as an obstacle. Turn the range of levels to your advantage. Focus on students' representations (words, graphs, equations, sketches). This gives everyone the chance to see that answers emerge in several ways. Slowing down deepens understanding and avoids facile responses. Having calculators available can even the playing field. Implement the suggestions in *Making the Lesson Easier* and *Making the Lesson Harder* of each lesson facilitation in the *Lesson Commentary* sections.

Q: *How do I deal with erratic attendance patterns?*

A: Uneven attendance can be disruptive. Students who miss class may feel disoriented; however, the lessons spiral back to the most important concepts. When the curriculum circles back, students will have a chance to revisit concepts and get a toehold.

Q: *What do I do if I run out of time, and there is no way to finish a lesson?*

A: Each activity is important, but reviewing it is equally important. It is better to cut the activity short so there is time to talk with students about what they noticed. Maximize the time by selecting a student or group whose work you feel will add to the class's understanding to report their findings. Be conscious of when you are letting an activity go on too long because the energy is high. Fun is good, but be sure important learning is happening. If you like to give time in class to reviewing homework, and you want to hear from everyone in discussions, you will run out of time. Schedule a catch-up session every three or four lessons.

Q: *How do I respond to comments such as "Can't we go back to the old way?"*

A: Change is unsettling, especially for students who are accustomed to math classes where their job is to work silently on a worksheet solving problems by following a straightforward example. Be clear about the reasons why you have chosen to de-emphasize some of the traditional ways of teaching in favor of this approach. Ultimately, you may need to agree to some changes to accommodate students' input. Meanwhile, stick with the curriculum. Reiterate for students what they have accomplished. When there is an "Aha!" moment, point it out.

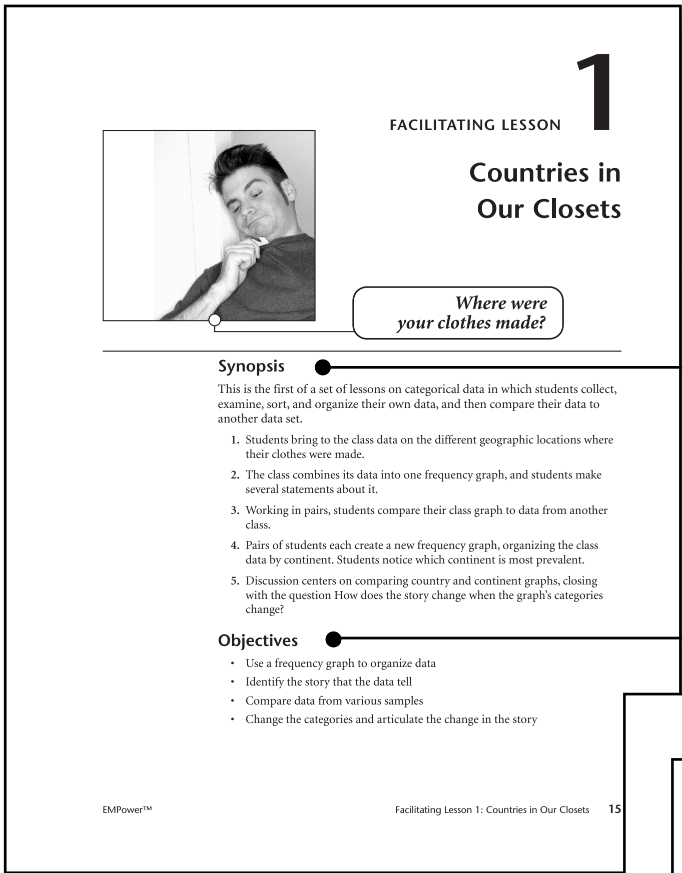
Q: *My own math background is not strong. Will I be able to teach this curriculum?*

A: Yes! Most teachers tend to teach the way they were taught. Adopting a different stance requires support, and the more types of support, the better. This curriculum offers support in a few ways. The teacher books for each unit list open-ended questions designed to keep the math on track. In the *Lesson Commentary* sections, *Math Background* helps teachers deepen their understanding of a concept. In addition, the *Lesson in Action* sections provide examples of student work with comments that illuminate the underlying mathematics.

The best support often comes from a colleague. If no one at your site is currently teaching EMPower, join the Adult Numeracy Network, http://shell04.theworld.com/std/anpn, and attend your regional NCTM conference. Look for others who are integrating the NCTM Principles and Standards through the use of a curriculum such as *Investigations in Number, Data, and Space* (TERC, 1997); *Connected Mathematics* (Lappan et al., 1998); or *Interactive Mathematics Program* (Alper et al., 1997).

Overview of EMPower Units

Features of the Teacher Book



FACILITATING LESSON **1**

Countries in Our Closets

Where were your clothes made?

Synopsis

This is the first of a set of lessons on categorical data in which students collect, examine, sort, and organize their own data, and then compare their data to another data set.

1. Students bring to the class data on the different geographic locations where their clothes were made.
2. The class combines its data into one frequency graph, and students make several statements about it.
3. Working in pairs, students compare their class graph to data from another class.
4. Pairs of students each create a new frequency graph, organizing the class data by continent. Students notice which continent is most prevalent.
5. Discussion centers on comparing country and continent graphs, closing with the question How does the story change when the graph's categories change?

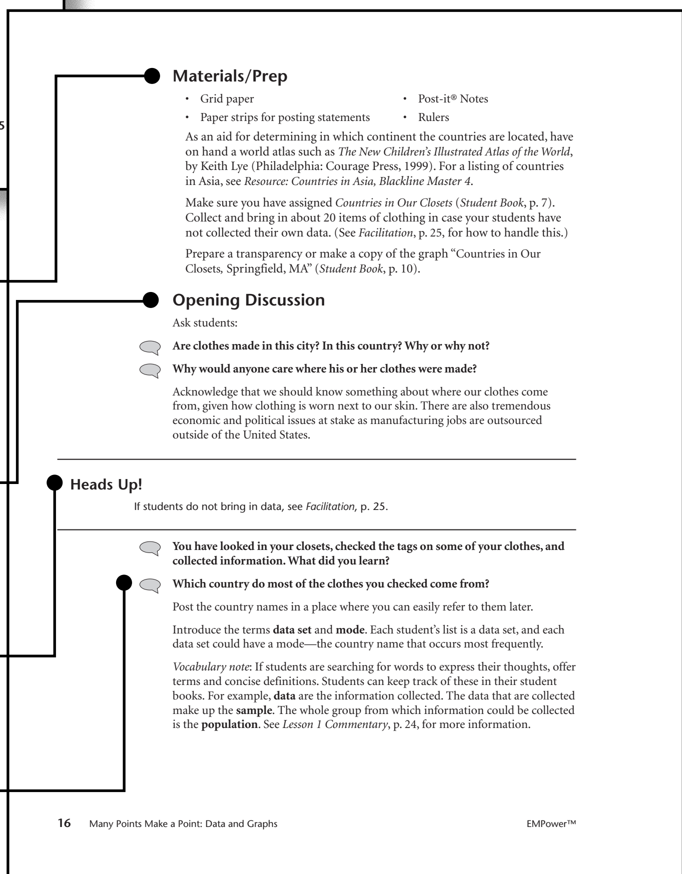
Objectives

- Use a frequency graph to organize data
- Identify the story that the data tell
- Compare data from various samples
- Change the categories and articulate the change in the story

EMPower™ Facilitating Lesson 1: Countries in Our Closets 15

Each lesson opens with a **synopsis** of the lesson activities.

Objectives summarize the skills students should gain from the lesson.



Materials/Prep

- Grid paper
- Paper strips for posting statements
- Post-it® Notes
- Rulers

As an aid for determining in which continent the countries are located, have on hand a world atlas such as *The New Children's Illustrated Atlas of the World*, by Keith Lye (Philadelphia: Courage Press, 1999). For a listing of countries in Asia, see *Resource: Countries in Asia, Blackline Master 4.*

Make sure you have assigned *Countries in Our Closets* (*Student Book*, p. 7). Collect and bring in about 20 items of clothing in case your students have not collected their own data. (See *Facilitation*, p. 25, for how to handle this.)

Prepare a transparency or make a copy of the graph "Countries in Our Closets, Springfield, MA" (*Student Book*, p. 10).

Opening Discussion

Ask students:

Are clothes made in this city? In this country? Why or why not?

Why would anyone care where his or her clothes were made?

Acknowledge that we should know something about where our clothes come from, given how clothing is worn next to our skin. There are also tremendous economic and political issues at stake as manufacturing jobs are outsourced outside of the United States.

Heads Up!

If students do not bring in data, see *Facilitation*, p. 25.

You have looked in your closets, checked the tags on some of your clothes, and collected information. What did you learn?

Which country do most of the clothes you checked come from?

Post the country names in a place where you can easily refer to them later.

Introduce the terms **data set** and **mode.** Each student's list is a data set, and each data set could have a mode—the country name that occurs most frequently.

Vocabulary note: If students are searching for words to express their thoughts, offer terms and concise definitions. Students can keep track of these in their student books. For example, **data** are the information collected. The data that are collected make up the **sample.** The whole group from which information could be collected is the **population.** See *Lesson 1 Commentary*, p. 24, for more information.

16 Many Points Make a Point: Data and Graphs EMPower™

The **Materials/Prep** feature tells teachers what materials they will need to prepare for the day's activities.

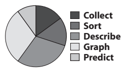
The **Opening Discussion** gives teachers ideas for engaging students at the beginning of the lesson.

Heads Up! alerts teachers to helpful hints and things to look for in facilitating the lesson.

Dialog balloons and bold text suggest statements and questions teachers can use to spark and guide class discussion.

Summary Discussion

DATA



Finish the lesson by posing the following questions. (Students may not have ready answers at this point, but the questions will be a thread through the rest of the lessons.)

What stories can be told from this set of data (or what questions can be asked and answered)?

How does organizing the information into different categories impact how the data may be interpreted?

Who would be curious about what the people in this class eat?

Direct student to the *Reflections* questions (*Student Book*, p. 145).

Practice

Take a Sample, p. 24
For practice choosing a representative sample.

Thirsty and *Ouch!*, pp. 26 and 28
For practice organizing information and making statements.

Extension

Friends and Family Drink Too, p. 30

Data Collection, p. 31
Students begin recording what they eat for a project they will do in a later lesson. For now, they just write down all the foods they eat throughout the day.

Test Practice

Test Practice, p. 32

Looking Closely

Observe whether students are able to

Organize data into consistent categories

Do students group similar foods together? Make sure students are moving index cards around and that they are writing down the category names. If students seem stuck, have them ask you specific questions.

If you see foods categorized incorrectly, ask a question such as "Do all group members agree with the categories?" Encourage students to think, question, and defend their reasoning. If students use overlapping categories such as "cereal" and "grain," present an example close to their own experience. Say, for instance, "It is

EMPower™ Facilitating Lesson 2: Most of Us Eat… 33

A **Summary Discussion** guides the conclusion of the lesson.

Practice, Extension, and **Test Practice** sections list and describe choices for additional work.

Looking Closely draws the teacher's attention to the goals and suggests teaching strategies.

LESSON 1 COMMENTARY

Rationale

The lesson takes students through the first steps of displaying data: collecting and organizing the information. As you work through this lesson, you and your students will notice that each time you categorize and recategorize the data set, you tell a different story.

Math Background

Data are information. This information may be numerical—e.g., salaries, test scores, heights, age, weight—as well as categorical—e.g., countries, foods we eat, types of product defects.

Most data are reported using statistics based on fractions and percents—one-third of pregnant mothers, half of native plants, etc. Commonly used, or benchmark, fractions and percents are often invoked to influence others to make decisions. When the headlines shout, "One-half of all pregnant women …," readers tend to form opinions based on the data. What readers often neglect to consider is the size of the sample, where the data were collected, and how the data were organized.

The media generally report data that are based on a sample. It is rare for any group to have the time or resources to ask every person in the population to contribute information. The idea of sampling is to study a part in order to gain information about the whole. The sample—its size and characteristics—influences the data and the conclusions of the study.

A representative sample by definition includes representation across the population being surveyed. For example, a representative sample of the community would include individuals from across town, not just one neighborhood. Likewise, a representative sample across America would include individuals from many different states and regions, not just one or two. The size of the sample is important when inferences are made. For example, 30 samples could be sufficient for a population of several hundreds. The deciding factor for
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EMPower™

Math Background helps teachers deepen their understanding of mathematical concepts.

EMPower™

WHAT TO LOOK FOR IN *LESSON 1*	WHO STANDS OUT? (LIST STUDENTS' INITIALS)			NOTES FOR NEXT STEPS
	STRONG	ADEQUATE	NEEDS WORK	
Concept Development • Records all data • Organizes data • Recognizes key features of frequency graph • Compares data • Changes categories and notes change in story				
Expressive Capacity • Uses precise language to describe the frequency graphs • Contributes to the discussion about the way changing categories affects the story the data tell				
Use of Tools • Uses graph paper • Uses atlas or other reference				
Background Knowledge • Draws on understanding of geography, relative size, and number sense for comparisons				

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The **What to Look For** grid allows teachers to track students' progress and think ahead about next steps.

The authors give ideas for **Making the Lesson Easier** and **Making the Lesson Harder**.

Context

Some students may know about *maquiladoras* in Mexican border towns, where women make clothes for very little money and with no benefits or environmental Occupational Safety and Health Administration (OSHA) workplace protections. CorpWatch (www.corpwatch.org) is one source for information on *maquiladoras*.

Facilitation

If students do not bring in data, or if their sample is too small, skip the second part of the *Opening Discussion*. Have available a pile of 20 clothing articles with labels. First, ask students to predict where the clothes were made. Post the list of their guesses. Note that it will be hard for them to answer this question unless they organize the information on the labels. Then divide up the 20 articles of clothing. Have students write the name of the country for each piece of clothing on a Post-it Note, one country name per note. Ask: "Where are most of our clothes made?" Then continue with the activity.

Making the Lesson Easier

Frequency graphs lend themselves to comparisons among categories. If students have little fluency stating comparisons, you may choose only to compare size, using terms like "greater," "fewest," or "less than." For students who are encountering data formally for the first time, the notion that collapsing data yields different stories may be difficult. Treat this lightly in the activity, and revisit such questions after students have more experience categorizing and recategorizing data in the homework and in *Lesson 2*.

Making the Lesson Harder

If your students can handle benchmark fractions and percents, get them to look critically at the data, including the source and sample size. You might ask:

If we asked another class what countries are in their closets, what do you think would happen to the categories? What if we asked the entire community?

How do you think your data would compare to data from another class of adult students in another community?

If students struggle with the idea of sample, you might try this: Have them each write their favorite color on a Post-it Note. If you have a small class, ask them to write the color on two Post-it Notes. Place all of the notes in a container. Have someone randomly (eyes closed) choose a few notes from the container and place them across a line to form a frequency graph. Ask the students how they think this sample compares to the actual total number of colors on notes in the container. You can have them do another frequency graph to compare the sample to the actual total.

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LESSON 1 IN ACTION

Alice articulates the mathematical principle behind compressed data.

I asked, "How did the change in categories affect what we noticed about the data?"

Alice answered, "Well, we keep losing information."

"How so?"

Patiently, Alice explained that when we started our work, every bit of data was visible. She added that we had lost details initially recorded. "At first, we knew every country in every person's closet and how many pieces of clothing came from that country. Then we combined the data, and we lost track of who had which countries. Then we did it by continent, and we lost track of all the countries."

Alice's realization quickly gained agreement from the rest of the class. After all, just the previous week a classmate had noted, "When you change the amount of data you look at, you find different things."

Sonia added her comment with increased conviction: "It is like politics. Politicians use a graph and tell you this is true, but you look at the graph, and it does not tell you everything."

Tricia Donovan
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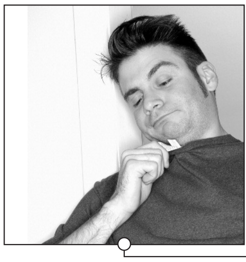
26 Many Points Make a Point: Data and Graphs EMPower™

In **Lesson in Action**, EMPower teachers share their classroom experiences.

Overview of EMPower Units

Features of the Student Book

LESSON **1**



Countries in Our Closets

Where were your clothes made?

In a world where information is at our fingertips, it is important to be able to make sense of it. What does it tell us? Why do we care?

Begin with your own clothes. Where were they made? Find out! You will collect **data** by reading the labels on your clothes. You will organize that data, make statements about it, and note where most of you[r clothes] were made.

You will then try organizing the data in a different way. Ho[w does grouping] the data in new groups change the story of the countries in[...] You will compare your class data to data of other classes. W[...] be the same?

On Your Own

I think the country in which most of my clothes were made[...]

Look at the labels of at least eight items of clothing. Make a[...] track of the information you are collecting. Research questi[on...] were your favorite clothes made?

EMPower™ Lesson 1: Countries in C[...]

An **introduction** explains the purpose and content of the lesson.

Activity 1: Organizing the Data

Look at the data you collected about countries where your clothes were made.

Organize the data.

1. What strikes you about your data?

2. What is the **mode** for your data?

3. What is the mode for the class data?

8 Many Points Make a Point: Data and Graphs EMPower™

Vocabulary words are easy to identify in bold type.

The class **activity** comes next. Often students work with a partner or in a small group.

Practice: Categorically Speaking

Sometimes we can organize categories into subgroups of other categories in order to better understand the data. For example, drug stores will often organize their aisles by category—skin care, beauty aids, and so on. Then the items in each of those aisles are also organized. This makes it easier to take inventory and place orders and for the customer to find products.

Can you think of other examples where information is categorized?

Use the chart below *or* create your own chart; draw a picture; make a list; or write about examples you see at home or at work. In the chart below, one example is given.

Location	What Stuff or Information?	How Is It Organized?
Drug store	Over-the-counter products they sell	Skin care Stationery Beauty aids First aid Seasonal Hair care

EMPower™ Lesson 1

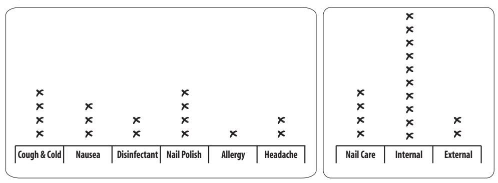
Practice pages invite students to use new skills.

Extension: Taking Inventory

Find a drawer or closet filled with many things. Use a frequency graph to show the contents. Start with six categories. Then do another frequency graph to show the same items, but this time use three categories.

For example:

My Medicine Cabinet



16 Many Points Make a Point: Data and Graphs EMPower™

Extension pages present a challenge or a new but related math idea.

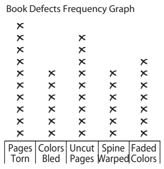
Ample space allows students to record most work in their books.

Test Practice

A **Test Practice** that reflects the format of the GED test concludes each lesson.

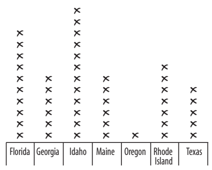
1. Freda worked at a printing press. She kept a tally of all the defects she found in the books she was processing. Based on the frequency graph she created, shownbelow, which of the following statements is true?
   (1) There were more defects related to colors than any other defect.
   (2) There were twice as many torn pages as there were warped spines.
   (3) There were more warped spines than colors that bled.
   (4) Half of all the defects were related to colors that bled.
   (5) Half of all the defects were related to pages that were torn.

Book Defects Frequency Graph



2. Tony, a tour guide, has been keeping a tally of visitors from different states. According to his tally, which of the following is *not* a true statement?
   (1) The fewest number of visitors came from Oregon.
   (2) There were as many visitors from Maine as there were from Georgia.
   (3) There were as many visitors from Georgia as there were from Oregon and Texas.
   (4) There were twice as many visitors from Florida as there were from Rhode Island.
   (5) There were twice as many visitors from Idaho as there were from Maine.

U.S. Tourist Frequency Graph



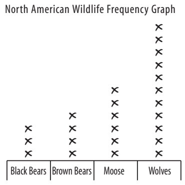
3. Clara is paid a commission on each of the large electronics devices that she sells. She tallied the different items she sold for the month. Based on her tally, what can she tell her boss about her sales?
   (1) She sold twice as many DVD players as she did TVs.
   (2) She sold twice as many DVD players as she did computers.
   (3) She sold more DVD players than she did TVs and computers combined.
   (4) She sold more TVs and VCRs than she did stereos and DVD players combined.
   (5) She sold half as many TVs as she did stereos.

Home Media Frequency Graph

EMPower™

4. Bronson, a forest ranger in the Green Mountains, kept track of the different animals that were reportedly seen in one of the campgrounds during the month of May. Based on his tally, which of the following statements could he tell the media?
   (1) There were twice as many wolves reportedly seen as bears.
   (2) Half of all the reported sightings were wolves.
   (3) There were more bears than wolves reportedly seen.
   (4) There were twice as many moose reportedly seen than there were black bears.
   (5) One-quarter of all reported sightings were brown bears.

North American Wildlife Frequency Graph



5. According to Global Exchange, some workers in China who make clothing for Disney are paid as little as $0.16 per hour. At this wage, what amount would a worker make for a 40-hour workweek?
   (1) $64.00
   (2) $6.40
   (3) $6.00
   (4) $80.00
   (5) $0.60

6. In 2003 the U.S. Department of Labor (DOL) issued new regulations for overtime pay. The DOL estimated that under the new regulations 1.3 million low-wage workers would become eligible for overtime pay, unless their wages were raised to $425 per week. The DOL estimated that 24.8 percent of those workers were Hispanic and 16.6 percent were African American. What percent were neither Hispanic nor African American?

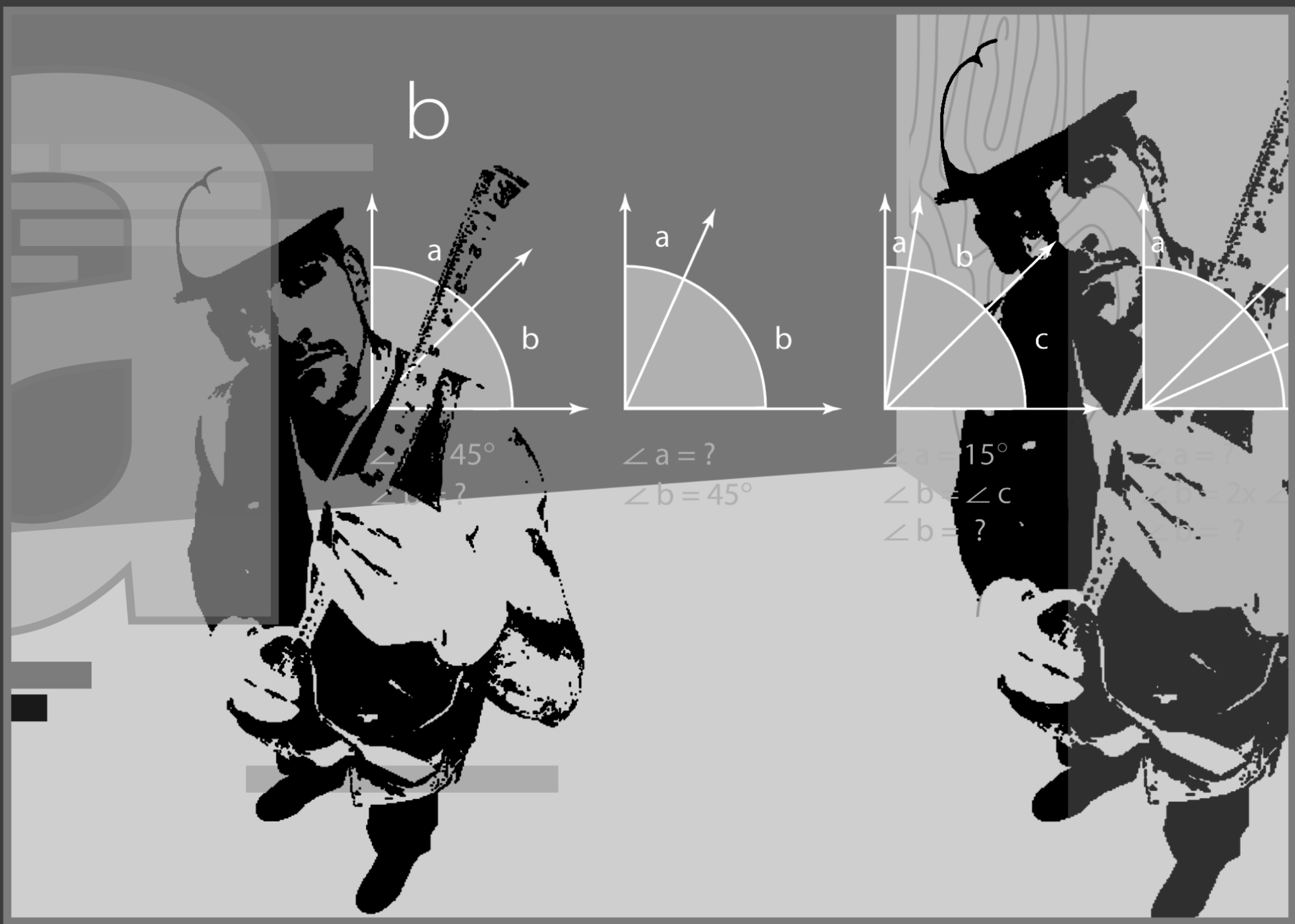
18 Many Points Make a Point: Data and Graphs EMPower™

EMPower

extending mathematical power

Over, Around, and Within

Geometry and Measurement



TEACHER BOOK

TERC

Over, Around, and Within: Geometry and Measurement

Everyone has some experience with geometry and measurement. In this unit, students build upon their knowledge as they encounter increasingly complex dilemmas about the nature of shapes, the measures of perimeter, area, and volume, as well as linear-, square-, and cubic-unit measurements, both metric and U.S. customary. They learn to speak the language of geometry as they share secret designs and become increasingly familiar with shape attributes. Angles, and in particular right (90°) and straight angles (180˚) take center stage as students explore optimum reading angles and the use of protractors. They then proceed on a series of investigations regarding perimeter, area, and volume. Along the way, they learn about similar shapes, scale, and units of measure. The unit closes with an examination of surface area and volume. Assessments involve general review as well as practical applications of knowledge where, for instance, students plan to re-decorate their classroom or are asked to design a box fitting established criteria.

With its heavy emphasis on hands-on activities and mathematical discourse, *Over, Around, and Within* offers a welcoming context in which students develop a firmly grounded understanding of the often mysterious – angle relationships, unit differences and conversions, and interplay of dimensions in determining perimeter, area, and volume measures. Gradual shifts in emphasis allow students to move from intuitive to formal methods of measuring and comparing shapes and objects. No one memorizes formulas. Everyone understands them. Students will never see the world and its objects in the same way after completing this unit.

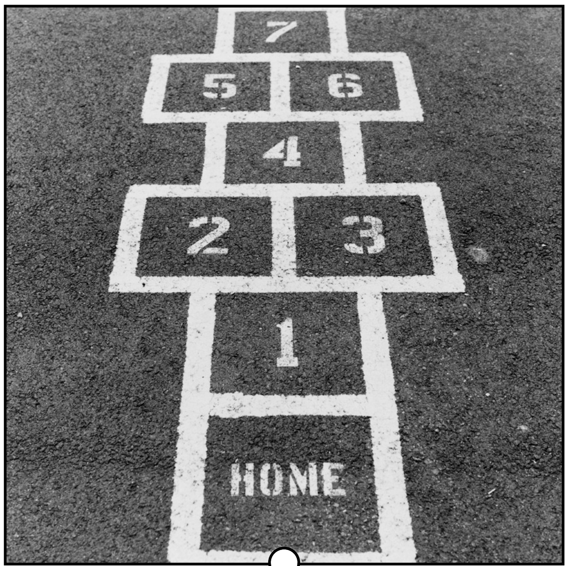
Correlations for *Over, Around, and Within: Geometry and Measurement*

Book Description: Students explore the features and measures of basic shapes. Perimeter and area of two-dimensional shapes and volume of rectangular solids provide the focus.		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/Topics Covered:**
Opening the Unit	Geometry Groundwork	• Shapes identified and sketched • Angles introduced • Geometry vocabulary list started • Prior Geometry knowledge assessed
Lesson 1:	Sharing Secret Designs	• Two-dimensional shape characteristics identifed • 12 Basic Geometric Shapes identified and described
Lesson 2:	Get It Right	• Angles identified and described with conventional notation • Right angles introduced • Angle measurements estimated with 90° benchmark and determined precisely with protractors
Lesson 3:	Get it Straight	• Straight (180°) angles explored • Sums of angles in triangles and rectangles established
Lesson 4:	Giant-Size	• Similar shapes identified and described • Length and width dimensions introduced and measured • Perimeters determined by adding
Interim Assessment 1	Shapes and Angles	• Attributes of shapes' and angle measurements' knowledge assessed
Lesson 5:	Line Up by Size	• Area and perimeter distinguished
Lesson 6:	Combining Rectangles	• Rectangle area calculated in square centimeters • Composite shapes' areas and perimeters compared
Lesson 7:	Disappearing Grid Lines	• Formulas for area and perimeter derived • Missing dimension values determined • Area of a right triangle calculated
Lesson 8:	Conversion Experiences	• Standard English Units introduced • Linear unit conversions established

Lesson 9:	Squarely in English	• Square units – square inches, feet, and yards constructed and connected with area measure • Square unit conversions established
Lesson 10:	Scale Down	• Scale drawings made and steps for scaling analyzed
Interim Assessment 2	A Fresh Look	• Area, perimeter, measurements, and scale knowledge applied and assessed
Lesson 11:	Filling the Room	• Volume explored as capacity • Third dimension – height becomes apparent
Lesson 12:	Cheese Cubes, Anyone?	• Cubic inch introduced then used to measure volume • Volume formula derived
Lesson 13:	On the Surface	• Surface area and volume compared • Surface area and shape relationship generalized
Closing the Unit	Design a Box	• Geometry and measurement knowledge applied and assessed

LESSON **6**

Combining Rectangles

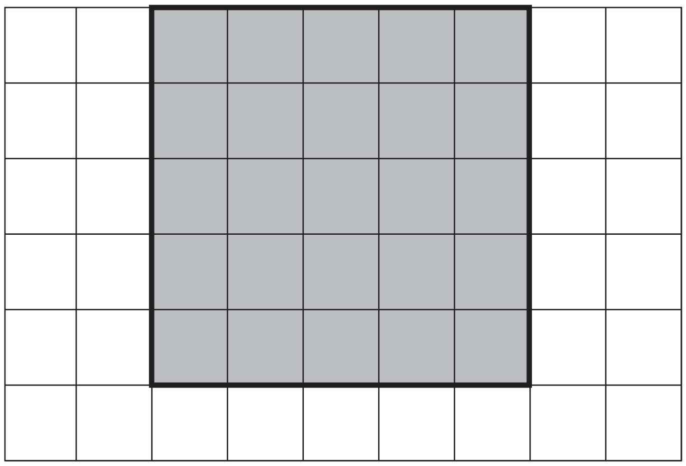


How much area does the hopscotch game take?

In this lesson, you will draw rectangles and then combine them to form a new, composite shape. When you compare measurements of the original rectangles with those of your new shape, you will see how combining rectangles affects area and perimeter.

Rectangles are among the most basic shapes in the world. You see them, for example, in boxes, buildings, street blocks, photographs, and magazines. When you know how to measure rectangles, you can use that information to make things, as well as to solve problems.

You cannot measure area with a ruler because rulers measure length in *lines*. Area is measured in *square* units. How many **square centimeters** (sq. cm) are inside this shape?



Activity 1: Drawing Four Rectangles

- Draw a 5 cm x 10 cm rectangle on square-centimeter grid paper. Label it "Rectangle 1." Draw three more rectangles of different dimensions on the grid paper. Label them "Rectangles 2, 3, and 4."
- Record the measurements in the table below. Use *cm* for length, width, and perimeter measures and *sq. cm* for area.

	Length (*l*) in cm	**Width (*w*) in cm**	**Area (*A*) in sq. cm**	**Perimeter (*P*) in cm**
Rectangle 1				
Rectangle 2				
Rectangle 3				
Rectangle 4				
All Rectangles Combined				

After you are done, ask a partner to check your measurements. Make sure you both agree the information you each recorded is accurate.

Activity 2: Making a Composite Shape

Imagine cutting out the four rectangles and then arranging them to make one combined shape.

1. Do you predict the area of the new combined shape will be larger, smaller, or the same as the total area of the four shapes you started with?

2. Do you predict the perimeter of the new combined shape will be larger, smaller, or the same as the sum of the perimeters of the four rectangles you started with?

Now actually cut out your four rectangles and tape them together carefully to make one new combined shape. *No overlaps, no gaps!*

3. What is the area of the new shape? How do you know?

4. What is the perimeter of the new shape? How do you know?

5. How many sides does the shape have?

6. How many angles does the shape have? Are they all 90° angles?

My Composite Shape (a sketch)	Area (*A*) sq. cm	Perimeter (*P*) cm	Number of Angles	Number of Sides

Practice: Area of 24 Sq. Cm

1. How many different rectangles can you design that have an area of 24 square centimeters?
   - Use square-centimeter grid paper to draw at least five rectangles that have an area of 24 sq. cm.
   - Find the perimeter of each of these shapes.
   - Complete the chart below.

2. Which rectangle has the smallest perimeter? ________
   Describe its shape.

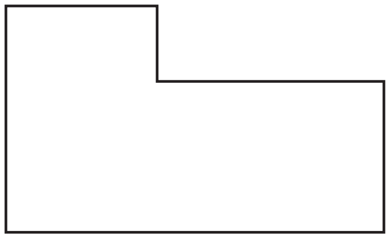
3. Which rectangle has the largest perimeter? ________
   Describe its shape.

Rectangle Dimensions (length and width)	Perimeter of Rectangle (Show or explain work.)	Area of Rectangle
A. l = w =		24 sq. cm
B.		
C.		
D.		
E.		

Practice: Divide the Shapes

Add lines to the shapes below to show how to make finding the area easy.

1.



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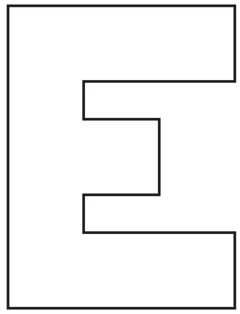

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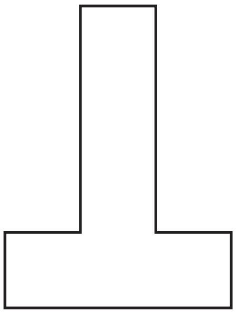
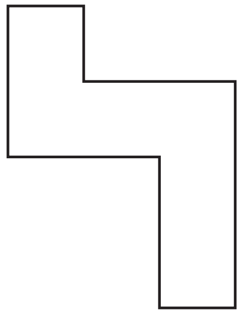


4.



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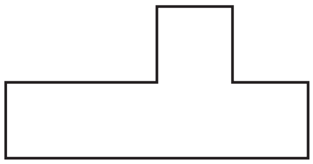

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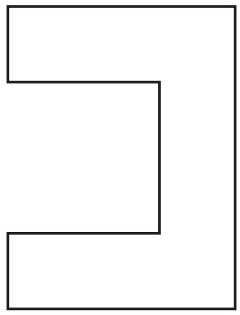
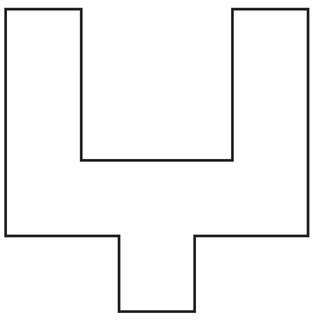


7.



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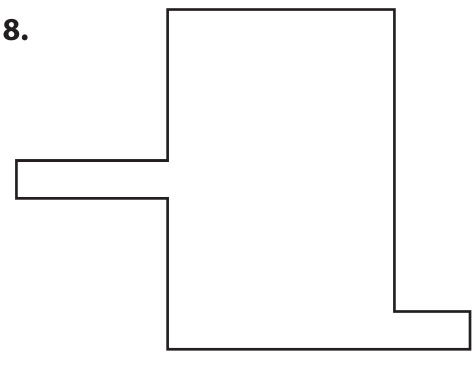


Extension: Sides and Angles

When you combined four rectangles making a **composite shape**, you kept track of the number of sides and angles in the shape.

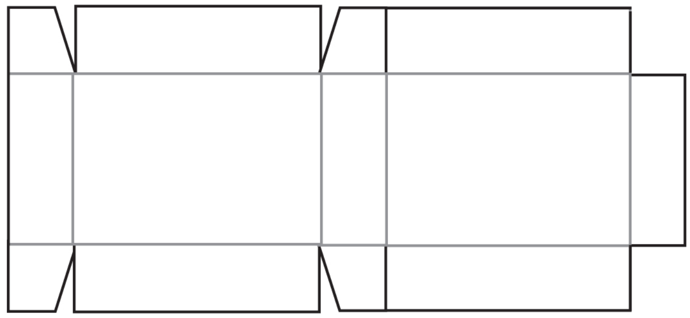
1. Create four new rectangles. Use them to make a two-rectangle shape, a three-rectangle shape, and a new four-rectangle shape. Trace these shapes onto square-centimeter grid paper.
2. Count the sides and angles in each shape. Record your data in the chart below.

Number of Rectangles	Number of Sides	Number of Angles
2 Rectangles		
3 Rectangles		
4 Rectangles		

3. Look at the numbers you recorded for sides and angles. What do you notice?
4. Does this surprise you? Why?
5. Are all the angles right angles? Why do you think this is so?
6. If a shape had eight angles, how many sides would you expect it to have? Why?

Think about putting eight angle demonstrators together to form a shape made of rectangles. What would happen?

Practice: Area in Packaging



- Find a small box, for example, a box for butter or raisins.
- Open up the box and lay it flat.
- Trace the box on grid paper.

1. About how many square centimeters of cardboard were needed to construct the box?

2. Make a sketch and explain how you arrived at the answer. You may use grid paper.

Draw lines to make rectangles within the box shape; label dimensions.

Practice: Cookie Cutter

You are starting a bakery and want to have unique cookie shapes.

1. Use grid paper to design two shapes for cookie cutters. All lines should be straight (no curves). Both shapes have the same area, 36 sq. cm. When you finish, measure the sides and find the perimeters for both cookie cutters.

Cookie Cutter 1	**Cookie Cutter 2**
Area: ______	Area: ______
Perimeter: ______	Perimeter: ______

2. You are ready to submit your designs to a metalsmith who will make the cookie cutter. Which number should you provide—perimeter or area? Why?

3. The sales person tells you that the cookie cutters will cost 10¢ for every centimeter. Which cookie cutter design is cheaper?

4. Which design would you choose? Why?

5. Which design is more practical for the baker? Why?

6. Which design is more practical for the metalsmith? Why?

Test Practice

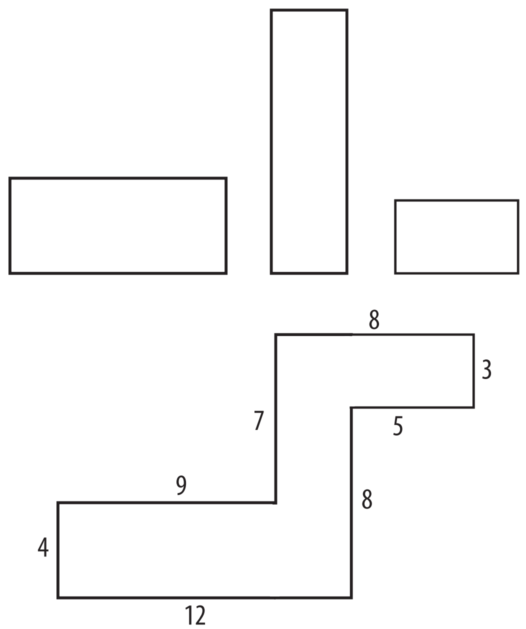
1. Which of the following is *always* true about a rectangle?
   (1) It has four angles that total 180˚.
   (2) It has 4 right angles and 4 sides.
   (3) It has only 2 right angles and 4 sides.
   (4) It has 4 right angles and 4 sides with equal widths.
   (5) It has only 2 right angles and 4 sides with opposite sides equal.

2. Which of the following is *not* a situation involving area?
   (1) Finding the amount of material needed to cover a pool
   (2) Finding the space in a parking lot
   (3) Finding the size of a lawn
   (4) Finding the length of wood needed for a picture frame
   (5) Finding how much of the Earth is covered in water

3. A rectangle's area measures 12 square centimeters. Which of the following might be the dimensions for the rectangle.
   (1) $l = 6$; $w = 3$
   (2) $l = 12$; $w = 2$
   (3) $l = 6$; $w = 2$
   (4) $l = 4$; $w = 2$
   (5) $l = 4$; $w = 4$

4. Candi and Claude each have a piece of chocolate. Candi's chocolate piece measures 6 cm by 4 cm, while Claude's piece measures 5 cm by 4 cm. How much bigger is Candi's chocolate than Claude's?
   (1) 4 cm
   (2) 4 sq. cm
   (3) 5 cm
   (4) 6 cm
   (5) 6 sq. cm

5. Sarah cuts out three rectangles. She finds the area and perimeter for each one. Sarah adds the areas of all the rectangles and adds all the perimeters. She then pastes the rectangles together (picture below). When she pastes them together into a new shape, what is true about the new shape?



(1) The number of sides is the same as before.

) The number of angles is the same as the number of sides.

(3) The area is larger than before.

(4) The perimeter is the same as before.

(5) The area is smaller than before.

6. Find the perimeter of Sarah's composite shape.

FACILITATING LESSON 6

Combining Rectangles

How much area does the hopscotch game take?

Synopsis

Students continue to explore area and perimeter of basic shapes, working with centimeter grid paper.

1. Students make and measure a composite shape made of rectangles.
2. They compare total areas and total perimeters of the individual rectangles and the composite shape.
3. Through a review of the investigation, students generalize that area is conserved in the new shape, but perimeter is not.

Objectives

- Calculate area of a rectangle using square centimeters
- Articulate what happens to area and perimeter when rectangles are combined
- Find the area of any shape composed of multiple rectangles

Materials/Prep

- Centimeter grid paper (*Blackline Master 29*)
- Centimeter rulers
- Markers and highlighters
- Tape or glue stick

Prepare overhead or enlarged version of the square on *Blackline Master 13*.

Prepare newsprint as follows:

Shape Name & Sketch	Areas of four rectangles (sq. cm)				Sum of Areas (sq. cm)	Area of Composite
Elisa's Cake	50	20	8	16	94	94 sq. cm
Generalization:						

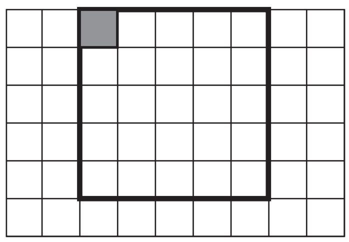
Shape Name & Sketch	Perimeters of four rectangles (cm)				Sum of Perimeters (cm)	Perimeter of Composite
Elisa's Cake	30	24	12	20	86	50 cm
Generalization:						

Opening Discussion

Refer to the last lesson by reminding students that they compared the areas of different shapes. Note that finding the areas of shapes that are rectangular is easier than finding the areas of other shapes, but the problems in life and on tests are rarely so straightforward.

Review definitions for perimeter and area and the ways to find them. If it was assigned, go over *See ing Perimeter and Area* (*Student Book*, p. 56), or do it together as a warm-up if you think students could use the review.

Introduce the idea of square units by showing the 5 x 5 square on centimeter grid paper.



Shade in one square and say:

This is one square centimeter. How many of these square centimeters cover the surface of the square?

Color in the squares as students count, reinforcing the idea of area as surface.

What are other ways to find the total number of square centimeters?

Record students' strategies as they share them, for example, counting by twos or fives, multiplying rows by columns, or adding repeatedly.

Highlight notation in one or more ways.

- Write the words on the transparency or the board (25 square centimeters; 25 sq. cm; 25 cm^2). Draw a line 25 cm long. Do students see the difference between 25 sq. cm and 25 cm? Do they acknowledge that writing “25” without saying whether the number refers to square centimeters or centimeters would be misleading?
- Note that the standard measurement for area is squares, not circles or hexagons.
- Ask everyone to make sure area is reported in square centimeters (sq. cm) and perimeter in centimeters (cm).

Heads Up!

Refrain at this point from teaching the formulas for perimeter and area of a rectangle. Allow students to figure area and perimeter out with the grid paper. This lesson will provide experiences that form a basis for understanding the formulas in *Lesson 7*.

Activity 1: Drawing Four Rectangles

Direct students to *Activity 1: Drawing Four Rectangles* (*Student Book*, p. 62). Distribute cm grid paper. All students draw four rectangles and find and record their dimensions, areas, and perimeters. When students finish, they should have a classmate check their work.

Before moving on, discuss ways to count and keep track of the number of squares. Ask:

What do you think will happen to the area if you combine these four shapes into a new shape?

What do you think will happen to the perimeter?

Write down your predictions on p. 62, and complete *Activity 2: Making a Composite Shape* to find out whether you are right.

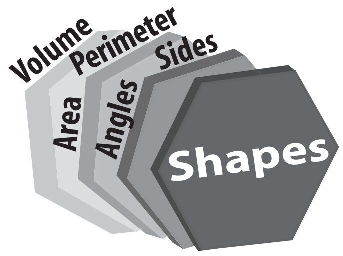
Activity 2: Making a Composite Shape

Students create their composite shapes. Once they have completed their charts (figured out area, perimeter, number of angles, and number of sides and recorded this information), ask them to exchange shapes and data with each other to verify calculations.

Ask students to post their composite shapes and findings on the newsprint sheets. They need information from both activity pages (pp. 62 and 63) to do this. If some finish earlier than others, assign *Extension: Sides and Angles* (*Student Book*, p. 68).

Ask students to talk about how they calculated area. Choose two or three students whom you observed using multiplication or repeated addition to share how they determined area of their shapes. Point out the relationship between repeated addition and multiplication.

Summary Discussion



Discussion should focus on the data the students posted. If students have trouble grasping all of the generalizations, focus on one and its implications.

Whose predictions about area and perimeter came true? What was unexpected?

Probe for reasons underlying accurate and inaccurate predictions. Draw attention to students' data on area.

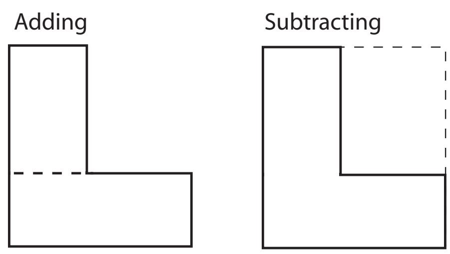
Looking at your data, what do you notice about the areas of the individual rectangles compared with the area of the composite shape?

Write students' statements on the board under "Generalizations."

Draw one or two multi-rectangular shapes and ask:

How would I find the area for something like this?

Make sure everyone understands that breaking the shape down into rectangles, finding the area for each, and adding the areas will yield a total. Or they can add lines to contain both rectangles, calculate the size of the new rectangle and of the additional "empty" space, then subtract the latter from the former.



Now review the perimeter data. Ask:

How does your method for finding area compare with your method for finding perimeter?

What do you notice about the perimeters of the individual rectangles compared with the perimeter of the composite shape?

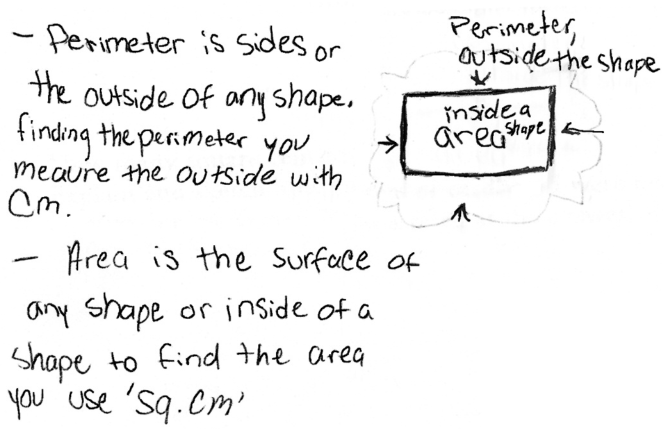
Post students' comments under "Generalizations." Ask:

Why would the perimeter of the composite shape be a smaller number than the one for the perimeters of the four rectangles added together?

Will this always be true? Why?

Ask students to give examples and demonstrations, for example, pushing together tables so everyone can see that the surface does not change, but the number of exposed edges is quite different.

Students record their thoughts in *Reflections* (*Student Book*, p. 154): "What do you want to remember about area and perimeter? How will you find the area of shapes that look like combined rectangles?"



Practice

Area of 24 Sq. Cm, p. 64
For practice comparing area and perimeter.

Divide the Shapes, p. 65
For practice sectioning off rectangles, the first step to finding the area of composite shapes.

Cookie Cutter, p. 66
For practice creating a shape and applying perimeter and area to a context with implications for cost.

Area in Packaging, p. 67
For practice finding area using a cardboard box.

Extension

Sides and Angles, p. 68
For practice generalizing about characteristics of composite shapes.

Test Practice

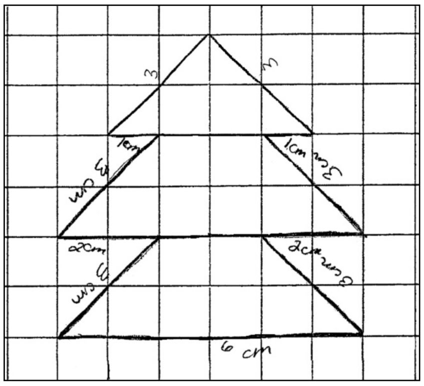
Test Practice, p. 69

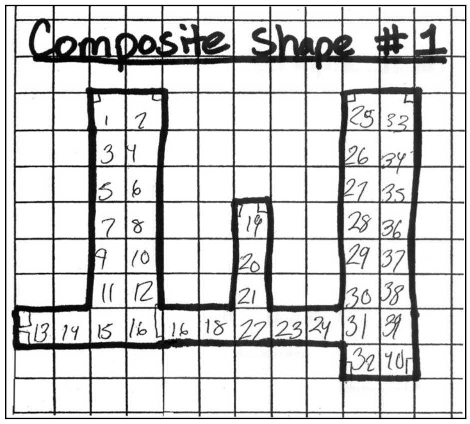
Looking Closely

Observe whether students are able to

Calculate area of a rectangle using square centimeters

How do students find area—by counting, skip counting, or multiplying? Ask students who are counting one by one whether they could also count by a larger number, for instance, two or five. The students' work will indicate their strategies, as in the examples provided. Counting one by one may feel like the most conscientious way to some students. These students need practice to become fluid with a more efficient approach. For those who are skip counting, ask whether there is a quicker way to find the number of square centimeters in, for instance, five rows of four squares.





Articulate what happens to area and perimeter when rectangles are combined

Perimeter changes are hard to explain. One way to see this is to highlight the perimeter of all the original rectangles in one color and then highlight the new shape's perimeter with another color. Ask what happened to the original perimeters when the shapes were combined. Do students notice that some have disappeared? Offer a real-life example, such as putting up fencing. Four separate plots require more fencing than combining the plots. Why?

Find the area of any shape composed of multiple rectangles

Do students understand area well enough to see the new shape made up of all the different areas combined? Suggest students do *Extension: Sides and Angles* (*Student Book*, p. 68), where they combine rectangles and record area and perimeter data for them.

WHAT TO LOOK FOR IN *LESSON 6*	WHO STANDS OUT? (LIST STUDENTS' INITIALS)			NOTES FOR NEXT STEPS
	STRONG	ADEQUATE	NEEDS WORK	
Concept Development • Finds area of rectangle using square centimeters • Combines rectangles to form composite shape • Finds area of composite shape • Finds perimeter of composite shape • Makes generalization about area and perimeter of composite shapes				
Expressive Capacity • Distinguishes between area and perimeter of rectangle • Distinguishes between area and perimeter of composite shape • Articulates what has happened to the sides when the rectangles are merged to form composite shape • Writes generalized statements about area and perimeter of composite shapes				
Use of Tools • Uses sq. cm paper to draw rectangles • Cuts and tapes together rectangles				
Background Knowledge • Refers to formulas of rectangle area and perimeter • Relates composite shapes to familiar objects				

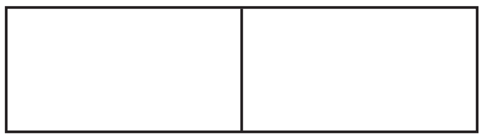
Rationale

By combining rectangles, comparing their areas and perimeters, noting relationships, and articulating generalizations, students acquire a fundamental understanding of the differences between area and perimeter.

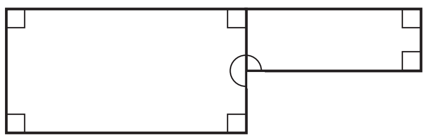
Math Background

The difference between area and perimeter of rectangles and composite shapes formed by those rectangles is that the inside surface amount remains the same—there is no overlap in the shapes that are merely set side by side—but the outside bounds of the sides are different. So, although area is conserved, perimeter is not. The same would be true if the shapes were triangles, hexagons, or other polygons: The area of the composite shape is the sum of the areas of the individual shapes, but the perimeter of the composite shape is *not* the sum of the perimeters of the individual shapes. Understanding the definitions for area and perimeter helps explain these differences.

The number of angles in a rectangle is four, and they are all right angles (90°). When two identical rectangles are set side by side or stacked, the resulting composite shape has four sides and four angles, not eight. The area of the resulting composite shape is the sum of the areas of each rectangle, and the perimeter is smaller than the sum of the perimeters of the two rectangles.



When two different-size rectangles are composed, they may or may not share a complete side. If they look like this, for example,



the area of the composite is the sum of the individual areas, but the perimeter is slightly smaller. Examining the angles, we find there are five right angles and one other 270° angle. Note that the composite shape has six sides.

Facilitation

Making the Lesson Easier

One field-test teacher reported how she helped her students who struggled to find the perimeter, number of angles, and sides. Her strategy made the task easier for students.

> *When my students cut out the shapes and taped them together, none seemed able to count the areas or perimeters correctly the first time around. They were skipping sides with the perimeters, skipping squares with the areas, and seemed at a loss for identifying corners and sides. I had to go around to each person (because they were working at different paces) and ask questions to help him or her think through the answers. For example, if someone said her shape had 6 sides when it had 12, I would say, "Okay, show me the sides." As she identified each side, I would trace it with a highlighter. When she reached the sixth, I would say, "Do you see any more sides we missed?" Most students understood.*

Making the Lesson Harder

A third generalization can be made based on a count of angles and sides. When students agree that the number of sides and the number of angles in a composite shape are the same, post that generalization. This insight might lead students to check shapes such as the octagon or hexagon to see whether the rule proves true for other shapes.

The area and perimeter findings can be expressed using algebraic notation. Tell students that mathematicians often record their generalizations in shorthand, using letters to stand for certain measurements.

Model this by saying the generalization for area aloud and writing it in shorthand, with A_c meaning the area of the composite shape and A_1, A_2, A_3, and A_4 standing for the areas of each of the four rectangles. Connect the algebraic equation to people's natural language.

A_c	$=$	$A_1 + A_2 + A_3 + A_4$
(The area of the big one)	(is the same as)	(the four areas put together),

or the area of the composite shape equals the sum of the areas of the rectangles.

Check the algebraic statement with test cases: Plug in numbers from the class's newsprint.

Write:

$$P_c \qquad P_1 + P_2 + P_3 + P_4$$

Ask students to complete the statement using an appropriate symbol. Symbols such as greater than (>), less than (<), or the symbol for inequality (≠) can make the statement true. This rule should also be true for every case. You could pose a challenge to students to come up with a design for which the rule would not be true.

Multiple experiences with area and perimeter can help students build confidence and intuition.

What I have noticed at this point is that my students have seen a relationship between length and width and perimeter; they are starting to double each and add them together. They have not noticed that they could measure length and width on the graph paper without a ruler. They see no relationship yet between length and width and area. One student was counting by ones and another student showed her how she was counting the rows by fives and twos.

I thought the student that counted by ones was having a major breakthrough; when asked to predict the area of the composite shape, the first thing she said was, "That's easy—it's just the area of all the shapes added together." But instead of adding the areas from the first page together, she started re-counting the entire area by ones. She also had problems on the first page because she did not draw her rectangles on the lines. Checking each other's work was great because her partner could point out her mistake. It is so much less intimidating to students who are not very comfortable with math for a student to point out their errors than for the teacher to point them out.

Phyllis Flanagan
Rock Valley Community College, Rockford, IL

Name ______________________________ Date ____________

Over, Around, and Within: Geometry and Measurement

INITIAL ASSESSMENT

Task 1: Identifying Angles

1. Use your angle demonstrator to identify and record an angle in the classroom.

2. Trace the angle and label it, for example, wall–floor.

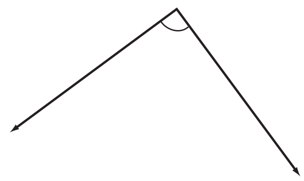
Look at each task and decide whether you can do it, don't know how to do it, or are not sure if you can. Check off the appropriate choice, and then complete the task, if you can.

3. Estimate the measure of each angle on the next page and write the number next to the angle.

___ Can do ___ Don't know how ___ Not sure

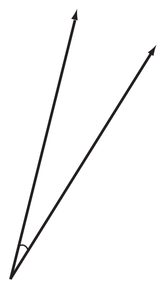
4. Identify each of the angles below as right angle, angle greater than 90˚, *or* angle less than 90˚.

___ Can do ___ Don’t know how ___ Not sure



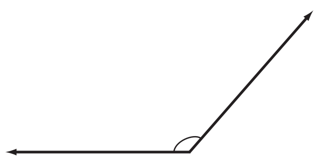
a.

☐ right angle
☐ angle greater than 90° ($>90^\circ$)
☐ angle less than 90° ($<90^\circ$)



b.

☐ right angle
☐ angle greater than 90° ($>90^\circ$)
☐ angle less than 90° ($<90^\circ$)



c.

☐ right angle
☐ angle greater than 90° ($>90^\circ$)
☐ angle less than 90° ($<90^\circ$)

Task 2: Measurement

1. How big is an inch? Draw an inch.

2. Complete the following measurement statements.

a. One foot = _____ inches.

___ Can do ___ Don’t know how ___ Not sure

b. Three feet = _____ inches.

___ Can do ___ Don’t know how ___ Not sure

c. One square yard = _____ square feet.

___ Can do ___ Don’t know how ___ Not sure

Task 3: Shapes and Name

1. Choose *one* of the two cards and follow the directions.

Draw the shape in the space below.

Card 1	**Card 2**
Draw a rectangle that measures 2″ by 4″. On one of the short ends, draw an isosceles triangle.	Draw a square that measures 3″ on each side. Find the center of the top edge and draw a line perpendicular to that edge.

2. What is the area of the square or rectangle you drew?

___ Can do ___ Don't know how ___ Not sure

3. What is the perimeter of the square or rectangle you drew?

___ Can do ___ Don't know how ___ Not sure

Task 4: Volume

Use the box your teacher gives you to answer the following questions:

1. What would you need to measure to find out how many cubic centimeters a shoe box could hold?

___ Can do ___ Don't know how ___ Not sure

2. How many paper-clip boxes (2 x 5 x 7 cm) could fit inside a shoebox? Explain how you know.

___ Can do ___ Don't know how ___ Not sure

INITIAL ASSESSMENT CLASS TALLY

Task 1, Problems 1 and 2 Notes:

Task	Can Do	Don't Know How	Not Sure	Percent Who Can Do
1. 1				
2				
3				
4a				
4b				
4c				
2. 1				
2a				
2b				
2c				
3. 1				
2				
3				
4. 1				
2				

Notes on Confidence Levels

INITIAL ASSESSMENT CHECKLIST

Use a ✓, ✓+, or ✓– to assess how well students met each skill. When you give feedback to students, note areas in which they did well in addition to areas for improvement.

Use ✓ to show work that is mostly accurate; some details, additional work needed

Use ✓+ to show work that is accurate, complete.

Use ✓– to show work that is inaccurate, incomplete.

Student's Name____

Task	Skills	Lesson Taught
1. Make an Angle Demonstrator 1 ____ 2 ____ 3 ____ 4a ____ 4b ____ 4c ____	Identifies an angle Records the angle by tracing Estimates the measure of the angle Identifies angles less than or greater than 90˚	*Opening the Unit* 2 2 2
2. Measurement 1 ____ 2a ____ 2b ____ 2c ____	Approximates the size of an inch Knows Standard English measurement unit conversions for linear inches, feet and square feet and square yards	8 8, 9
3. Shapes and Names 1 ____ 2 ____ 3 ____	Interprets terminology (e.g., parallel, perpendicular, forms a right angle) Measures and draws accurately Finds the area of a square or rectangle Distinguishes between area and finds perimeter	1 4, 8 6 5
4. Volume 1 ____ 2 ____	Identifies length, width, and height as dimensions for figuring out capacity Explains volume Uses terms such as layers	11 12

OVERALL NOTES

Strengths

Areas for Improvement

EMPower
extending mathematical power

Many Points Make a Point

Data and Graphs

TEACHER BOOK

TERC

Many Points Make A Point: Data and Graphs

The world of data sparks to life for students when they engage with numerous high-interest, real-world data sets, and construct as well as interpret a variety of graphs. Using personal data about the clothes they wear, the foods they eat, and the hours they spend watching television as well as social data about amusement parks, weather trends, and stock prices, students work individually, in pairs and in groups to make frequency, bar, circle, and line graphs. Along the way, they explore graph elements, such as axes, scales, and slope direction. They also encounter three summarizing statistics—mode, mean and median.

Throughout the series of carefully crafted lessons, students hone their graph/data interpretation skills and ability to connect the narrative story of a situation to its graphic display. They begin by making verbal statements about frequencies. As the lessons progress, they refine their observations. By the close of the unit, students are able to describe a graph or data set with benchmark fractions and percents as well as mean and/or median statistics. Concurrently, they develop the capacity to make reasoned arguments and decisions based on data. As well, they learn to question data and graphic representations.

Correlations for *Many Points Make a Point: Data and Graphs*

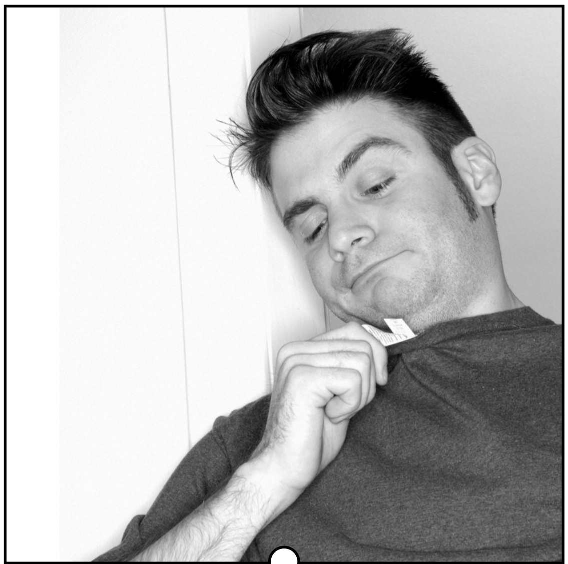
Book Description: Students collect, organize, and represent data using frequency, bar, and circle graphs. They use line graphs to describe change over time. They use benchmark fractions and percents and the three measures of central tendency—mode, median, and mean—to describe sets of data.

Lesson Number:	Lesson Name:	Mathematical Concepts/Topics Covered:
Opening the Unit	Many Points Make a Point	• Assess familiarity with graph formats, features, and purposes • Graph terms
Lesson 1:	Countries in Our Closets	• Categorize and compare data with frequency graphs • Identify graph 'story' • Change data display to see change in graph 'story'
Lesson 2:	Most of Us Eat	• Organize data for specific purposes • Describe data numerically with benchmark fractions and percents
Lesson 3:	Displaying Data in a New Way	• Bar and circle graph construction • Axes intervals • Bar and circle graph formats compared and contrasted
Lesson 4:	A Closer Look at Circle Graphs	• Parts and wholes in circle graphs • Benchmark percents to estimate circle graph portions • Circle Graph interpretation
Midpoint Assessment	The Data Say	• Bar and circle graph knowledge assessed
Lesson 5:	Sketch This	• Line graphs sketched • Correlation of graph line shape and graph story over time
Lesson 6:	Roller-Coaster Rides	• Line graph construction and description • Points of change
Lesson 7:	A Mean Idea	• 'Average' (mean) defined and determined given all values or missing values
Lesson 8:	Mystery Cities	• Multiple data lines • Scale variation impact • Graph and text alignment

Lesson 9:	Median	• Median detemined with odd and even data sets • Data set determined from given median
Lesson 10:	Stock Prices	• Tables connected to and generated from graphs • Scale generalizations
Closing the Unit	Stock Picks	• Application of graph knowledge for evaluations, recommendations, problem solving and presentations

LESSON 1

Countries in Our Closets



Where were your clothes made?

In a world where information is at our fingertips, it is important to be able to make sense of it. What does it tell us? Why do we care?

Begin with your own clothes. Where were they made? Find out! You will collect **data** by reading the labels on your clothes. You will organize that data, make statements about it, and note where most of your clothes were made.

You will then try organizing the data in a different way. How will placing the data in new groups change the story of the countries in your closet? You will compare your class data to data of other classes. Will the stories be the same?

On Your Own

I think the country in which most of my clothes were made is

____.

Look at the labels of at least eight items of clothing. Make a list to keep track of the information you are collecting. Research question: Where were your favorite clothes made?

Activity 1: Organizing the Data

Look at the data you collected about countries where your clothes were made.

Organize the data.

1. What strikes you about your data?

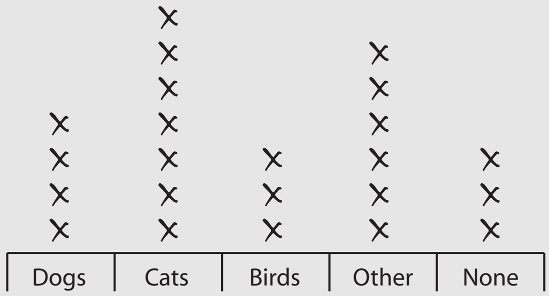
2. What is the **mode** for your data?

3. What is the mode for the class data?

Frequency Graphs

Create an easy-to-use **frequency graph** by making a line. Use equal-size x’s and equal spacing for each **category**. Line up your x’s or use graph paper so it is easy to note which category contains the most data items.

Some people can look at a frequency graph and make true statements by “eyeballing,” or visually estimating, the size of each category. For example:



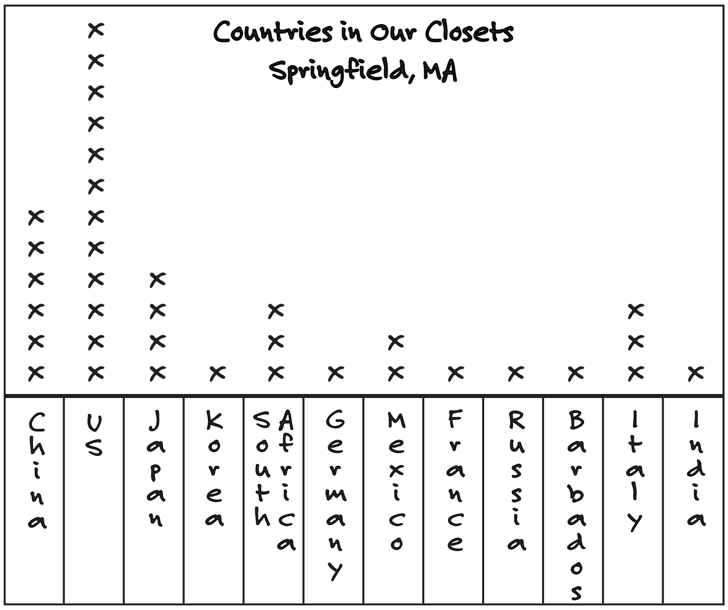
If you have a hard time visually estimating, try this: Count all the x’s. Make one tall tower showing every x on the frequency graph. Compare each of the categories to the tower with all the data.

Does that category represent more than half, less than half, or exactly half of the data?

Compare two categories: Is one category more than half of the other?

Activity 2: Statements about Data

Use the data in the following frequency graph to fill in the blanks:



Use the following words to fill in the blanks:

half	six	United States	twice	three
one-fourth	four	one-third	Russia	China

1. There are ______________ times as many clothes from South Africa as from Barbados.
2. The number of clothes from Mexico is ______________ of the number of clothes from China.
3. ______________ as many clothes come from the United States as from China.
4. The clothes from Korea are ______________ as many as those from Japan.
5. _______________ countries have only one article of clothing.
6. _______________ of the clothes come from _____________ and _______________.
7. Japan has ______________ times as many clothes as Germany.

Activity 3: Changing the Categories

On grid paper, make a new frequency graph using the class data. Group the clothes by continent this time, instead of by country.

Frequency graphs have three components: a line, frequencies (marked with x's), and labels. When making your own graph, be sure to include all three parts.

1. What do you notice about these new groupings (continents)?

2. How is the story of this graph different from the story of the whole-class frequency graph by country?

3. What advantages do you see to this way of grouping? What disadvantages?

4. Make a numerical statement about the new organization of the data.

Practice: Clothes by Continent in Springfield, MA

Refer to the Springfield data on page 10.

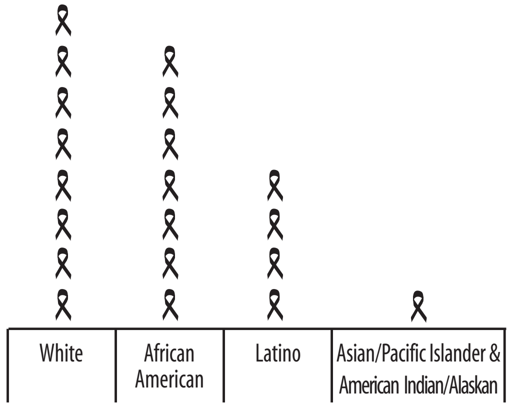
1. Sort the data by continent and make a frequency graph on the graph paper below:

2. How do the data represented in the new graph compare to the data from the graph your class created?

3. Write one statement about the Springfield clothes frequency graph organized by continent.

Practice: Reporting Data 1

The race/ethnicity of 20 HIV/AIDS patients at a clinic in the United States is shown in the following frequency graph:



Source: Based on data from the AIDSHotline.org, 2003

1. Write a statement comparing two of the categories.

Ana reports the data from her clinic using only two categories: White and non-White.

2. Use grid paper to show what her frequency graph will look like.
3. How does using only two categories change the story of the HIV/AIDS data at her clinic?
4. Would you expect the data to be similar if the clinic were in your city, a city in Alaska, or a city in Florida?
5. Choose one city and make a frequency graph on grid paper. Show 20 patients by race. Explain your choices.

Practice: Reporting Data 2

The mayor wants to cut commuting time. He commissioned a survey to find out how long it takes people to get to work. The results were shown in five travel-time categories.

Regroup the data to show only three categories.

Travel Time to Work	Percent of Commuters	Number Based on 25 People
Less than 10 minutes	16%	4
10–19 minutes	32%	8
20–29 minutes	20%	5
30–44 minutes	20%	5
45 minutes or more	12%	3
Total	100%	25

Travel Time to Work	Percent of Commuters	Number Based on 25 People

Compare the two ways of organizing the information.

1. What is the travel-time category with the biggest percent of commuters?

   Five categories ___________ Three categories ___________

2. Which category has the smallest percent of people?

   Five categories ___________ Three categories ___________

3. How does regrouping the categories change your impression of people's travel time to work?

4. Which group would you recommend the mayor focus on if he starts a program to cut commuting time? Why?

Practice: Categorically Speaking

Sometimes we can organize categories into subgroups of other categories in order to better understand the data. For example, drug stores will often organize their aisles by category—skin care, beauty aids, and so on. Then the items in each of those aisles are also organized. This makes it easier to take inventory and place orders and for the customer to find products.

Can you think of other examples where information is categorized?

Use the chart below *or* create your own chart; draw a picture; make a list; or write about examples you see at home or at work. In the chart below, one example is given.

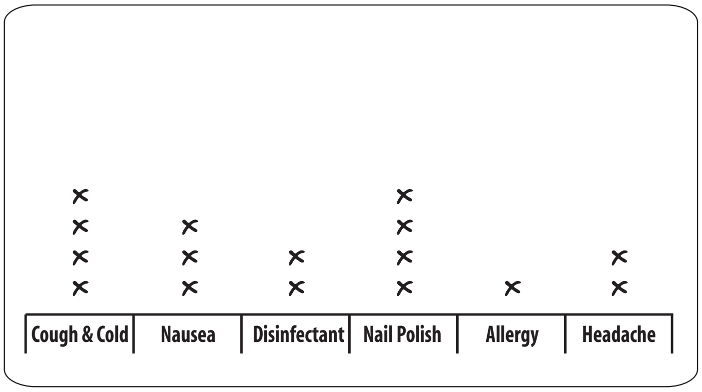
Location	**What Stuff or Information?**	**How Is It Organized?**
Drug store	Over-the-counter products they sell	Skin care, Stationery Beauty aids, First aid Seasonal, Hair care

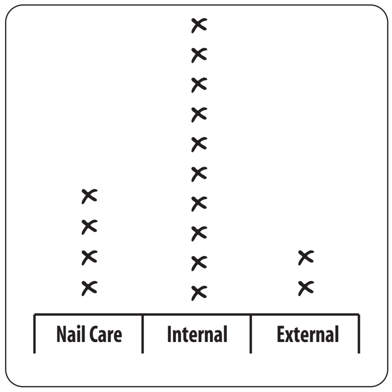
Extension: Taking Inventory

Find a drawer or closet filled with many things. Use a frequency graph to show the contents. Start with six categories. Then do another frequency graph to show the same items, but this time use three categories.

For example:

My Medicine Cabinet





Test Practice

1. Freda worked at a printing press. She kept a tally of all the defects she found in the books she was processing. Based on the frequency graph she created, shown below, which of the following statements is true?

   (1) There were more defects related to colors than any other defect.

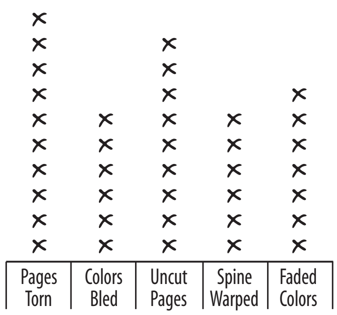
   (2) There were twice as many torn pages as there were warped spines.

   (3) There were more warped spines than colors that bled.

   (4) Half of all the defects were related to colors that bled.

   (5) Half of all the defects were related to pages that were torn.

Book Defects Frequency Graph



2. Tony, a tour guide, has been keeping a tally of visitors from different states. According to his tally, which of the following is *not* a true statement?

   (1) The fewest number of visitors came from Oregon.

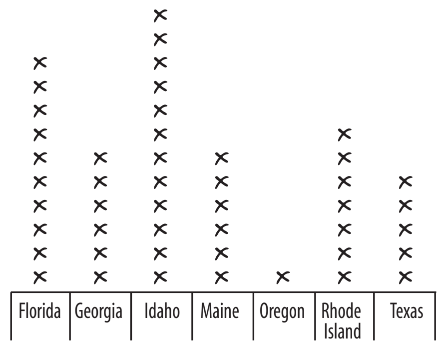
   (2) There were as many visitors from Maine as there were from Georgia.

   (3) There were as many visitors from Georgia as there were from Oregon and Texas.

   (4) There were twice as many visitors from Florida as there were from Rhode Island.

   (5) There were twice as many visitors from Idaho as there were from Maine.

U.S. Tourist Frequency Graph



3. Clara is paid a commission on each of the large electronics devices that she sells. She tallied the different items she sold for the month. Based on her tally, what can she tell her boss about her sales?

   (1) She sold twice as many DVD players as she did TVs.

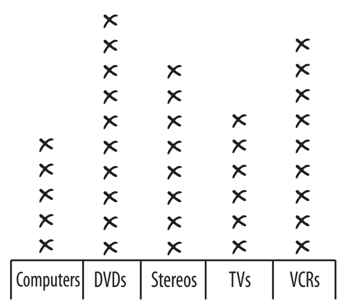
   (2) She sold twice as many DVD players as she did computers.

   (3) She sold more DVD players than she did TVs and computers combined.

   (4) She sold more TVs and VCRs than she did stereos and DVD players combined.

   (5) She sold half as many TVs as she did stereos.

Home Media Frequency Graph



4. Bronson, a forest ranger in the Green Mountains, kept track of the different animals that were reportedly seen in one of the campgrounds during the month of May. Based on his tally, which of the following statements could he tell the media?

    (1) There were twice as many wolves reportedly seen as bears.

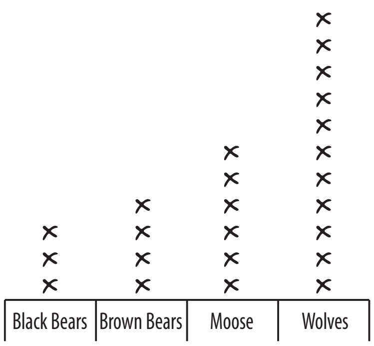
    (2) Half of all the reported sightings were wolves.

    (3) There were more bears than wolves reportedly seen.

    (4) There were twice as many moose reportedly seen than there were black bears.

    (5) One-quarter of all reported sightings were brown bears.

North American Wildlife Frequency Graph



5. According to Global Exchange, some workers in China who make clothing for Disney are paid as little as \$0.16 per hour. At this wage, what amount would a worker make for a 40-hour workweek?

    (1) \$64.00

    (2) \$6.40

    (3) \$6.00

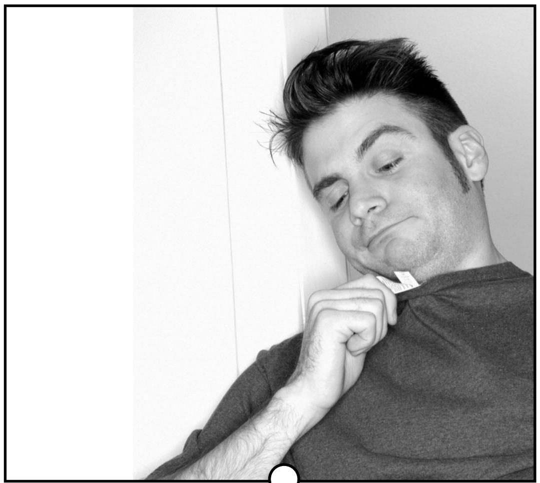
    (4) \$80.00

    (5) \$0.60

6. In 2003 the U.S. Department of Labor (DOL) issued new regulations for overtime pay. The DOL estimated that under the new regulations 1.3 million low-wage workers would become eligible for overtime pay, unless their wages were raised to \$425 per week. The DOL estimated that 24.8 percent of those workers were Hispanic and 16.6 percent were African American. What percent were neither Hispanic nor African American?

FACILITATING LESSON 1

Countries in Our Closets



Where were your clothes made?

Synopsis

This is the first of a set of lessons on categorical data in which students collect, examine, sort, and organize their own data, and then compare their data to another data set.

1. Students bring to the class data on the different geographic locations where their clothes were made.
2. The class combines its data into one frequency graph, and students make several statements about it.
3. Working in pairs, students compare their class graph to data from another class.
4. Pairs of students each create a new frequency graph, organizing the class data by continent. Students notice which continent is most prevalent.
5. Discussion centers on comparing country and continent graphs, closing with the question How does the story change when the graph's categories change?

Objectives

- Use a frequency graph to organize data
- Identify the story that the data tell
- Compare data from various samples
- Change the categories and articulate the change in the story

Materials/Prep

- Grid paper
- Paper strips for posting statements
- Post-it® Notes
- Rulers

As an aid for determining in which continent the countries are located, have on hand a world atlas such as *The New Children's Illustrated Atlas of the World*, by Keith Lye (Philadelphia: Courage Press, 1999). For a listing of countries in Asia, see *Resource: Countries in Asia, Blackline Master 4*.

Make sure you have assigned *Countries in Our Closets* (*Student Book*, p. 7). Collect and bring in about 20 items of clothing in case your students have not collected their own data. (See *Facilitation*, p. 25, for how to handle this.)

Prepare a transparency or make a copy of the graph "Countries in Our Closets, Springfield, MA" (*Student Book*, p. 10).

Opening Discussion

Ask students:

Are clothes made in this city? In this country? Why or why not?

Why would anyone care where his or her clothes were made?

Acknowledge that we should know something about where our clothes come from, given how clothing is worn next to our skin. There are also tremendous economic and political issues at stake as manufacturing jobs are outsourced outside of the United States.

Heads Up!

If students do not bring in data, see *Facilitation*, p. 25.

You have looked in your closets, checked the tags on some of your clothes, and collected information. What did you learn?

Which country do most of the clothes you checked come from?

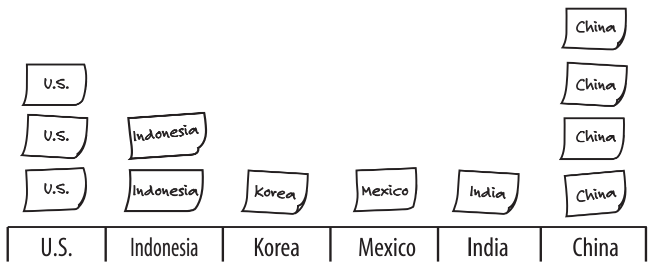
Post the country names in a place where you can easily refer to them later.

Introduce the terms **data set** and **mode**. Each student's list is a data set, and each data set could have a mode—the country name that occurs most frequently.

Vocabulary note: If students are searching for words to express their thoughts, offer terms and concise definitions. Students can keep track of these in their student books. For example, **data** are the information collected. The data that are collected make up the **sample**. The whole group from which information could be collected is the **population**. See *Lesson 1 Commentary*, p. 24, for more information.

Activity 1: Organizing the Data

Explain the motivation for organizing data: It is easy to identify the mode from a small sample, but when working with larger samples of data, it helps to have a strategy. In addition, a graph will illustrate the data, enabling everyone to see the number of articles of clothing from each country.



Distribute Post-it Notes to students. Ask them to write on a separate Post-it Note the name of the country each article of clothing came from. On the board, draw a horizontal line, long enough to fit many different country names. Ask students to place their Post-it Notes above the line, organizing the data by country. Volunteers should write the names for countries along the bottom of the line. This graph is called a **frequency graph** because it shows how often (frequently) each country occurs.

Heads Up!

Leave the whole-class frequency graph posted for use in the following activity. If you have a copier available, make and distribute to each student a copy of the class frequency graph done on graph paper.

If you do not have a copier, distribute graph paper for students to copy the whole-class data for themselves—be aware that this could be a time-consuming task.

When students finish, ask:

Which country did most of our clothes come from?

How does the mode for the whole-class data compare to the mode for your own data?

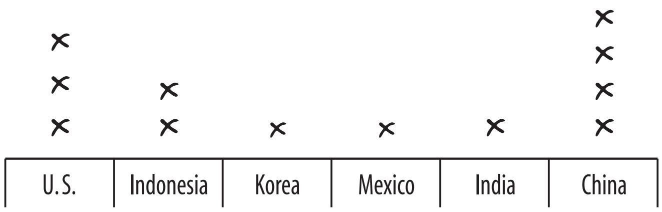
Ask students to create a few paired statements about the class data. Model this with a statement such as

One article of clothing was made in each of the following countries: China, Mexico, and Korea.

There are twice as many clothes from China as from Mexico.

Referring to the whole-class frequency graph, you might say:

All these Post-it Notes could fly off, and we would lose our data. One way to avoid that problem is to use x's in place of each Post-it Note like this (model it):

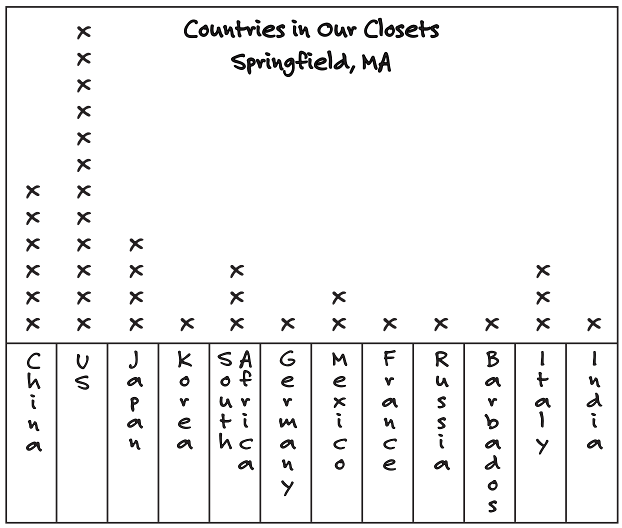


Note the components of a frequency graph:

- Line
- x's
- Labels

Activity 2: Statements about Data

Students now compare their class data with data from another class. Post the data and ask students to comment on similarities and differences between the country data from their class and from a pre-GED class in Springfield, Massachusetts.



Start by describing the Springfield data:

In the Springfield data, four times as many clothes come from the United States as from South Africa. Explain that statement.

As students explain, make sure "3 × 4 equals 12" is made explicit.

What other statements can be made about the Springfield data?

Prompt students to provide numerical examples (e.g., twice as many clothes came from China as from Italy; half as many clothes came from China as from the United States; nine more articles of clothing came from the United States than from Italy). Refer students to *Activity 2: Statements about Data* (*Student Book*, p. 10). Ask them to work in pairs to answer these questions:

We have data from two sets of students—one in Springfield and our own. How are they similar? Different?

Do our two classes have the same mode? Why do you think that might be?

What if we asked students from another state to make a data set? How do you think their data would compare to these two data sets? Why?

What if we included people other than adults in our program? How do you think their data would compare to these two sets?

When researchers collect data, they ask themselves these kinds of questions.

Heads Up!

The important thing to note is that the statements made are valid *only* for the group from which the data are collected. We can hypothesize that other groups might have similar or different data, but we do not know for sure unless we collect data. This is meant to open a conversation about the concept of sample. Draw from the *Math Background*, p. 24, if your students are interested. *Lesson 2* addresses the issue of sample in more depth.

Activity 3: Changing the Categories

Next, the data will be grouped in another way—by continent. Students will work with the class data and might need additional reference materials. Refer students to *Activity 3: Changing the Categories* (*Student Book*, p. 11). Ask pairs of students to make a frequency graph showing the continents in their closets.

When you make a frequency graph, remember the three parts: the line, the x's, and the labels.

Ask the student pairs to post their graphs and to make one numerical statement describing the data in their graph. For example, "Less than half of the clothes came from Europe"; or "A quarter of our clothes came from South America."

What do you notice about this new way of sorting our clothes?

Be sure to highlight the following:

- It is often easier to remember and manage fewer categories.
- All the clothes from one continent could have been produced by many countries or by just one. The continent label hides this information.
- The "part of the world" many of our clothes came from becomes clear.
- The continent labels might change the relationships among categories. For example, the frequency graph by country could show that more than half the clothes were made in China; whereas the graph by continent could show that more than three-quarters of the clothes were made in Asia.

Open a discussion about the sample:

What do you imagine you would find if a different group collected and shared their data?

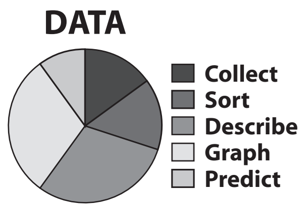
Would it be appropriate to make statements based on these data that would apply to all adults' clothes? Why?

Heads Up!

Generalizations from data are affected by how representative the sample is and by its size. You do not need to go into details about this now, but if students wonder about whether they *can* generalize, give an example such as "If all our data were collected in (*name a wealthy area*), the countries might not be the same as if we collected data from (*name a non-wealthy area*)"; or, "If we only looked at clothes bought at Wal-Mart, we might be missing some countries."

Summary Discussion

Encourage students to share—orally or in writing—what they have learned about collecting, sorting, graphing, and grouping and regrouping data using a frequency graph.



Say:

You are in charge of making statements. The data are just pieces of information until you interpret them. The data stay static (that is, the data do not change)—the way they are represented does not. You can change the categories and/or combine them; this might yield a new story.

Circle back to the larger conversation about where the clothes were made. The data will likely show that most of our clothes were made far from home. Engage students in a conversation about why this is. You might jump-start the discussion by saying:

Although we have not talked about the cost of shipping or the cost of each item, companies seem to want to buy clothes made in other countries. Why might that be?

You might also pose the question, "What does 'Made in the USA' really mean?"

This is a good time to remind students about the three components of a frequency graph. Go over the steps they took to represent the data, i.e., collecting, sorting, organizing, labeling, and making statements and generalizations about the data. Direct students to *Reflections* (*Student Book*, p. 144), where they can record their answers.

Practice

Clothes by Continent in Springfield, MA, p. 12
For practice sorting data by continent.

Reporting Data 1 and 2, pp. 13–14
For practice seeing what effect collapsing data has on the story the graph tells.

Categorically Speaking, p. 15
For practice analyzing how things in our lives are organized in categories.

Extension

Taking Inventory, p. 16
For practice collecting new data, then sorting, categorizing, and reorganizing them.

Test Practice

Test Practice, p. 17

Looking Closely

Observe whether students are able to

Use a frequency graph to organize data

How do students manage the logistics of constructing a frequency graph? Make sure they use the grid lines as a guide so that that the x's are uniform. This way, when they compare countries, the comparison will be precise. Help students make the connection between one article of clothing and one x. Ask them to double-check their graphs to make sure they included every data point, not adding or omitting any. The category labels are also important. Assist students with spelling if necessary.

Identify the story that the data tell

Do students make statements based on the data displayed on the graph? Some students might feel the job is done when the graph is complete. Not so. If students are hesitant, pose questions about what the data tell: "Is there one country that stands out in the graph? How so? Why might that be?" Accept either statements that quantify the relative value ("Asia has three times as many") or the total value ("India has nine more than …").

Compare data from various samples

Do students notice the big differences or similarities between two data sets? If they say, for example, "This data set includes Japan and the other one does not," that is a good start. Probe for more by asking, "Do they both have the same mode? How do you know?" You are only looking for a big-picture comparison.

Change the categories and articulate the change in the story

Do students recognize the differences between the frequency graphs and the accompanying stories for countries and continents?

While some of the graphs might look similar, the labels are different, as are the groupings. Ask students who have trouble articulating specific questions: "What are the categories (groups) in this graph? And in the other one? Are there continents that stand out? How are they related to the countries that stood out?"

WHAT TO LOOK FOR IN *LESSON 1*	WHO STANDS OUT? (LIST STUDENTS' INITIALS)			NOTES FOR NEXT STEPS
	STRONG	ADEQUATE	NEEDS WORK	
Concept Development • Records all data • Organizes data • Recognizes key features of frequency graph • Compares data • Changes categories and notes change in story				
Expressive Capacity • Uses precise language to describe the frequency graphs • Contributes to the discussion about the way changing categories affects the story the data tell				
Use of Tools • Uses graph paper • Uses atlas or other reference				
Background Knowledge • Draws on understanding of geography, relative size, and number sense for comparisons				

Rationale

The lesson takes students through the first steps of displaying data: collecting and organizing the information. As you work through this lesson, you and your students will notice that each time you categorize and recategorize the data set, you tell a different story.

Math Background

Data are information. This information may be numerical—e.g., salaries, test scores, heights, age, weight—as well as categorical—e.g., countries, foods we eat, types of product defects.

Most data are reported using statistics based on fractions and percents—one-third of pregnant mothers, half of native plants, etc. Commonly used, or benchmark, fractions and percents are often invoked to influence others to make decisions. When the headlines shout, "One-half of all pregnant women …," readers tend to form opinions based on the data. What readers often neglect to consider is the size of the sample, where the data were collected, and how the data were organized.

The media generally report data that are based on a sample. It is rare for any group to have the time or resources to ask every person in the population to contribute information. The idea of sampling is to study a part in order to gain information about the whole. The sample—its size and characteristics—influences the data and the conclusions of the study.

A representative sample by definition includes representation across the population being surveyed. For example, a representative sample of the community would include individuals from across town, not just one neighborhood. Likewise, a representative sample across America would include individuals from many different states and regions, not just one or two. The size of the sample is important when inferences are made. For example, 30 samples could be sufficient for a population of several hundreds. The deciding factor for determining sample size is how confident you want to be about the inference.

It is not the concern of this unit to determine appropriate sample size, but it is important to note that statisticians use formulas derived from repeated surveys to decide sample size. Regardless, small samples are used in many studies. Factors such as availability of subjects or scarcity of time or financial resources might lead researchers to use a small sample and to base policy or claims upon the outcome.

Throughout this unit, it will be important to help students begin to think critically about data, whether the data are grouped into categories of information or into graphs. They will need to understand that a random sample will often yield different results from one that is not random. This topic resurfaces in *Lesson 2*.

Context

Some students may know about *maquiladoras* in Mexican border towns, where women make clothes for very little money and with no benefits or environmental Occupational Safety and Health Administration (OSHA) workplace protections. CorpWatch (www.corpwatch.org) is one source for information on *maquiladoras*.

Facilitation

If students do not bring in data, or if their sample is too small, skip the second part of the *Opening Discussion*. Have available a pile of 20 clothing articles with labels. First, ask students to predict where the clothes were made. Post the list of their guesses. Note that it will be hard for them to answer this question unless they organize the information on the labels. Then divide up the 20 articles of clothing. Have students write the name of the country for each piece of clothing on a Post-it Note, one country name per note. Ask: "Where are most of our clothes made?" Then continue with the activity.

Making the Lesson Easier

Frequency graphs lend themselves to comparisons among categories. If students have little fluency stating comparisons, you may choose only to compare size, using terms like "greater," "fewest," or "less than." For students who are encountering data formally for the first time, the notion that collapsing data yields different stories may be difficult. Treat this lightly in the activity, and revisit such questions after students have more experience categorizing and recategorizing data in the homework and in *Lesson 2*.

Making the Lesson Harder

If your students can handle benchmark fractions and percents, get them to look critically at the data, including the source and sample size. You might ask:

If we asked another class what countries are in their closets, what do you think would happen to the categories? What if we asked the entire community?

How do you think your data would compare to data from another class of adult students in another community?

If students struggle with the idea of sample, you might try this: Have them each write their favorite color on a Post-it Note. If you have a small class, ask them to write the color on two Post-it Notes. Place all of the notes in a container. Have someone randomly (eyes closed) choose a few notes from the container and place them across a line to form a frequency graph. Ask the students how they think this sample compares to the actual total number of colors on notes in the container. You can have them do another frequency graph to compare the sample to the actual total.

Alice articulates the mathematical principle behind compressed data.

I asked, "How did the change in categories affect what we noticed about the data?"

Alice answered, "Well, we keep losing information."

"How so?"

Patiently, Alice explained that when we started our work, every bit of data was visible. She added that we had lost details initially recorded. "At first, we knew every country in every person's closet and how many pieces of clothing came from that country. Then we combined the data, and we lost track of who had which countries. Then we did it by continent, and we lost track of all the countries."

Alice's realization quickly gained agreement from the rest of the class. After all, just the previous week a classmate had noted, "When you change the amount of data you look at, you find different things."

Sonia added her comment with increased conviction: "It is like politics. Politicians use a graph and tell you this is true, but you look at the graph, and it does not tell you everything."

Tricia Donovan
Pioneer Valley Adult Education Center, Northampton, MA

EMPower
extending mathematical power

Seeking Patterns, Building Rules

Algebraic Thinking

TEACHER BOOK

TERC

Seeking Patterns, Building Rules: Algebraic Thinking

This unit demystifies basic algebra, as students explore the meanings revealed by tables, graphs, verbal rules, and equations. By investigating patterns in their own lives, In-Out tables, banquet table and patio tile arrangements, calorie-burning tables, graphs, and equations, the relationship between diameter and circumference, pay and accumulated earnings, gas price increases, or cell phone use-cost patterns, students learn to connect algebraic representations with the linear (and occasionally non-linear) patterns or functions they describe. They see that algebraic tools and symbols serve to describe and interpret a situation; the situation itself is always central. Early lessons introduce students to ways of 'reading' tables, graphs, and equations through construction of these representations. Graph, table, verbal rule, and equation conventions become familiar through varied and meaningful use.

Students gradually gain proficiency in representing situations graphically and symbolically then deepen their understanding, as they explore concepts and representational conventions related to rates of change. They come to recognize equivalent expressions and to compare expressions. Students grasp what a y-intercept, flat-line graph, straight- or curved-line graph, or a point of intersection reveal about situations. All of this learning occurs in a lively, practical way that takes the fear out of approaching algebra and replaces it with a sense of wonder and mastery.

Correlations for *Seeking Patterns, Building Rules: Algebraic Thinking*

Book Description: Students use a variety of representational tools—diagrams, words, tables, graphs, and equations—to understand linear patterns and functions. They connect the rate of change with the slope of a line and compare linear with nonlinear relationships. They also gain facility with and comprehension of basic algebraic notation.

Lesson Number:	Lesson Name:	Mathematical Concepts/Topics Covered:
Opening the Unit	Seeking Patterns, Building Rules	• Personal patterns described and term 'pattern' explored for assessment purposes • Algebra vocabulary list initiated • Prior algebra knowledge assessed
Lesson 1:	Guess My Rule	• Patterns/relationships between two variables in a visual pattern • Expressing patterns in equation form
Lesson 2:	Banquet Tables	• Tracking table data • Multiple representations of a situation to predict outcomes
Lesson 3:	Body at Work—Tables and Rules	• Verbal and symbolic rule practice
Lesson 4:	Body at Work—Graphing the Information	• Graph features identified and compared • Graph generated from tables and/or equations
Lesson 5:	Body at Work—Pushing It to the Max	• Graph construction and connections practiced • *x-y* relationships explored
Lesson 6:	Circle Patterns	• Diameter and circumference relationship explored • Rule and formula application
Midpoint Assessment	Using the Tools of Algebra	• Production and interpretation of representations assessed • Symbolic notation use assessed
Lesson 7:	What Is the Message?	• Translating equations • Equivalent expressions • Coefficients – meaning and representations

Lesson 8:	Job Offers	• Algebraic problem solving • Point of intersection • y-intercept
Lesson 9:	Phone Plans	• Features of graphs • Matching representations • Supporting decisions with mathematical information
Lesson 10:	Signs of Change	• Constant rate of change identified and compared in representations
Lesson 11:	Rising Gas Prices	• Linear and non-linear patterns/rates of change compared
Lesson 12:	The Patio Project	• Algebraic knowledge applied
Closing the Unit	Putting It All Together	• Algebraic knowledge assessed

LESSON **9**

Phone Plans

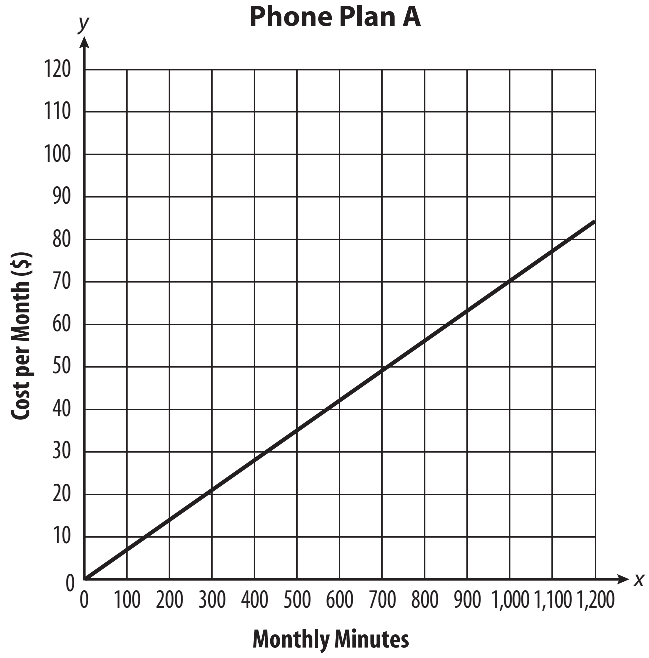
How do I know which plan is best for me?

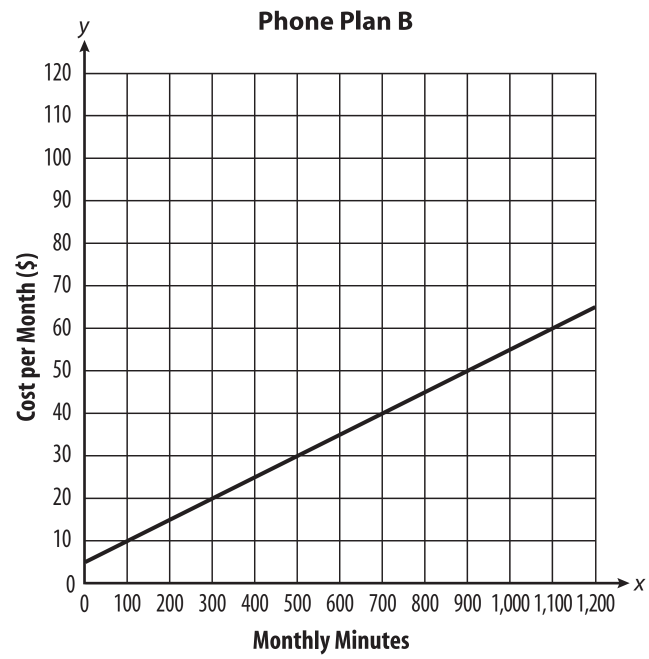
Ads are everywhere—luring us to choose a long-distance phone plan or to take a loan on credit, for example. It is not always easy to figure out which company offers the best deal. Your algebra tools will help you see more clearly which deal is best.

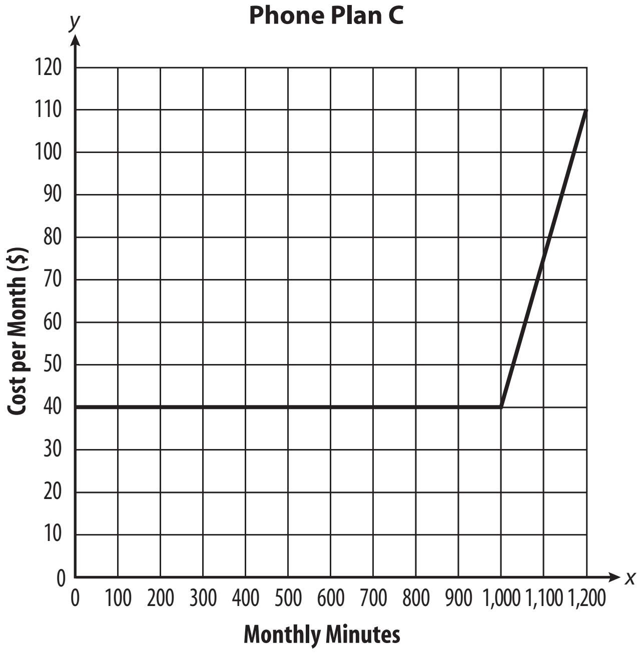
You will piece together information about four phone plans. To find the best deal, you will have to make sense of tables, graphs, words, and equations. Then you will think about the advantages and disadvantages of the plans for particular customers.

Activity 1: Phone Plans

Four ads for cellular and long-distance phone plans came in the mail today, but the ads got ripped up. Can you put the pieces back together? Your teacher will give you all of the words, tables, and equations from the ads. Reattach them to their matching graphs.



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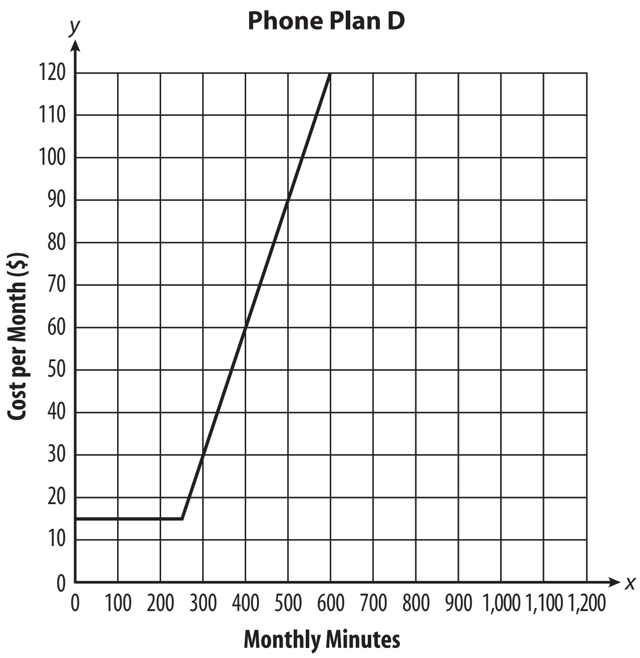
Monthly Minutes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Cost per Month ($)
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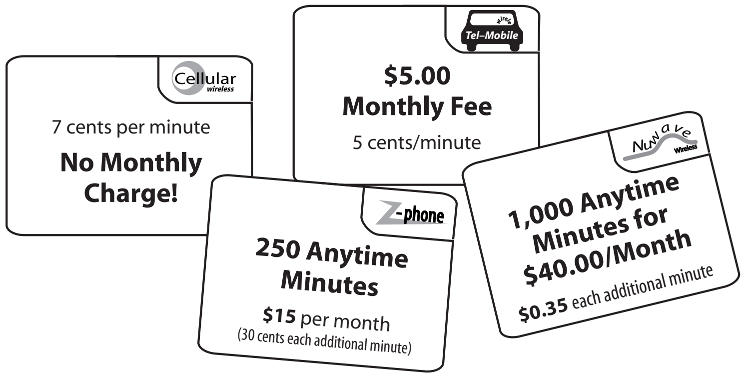
0 100 200 300 400 500 600 700 800 900 1,000 1,100 1,200
Monthly Minutes

Activity 2: It Would Depend on the Person



Use the pieced-together advertisements to answer the following questions about which plans are best and worst for specific customers:

1. Mary Jane loves to talk to her family, many of whom live out of state. She wants as many minutes as she can get, but definitely does not want to pay more than $50.00. Which plan should she choose? Why? Which plan would be the worst for her? Why?

2. Jenny, who lives in New York City, talks to her best friend in Miami every night for about a half-hour. Other than that, she makes very few long distance calls. Which plan is best for her and which is the worst? Why?

3. Tricia wants to get a phone she will only use in emergencies, for instance, if her car breaks down. Which plan is best for her and which is the worst? Why?

4. Which phone plan would be the best for you? Which would be the worst for you? Why?

Practice: I Am Changing the Rules!

You are the new Chief Executive Officer (CEO) of Cellular Wireless that has advertised Plan A, and you think it is time to change the rules.

The Old Rule

7 cents per minute

No Monthly Charge!

The New Rule

Only 7 cents per minute

Unlimited Minutes!!! *

(*$20.00 one-time joining fee)

Change the old equation, table, and graph to go along with your new rule for the first month.

The Old Equation

C = $0.07M

The New Equation

____

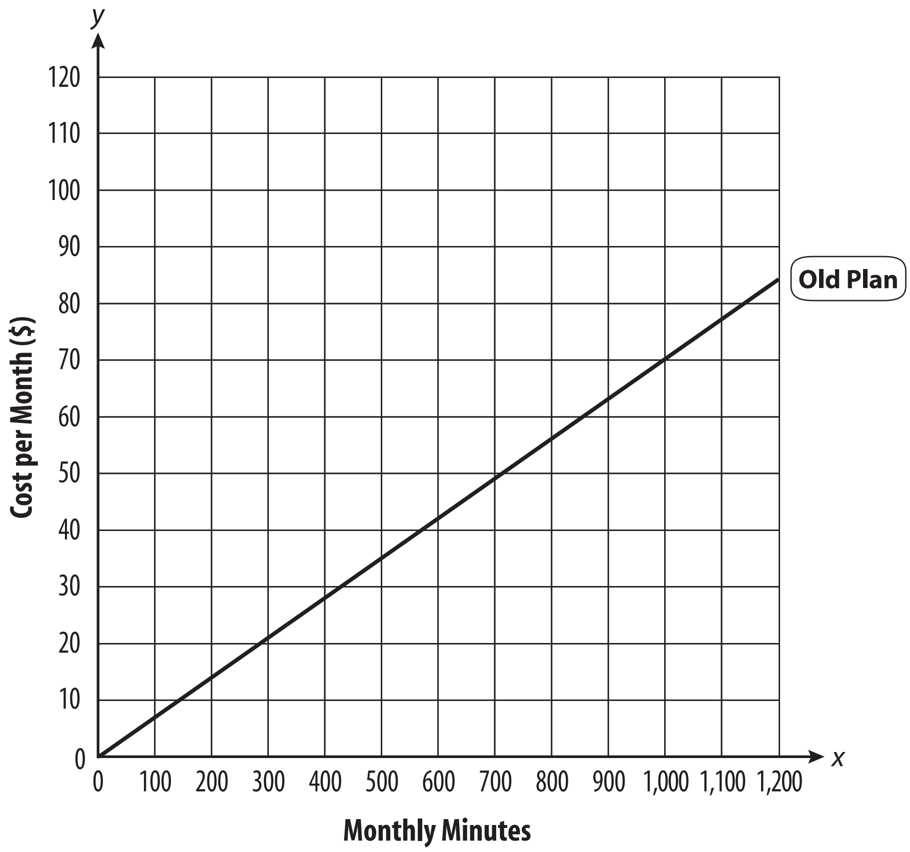
The Old Table

Monthly Minutes	Cost
50	$ 3.50
100	$ 7.00
200	$14.00
500	$35.00

The New Table

This is a graph of the old plan.

Graph the new plan here in another color.



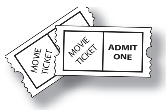
Symbol Sense Practice: Greater Than, Less Than

Math Symbol	In Words
=	equals
>	is greater than
<	is less than
≥	is greater than or equal to
≤	is less than or equal to

Use the above math symbols to rewrite the advertisements below where cost depends upon age. Let C stand for cost, and let A stand for age.

Advertisements / Math Symbols

1. Movie tickets



Under 12 years: $5.00 — When $A < 12$, $C = \$5.00$

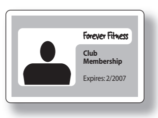
12 and over: $9.00 — When $A \geq 12$, $C = \$9.00$

2. Coffee at a diner

Cup of coffee: $1.25

Senior citizens (65 and over): $0.05

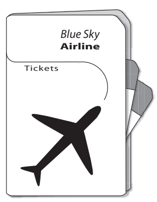
3. Club membership



Under 21: free

21 and over: $10.00

4. Airline tickets



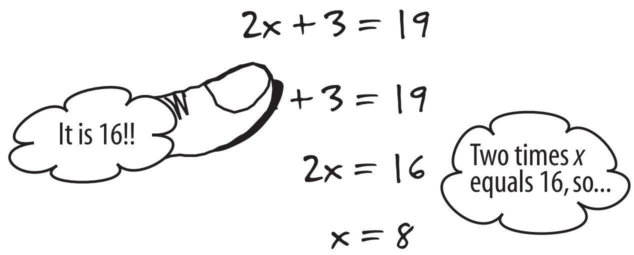
Five and over: $250

Under five years: half price

Symbol Sense Practice: Solving Two-Step Equations

One easy way to solve two-step equations is to cover part of the equation using the “finger cover-up” method.

Faced with an equation such as $2x + 3 = 19$, put your finger over the “$2x$” part of the equation like this:



Then ask yourself: What number would I add to 3 to get 19? You would add 16. That means that 2 times the missing number (x) must equal 16. The missing number must be 8!

Does it work? Replace the “x” with an 8 to see whether it makes sense:

$$2(8) + 3 = 19$$

It works! Try this method with the problems below. Remember to rethink your questions when you are subtracting, not adding, a number.

Balance the amounts. For Questions 1–10, find the missing number using the cover-up method.

1. $2x + 10 = 20$ $\qquad x =$ _______

2. $5x + 9 = 49$ $\qquad x =$ _______

3. $2x - 50 = 150$ $\qquad x =$ _______

4. $4x - 5 = 15$ $\qquad x =$ _______

5. $4x + 20 = 100$ $\qquad x =$ _______

6. $10x + 900 = 1{,}000$ $\qquad x =$ _______

7. $100x - 50 = 150$ $x =$ _______

8. $8x + \frac{1}{2} = 56.5$ $x =$ _______

9. $\frac{x}{2} + 7 = 27$ $x =$ _______

10. $\frac{x}{10} + 8 = 88$ $x =$ _______

For Questions 11–15, write the equation first, and then solve it using the finger cover-up method.

11. If you double a certain number and add 3, you will get 7.

The equation is ________________________. $x =$ _______

12. Multiply a certain number by 5 and add 3 to get 28.

The equation is ________________________. $x =$ _______

13. If you multiply a certain number by 1,000 and add 250, you will get 6,250.

The equation is ________________________. $x =$ _______

14. Multiply a certain number by 7 and subtract 4 to get 59.

The equation is ________________________. $x =$ _______

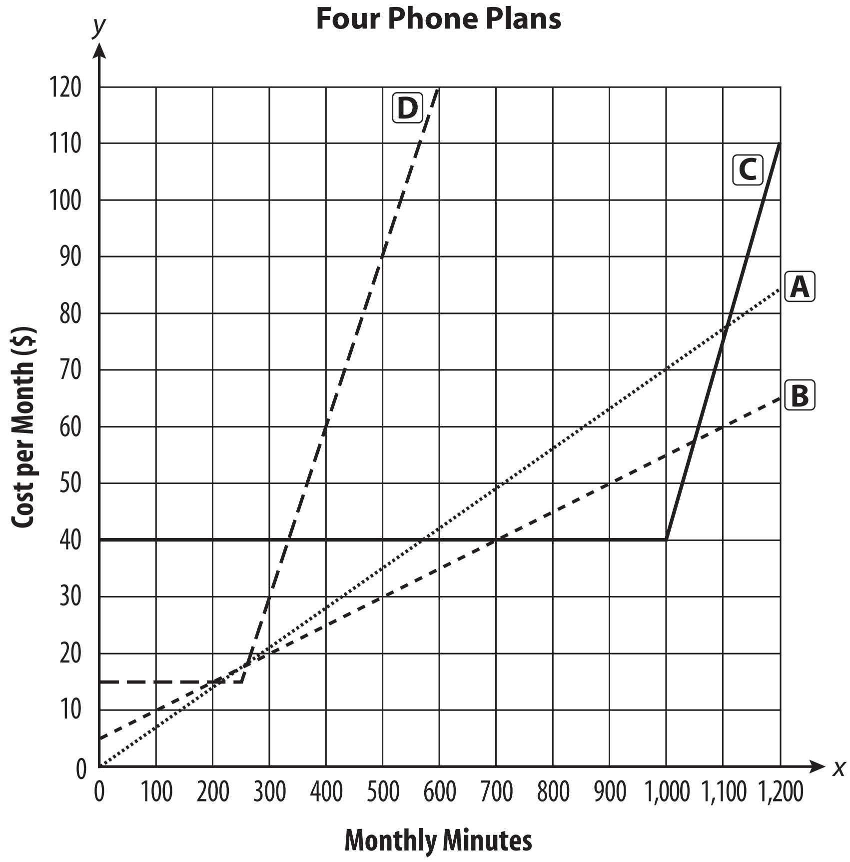
15. Half a certain number minus 10 is 41. Find the number.

The equation is ________________________. $x =$ _______

Extension: Looking at Four Graphs

Juania says: "I really do not think it makes much difference for me. On Plans A, B, and D, I will pay about the same … But Plan C is definitely out."

What do you think her calling pattern is? Use the graph to support your conclusion.



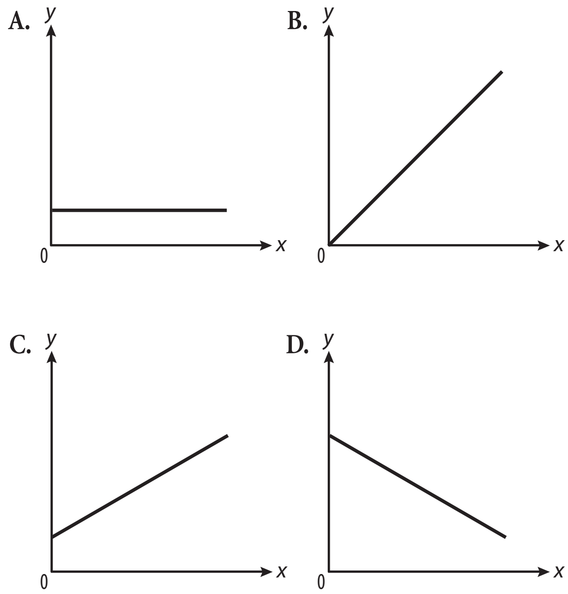
Test Practice

1. Which is a true statement?
   (1) $15 + 10 < 20$
   (2) $20 > 5 + 10$
   (3) $4(9) > 9(4)$
   (4) $4 + 9 > 9 + 4$
   (5) $20 + 5 < 15$

2. Which equation below would have the steepest graph?
   (1) $y = 2x + 20$
   (2) $y = 2x + 10$
   (3) $y = x + 1{,}000$
   (4) $y = 3x + 5$
   (5) $y = x - 1{,}000$

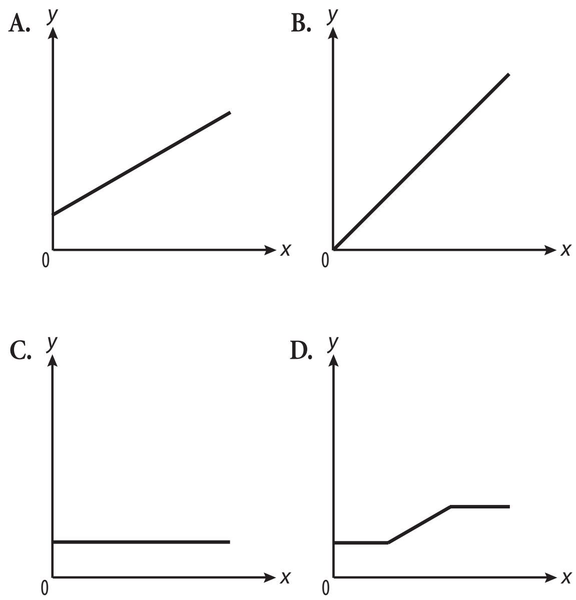
3. In which equation is 40 the value of x?
   (1) $x + 5 = 35$
   (2) $x - 5 = 45$
   (3) $45 = 2x - 5$
   (4) $x = 45 - 5$
   (5) $\frac{x}{2} = 90$

4. Which graph might represent the equation $y = 10$?



   (1) A
   (2) B
   (3) C
   (4) D
   (5) None of the above

5. Pedro likes to talk every day as much as he can on the phone. Which graph below represents a phone plan he would want? Cost of service is listed on the *y*-axis and minutes used on the phone is listed on the *x*-axis. The scales and intervals are the same on all graphs.



(1) A

(2) B

(3) C

(4) D

(5) None of the above

6. If $x = 5$, what is the value of $3x - \frac{1}{2}$?

Blackline Master 6
Phone Plans

Cellular wireless

7 cents per minute

No Monthly Charge!

Z-phone

250 Anytime Minutes

$15 per month

(30 cents each additional minute)

$C = \$5.00 + \$0.05M$

$C = \$0.07M$

Tel–Mobile Wireless

$5.00 Monthly Fee

5 cents/minute

NuWave Wireless

1,000 Anytime Minutes for $40.00/Month

$0.35 each additional minute

Monthly Minutes	Cost
50	$3.50
100	$7.00
200	$14.00
500	$35.00

Monthly Minutes	Cost
50	$7.50
150	$12.50
250	$17.50
350	$22.50

Monthly Minutes	Cost
100	$15.00
150	$15.00
250	$15.00
300	$30.00

Monthly Minutes	Cost
500	$40.00
1,000	$40.00
1,100	$75.00
1,200	$110.00

for $M \le 250,\ C = \$15$
for $M > 250,\ C = \$15 + \$0.30(M - 250)$

for $M \le 1{,}000,\ C = \$40$
for $M > 1{,}000,\ C = \$40 + \$0.35(M - 1{,}000)$

9

FACILITATING LESSON

Phone Plans

How do I know which plan is best for me?

Synopsis

Students piece together information about four long-distance phone plans and decide which plan best suits three customers with different needs. Previously, situations revealed patterns with a constant increase or decrease; here students also consider a situation where there is no change.

1. Pairs match ads, tables, graphs, and equations for four phone plans.
2. The class comes together to discuss graph and equation features that aided the matching process.
3. Pairs consider three consumer scenarios to decide which plan would be best and which would be worst for the customer, and justify their reasoning.
5. Everyone reflects on the representations, and the class discusses which tool they would use if faced with a similar situation.

Objectives

- Match graphs, tables, equations, and verbal rules by identifying the related features in each representation
- Connect the flatness of a horizontal line on a graph to a situation in which there is no change over time
- Use information from tables, graphs, rules, and equations to support consumer decisions

Materials/Prep

- Calculators
- Newsprint or transparencies
- Rulers
- Tape or glue

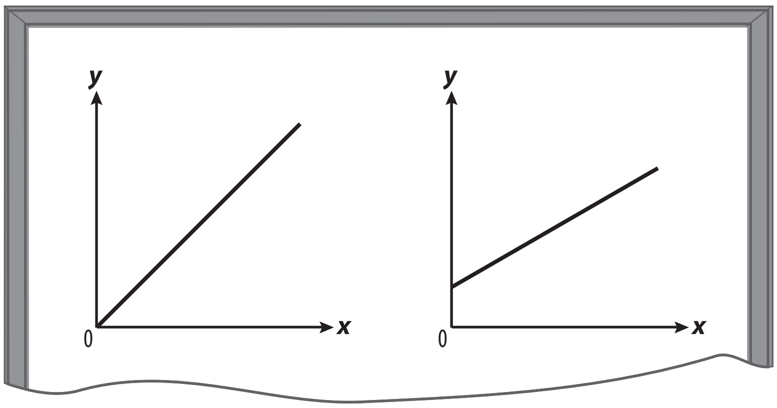
Make an enlarged version of each phone-plan graph (*Student Book*, pp. 114–15), on newsprint, or reproduce the graphs on transparencies.

Photocopy *Phone Plans*, *Blackline Master 6*, one for every pair of students. Cut the 12 pieces apart and put them in an envelope. (Students could do the cutting as well, but the scrambled pieces with unknown equations are more dramatic.)

Opening Discussion

(Optional: You might want to briefly review inequality symbols, which will be used in the phone-plan description. Write on the board: >, <, =. Ask volunteers to come to the board to read and write statements using one of the three symbols. Students share ways they remember the direction of the "is greater than" and "is less than" signs.)

Open by displaying a sketch of the salary graphs from *Lesson 8*.



Ask:

What on the graph told you about the starting bonus?

What on the graph told you who was getting a higher weekly rate of pay?

Say:

Today you will have four phone plans to compare in order to make some consumer decisions.

Activity 1: Phone Plans

Pair up students and distribute an envelope of cards (*Phone Plans*, *Blackline Master 6*) to each. Ask for a volunteer to read aloud the directions for *Phone Plans* (*Student Book*, p. 114). Allow everyone to work independently for 10 minutes, attaching the graphs to their matching representations. Observe which features draw students' attention and are used to help them connect representations.

Challenge those who finish quickly with an additional question:

Which monthly plan is the best for $50? Why?

Then ask students to pair up again to share their reasoning. Ask some pairs that come to agreement to write the equation, table, and ad for one plan on the enlarged versions of the graphs that you prepared earlier.

Draw the class together to address each plan. Pose questions that illuminate the graph features:

What tells you these go together?

Where do you see the table data in the graph?

Where do you see the equation in the graph—the coefficient, or the constant number added?

Invite students to the board to demonstrate how they made the connections. In particular ask:

Phone Plan A

Why does this line start here? (Point to the origin.)

Phone Plan B

Why does this line start here? (Point to (0, 5).)

Which graph is steeper—Plan A's or Plan B's? What does that tell you?

Phone Plan C

How is this graph different from those for Plans A and B?

Why is part of the graph flat? (Encourage references to the tables and equations to support statements made.)

What is happening at this point? (Point to (1,000, 40).)

What would the ad say if the graph looked like this? (Draw a flat line: $y = 40$.)

Phone Plan D

Compare the graphs of Plan D and Plan C.

Clear up any confusion about the equations by asking questions such as

Why do you subtract 1,000 from *M* in the equation for Plan C and subtract 250 from *M* in the equation for Plan D?

Ask for some examples to emphasize the connection between the situation described verbally in the ad and symbolically in the equation. Then ask about the rate of change:

Which plan shows the fastest rate of increase in cost for any period of time? What tells you that information?

Expect to hear that some see the rate of increase in the graph and others in the equation for Plan C.

Heads Up!

The conversation in this first activity can get very involved. Move on to the next activity. In the summary, continue discussing connections between representations.

Activity 2: It Would Depend on the Person

Turn to *It Would Depend on the Person* (*Student Book*, p. 116).

This activity uses the phone-plan representations to support a consumer decision. Students will need all four re-pieced ads to come to a good recommendation. Three kinds of customers are portrayed.

Suggest students count off: 1, 2, 3, 1, 2, 3 … to determine the number of the problem they will work on. Allow them to work independently for about 15 minutes.

When most students have completed the problem, ask all those with the same numbers to form groups to share their answers and the ways they made their decisions. Allow time for each group to come to a common agreement for a phone plan for its customer. Then ask a spokesperson from each group to make a persuasive recommendation to their customer as to the *best* and the *worst* plan for that person. Every statement should be justified using at least one type of representation.

If the following have not been answered by the group's spokesperson, ask:

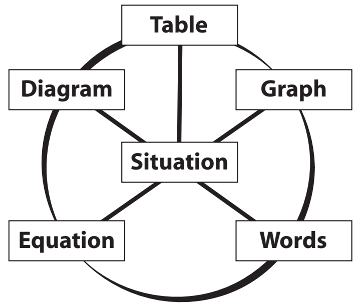
Which representation(s) did you use to help you decide?

What information in that representation swayed your decision?

How is your decision supported by another representation?

Assign Problem 4 to groups who finish early or for homework.

Summary Discussion



Discuss the different representations everyone used to help make decisions and what information was gleaned from each representation. Ask:

Which type of representation would you turn to first if you had to make this kind of decision again? Why?

Encourage specificity in responses, using today's work for points of reference. Refer students to their *Reflections* section (*Student Book*, p. 162).

Practice

I Am Changing the Rules!, p. 117
Gives students an opportunity to adjust one of the plans and its representations accordingly.

Symbol Sense Practice

Greater Than, Less Than, p. 119
Asks students to express conditions in mathematical notation using inequality symbols.

Solving Two-Step Equations, p. 120
Suggests some ways to solve equations in the form $ax \pm b = c$, using a simple strategy.

Extension

Looking at Four Graphs, p. 122
Asks for justification based on intersections of the four phone-plan graphs.

Test Practice

Test Practice, p. 123

Looking Closely

Observe whether students are able to

Match graphs, tables, equations, and verbal rules by identifying the related features in each representation

If students gravitate toward certain numbers and make decisions based on them—such as the number 1,000, which surfaces in the equation, the table, the graph, and the ad for Plan C—ask where the number occurs in other representations. What explains the flatness of this line? What tells you the graph starts on the *y*-axis?

Look for informal connections that arise, such as

- When Out numbers increase in a table as the In numbers increase, the line goes up to the right.
- When Out numbers stay the same in a table as the In number increases, the line is flat.
- The number multiplying x in an equation is the difference between the y numbers in the table.
- When the graph does not start at the origin, a number is added to another in the equation.
- If one line is steeper than another, the cost is increasing at a faster rate in the first line.

Connect the flatness of a horizontal line on a graph to a situation in which there is no change over time

Label a few points on the graph, and connect these to numbers in the tables and the ads, if students do not understand what the flatness indicates.

Ask students to think of other situations where there is no change over time (flat rates). For instance, if a heart-monitor graph shows a **flat line**, what does that mean?

Use information from tables, graphs, rules, and equations to support consumer decisions

Ask students what each person would be looking for in a phone plan. Students should be able to explain their decisions by referencing information presented in the various formats.

WHAT TO LOOK FOR IN *LESSON 9*	WHO STANDS OUT? (LIST STUDENTS' INITIALS)			NOTES FOR NEXT STEPS
	STRONG	ADEQUATE	NEEDS WORK	
Representations Graphs • Explains reasons for shape of each graph				
Connections • Matches representations with confidence • Flexibly moves among representations, connecting key features • Explains how table information shows up on each graph • Explains how features of the equations connect to the graph and the table				
Problem Solving • Uses information in the representations to solve problems and support conclusions				

Rationale

This lesson focuses attention on the different representations of a consumer scenario, and the capacity of algebra to simplify complicated situations. Learning to use the tools of algebra efficiently helps students evaluate situations that involve more than one set of conditions.

Math Background

In this lesson, the slant of a constant function, which appears in a graph as a flat line, is considered (a constant function with zero slope). In more formal algebra, the disappearance of x in such situations is explained by the fact that $y = mx + b$, and $m = 0$. Here, however, students focus on the constancy of the output, no matter the value of the input.

Notation is introduced for a situation where the rule changes at a certain point. [When $M \leq 1{,}000$, $C = 40$. When $M > 1{,}000$, $C = 40 + 0.35\ (M - 1{,}000)$].

Context

Phone plans are evolving, and many people have to make decisions about cell-phone and long-distance plans. The pricing structure for cell-phone plans used in this lesson was common in 2004. You might want to consider other pricing scenarios that have evolved since.

Facilitation

Making the Lesson Easier

Start with matching the graphs and tables first. Then match the ads with the equations.

Making the Lesson Harder

Change aspects of the graph, and ask people to predict how the changes might affect the equation and vice versa.

Gather some advertisements, such as grocery ads from three stores, and ask students to develop representations for one product.

Activity 2: It Would Depend on the Person motivated students to bring their everyday decision-making skills to bear when choosing the phone plan best suited to each person's particular phone habits.

For the final question on their own best phone plan, some students were sure of their monthly usage—400 minutes, for example—and compared plans accordingly:

A = $ 28 per 400
B = $ 30 per 400
C = $ 30 per 400
D = $ 40 per 400

The flat rate of $40.00 for 1,000 minutes appealed to many:

Plan C will be better for me, I make call anytime and 1000 minutes is a lot of minute at the rate of $40- per month.

Barbara Tyndall used a two-part rubric ("What I notice" and "My interpretation and next steps" to help her students examine their work. Below are examples of two students' written work from Practice: I Am Changing the Rules!, along with Barbara's written comments.

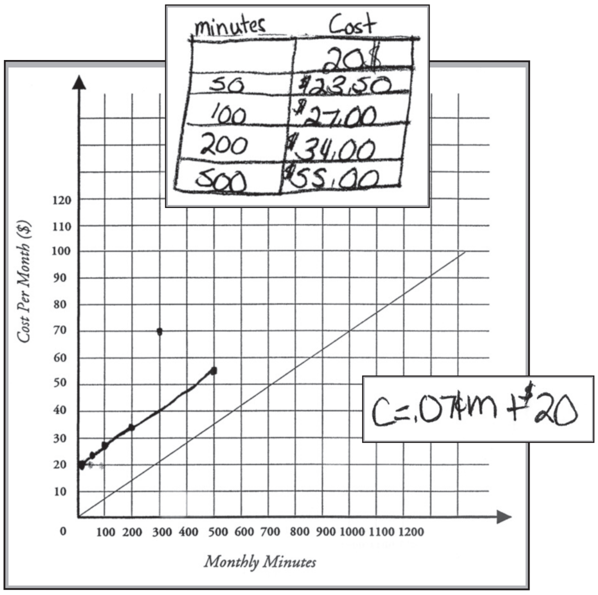
What I notice:

- Can write an equation to go with the verbal description.
- Creates a table that is accurate and reflects the initial cost.
- Correctly graphs the equation for the new rule, new equation, new table!

My interpretation and next steps:

- Can be alerted to correct notation ($0.07 or 7¢, not $0.07¢).
- Could put "0" in minutes column for first entry (student had erased the zero).
- In response to my asking what the student notices about the two line graphs, he or she mentions "constant rate of change" and "slope."
- Student is ready to go to next lesson!

Student 1 Response to I Am Changing the Rules!



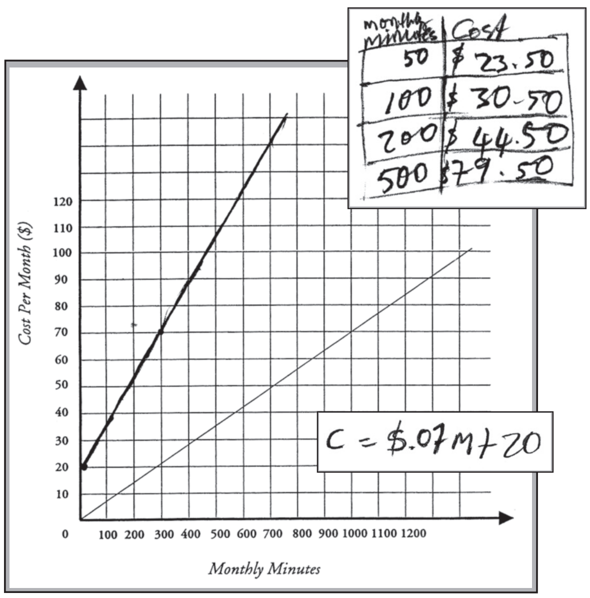
What I notice:

- Writes a new equation that shows the new rule.
- Correctly calculates the cost for 50 minutes, but subsequent costs are incorrect in the new table.
- Graph starts out at the correct point (0, 20), but does not accurately reflect the data in the table.

My interpretation and next steps:

- Understands the connection between the verbal description and the equation.
- Can use the equation to calculate the cost between the first value (of 50 min.) for time, but seems to use a different pattern to complete the table.
- May have difficulty estimating the values on the scale.
- Needs more practice before going to the next lesson.
- Should be urged to check that the values in the graph all follow the rule described.

Student 2 Response to I Am Changing the Rules!



Lancaster-Lebanon Intermediate Unit/13 Career Link, Lancaster, PA

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Everyday Number Sense

Mental Math and Visual Models

TEACHER BOOK

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Everyday Number Sense: Mental Math and Visual Models

Learning about numbers and operations needn't produce a class of yawning faces or tense handgrips on pencils, as you will discover when students tackle the engaging problems presented in *Everyday Number Sense.* Problems involving travel distances, historical dates, temperature fluctuations, mortgage payments, shopping questions, calculator conundrums, and mathematical puzzles, allow students to build upon their own robus strategies for adding, subtracting, multiplying, and dividing. However, the problems presented also strengthen students' number and operation sense by encouraging them to solve problems using mental math strategies such as estimating and adjusting, as well as grouping, visualizing, and decomposing numbers. These strategies expose the structure of the number system in ways that lead to 'algebrafying' arithmetic and they help students more easily manage numbers. Along the way students see how mathematical tools—number lines, arrays, diagrams, and calculators—can ease mathematical problem solving.

Everyday Number Sense focuses on the whole number benchmarks of 1, 10, 100, and 1,000 in a variety of world-life situations where mental math, estimation, and calculator skills prove useful. Multiple strategies are key. The lessons guide students toward development of computational fluency, flexibility, and accuracy and an understanding of operation meanings. Students come to understand the relationship between addition and subtraction as they count and compare quantities, and to understand the multiplication and division relationship in terms of equal group problems. These unique lessons move students beyond the anxious world of remembered (or forgotten) algorithms and into the world of mathematical problem solving, reasoning, connecting, and communicating

Correlations for *Everyday Number Sense: Mental Math and*

Book Description: Students solve problems with whole numbers using mental math strategies with benchmarks of 10, 100, and 1000. Number lines, arrays, and diagrams support conceptual understanding of number relationships and the four operations.		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/Topics Covered:**
Opening the Unit	Everyday Number Sense	• Personal math experiences shared • Mental math skills demonstrated • Fluency with visual models and symbolic expressions demonstrated
Lesson 1:	Close Enough with Mental Math	• Totals estimated using mental math strategies
Lesson 2:	Mental Math in the Checkout Line	• Totals computed mentally by rounding and then adjusting • Mental math processes described with mathematical notation
Lesson 3:	Traveling with Numbers	• Numbers on a number line are located, put in order, and operated on • Numbers rounded to the nearest 10 and 100
Lesson 4:	Traveling in Time	• Mental math strategies explained with a number line • Addition and subtraction problems solved by counting up and down by 10's and 1's • Mental math and number-line actions recorded with equations
Lesson 5:	Extending the Line	• Negative and positive numbers located on a number line • Difference between two numbers determined
Lesson 6:	Taking Your Winnings	• Composition of numbers in terms of 10's, 100's, and 1,000's examined and identified • Parentheses in expanded notation used • Multiples of 10, 100, and 1,000 added and subtracted mentally • Mental calculations checked with calculator
Midpoint Activity:	What Math Means to Me	• Personal feelings and perspectives about math shared • Another's perspective interpreted
Lesson 7:	Patterns and Order	• Patterns for multiplying and dividing by 10, 100, 1,000 identified • Problems solved using order of operations

Lesson 8:	Picture This	• Arrangements of objects in groups and arrays reflected in written expressions • Expressions represented using arrays and/or equal groups arranged to correspond with numbers and operations • Equivalent expressions identified
Lesson 9:	What's the Story?	• Words problems represented with pictures and mathematical equations • Problem-solving strategies recorded with equations and pictures
Lesson 10:	Deal Me In	• Division connected to the act of splitting or dealing out an amount • Verbal language and symbolic notation matched for division to a concrete model • Mental math strategies for division applied to situations calling for splitting dollar amounts over 4, 8, 12, 24, 10, or 100 time periods
Lesson 11:	String It Along	• Direct measurements and scale used to find number of groups of a given size in a total • Mathematical symbols used to express the action of division • Division related to multiplication and factors of 48 and 72 identified
Lesson 12:	Making Do	• Remainders dealt with sensibly, given the context of the problem • Remainders written and understood as decimals, fractions, and whole numbers
Closing the Unit:	Computer Lab	• Content taught in unit consolidated • Areas of strength and weakness assessed

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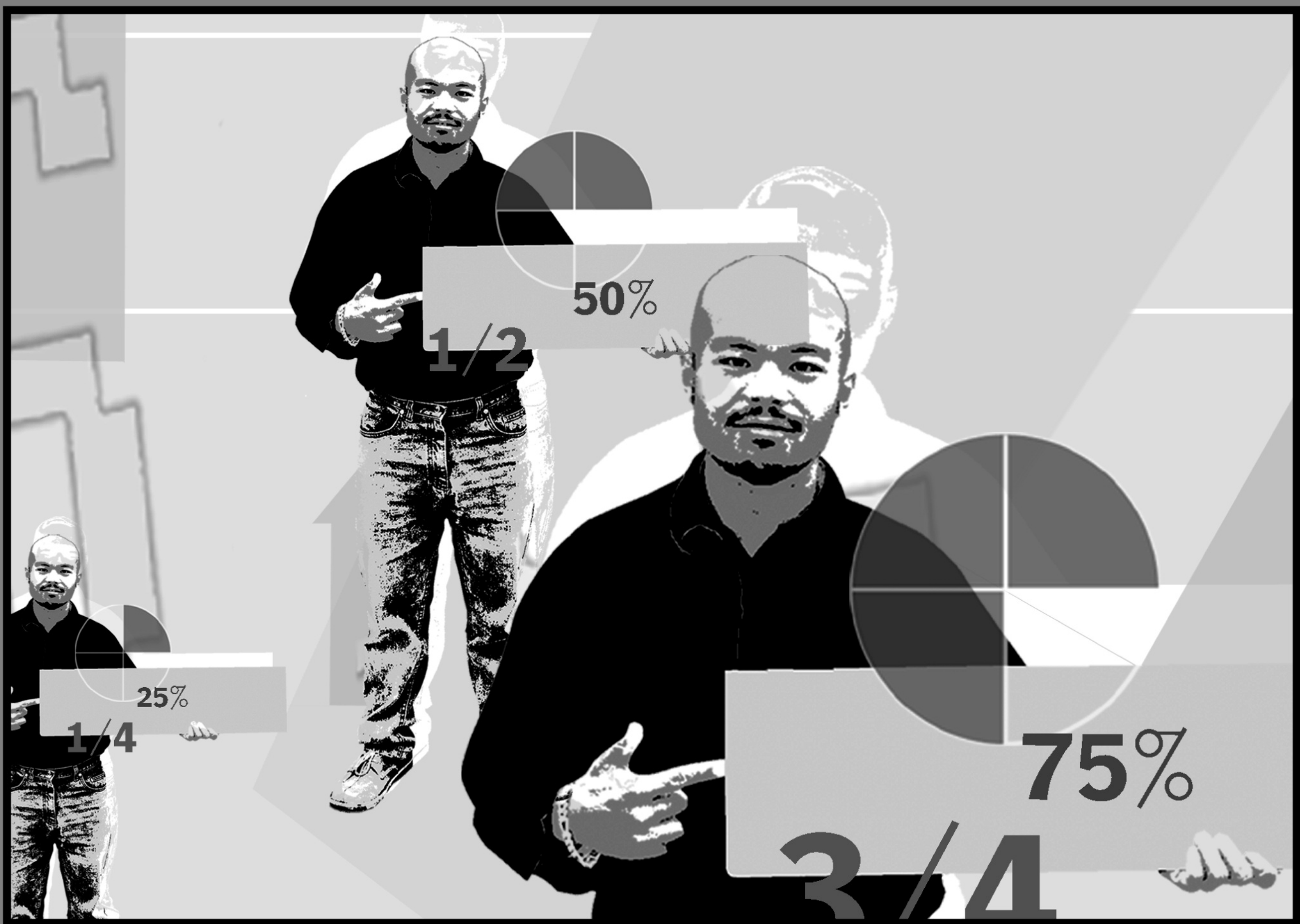
Using Benchmarks

Fractions, Decimals, and Percents



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Using Benchmarks: Fractions, Decimals, and Percents

Fractions may be interpreted many ways. In *Using Benchmarks*, students focus on the concept of fractions as representations of part/whole relationships. This opens the door for them to compare fractional quantities and make useful estimations about the size of amounts in a wide array of real-world situations. Understanding of the benchmark fractions, 1/2, 1/4, 3/4 and 1/10, develops gradually as students count and draw objects and answer the ever recurring questions: "What's the whole?" "What's the part?" As they move flexibly between finding a fractional part of an amount, as well as finding the whole when the part or fraction is known, students learn to use drawings and objects to support their reasoning and communication with others. By using the fundamental, familiar tools called benchmark fractions, decimals, and percents, students gain the dexterity necessary to explore the larger world of rational numbers.

Connections between the benchmark fractions and their decimal and percent equivalents are reinforced continually, so that students acquire flexibility, as well as fluency, with the use of benchmark terms. In addition, they learn to consider the part counted and the part remaining as complements that make a whole, so 3/4 is first introduced as the part left over when 1/4 is used and 9/10 is seen as the remaining amount when 1/10 is taken. Through a well-sequenced series of accessible lessons, students encounter deep mathematical ideas related to rational numbers. Along the way, they cement an understanding of the crucial and frequently used benchmark fractions.

Correlations for *Using Benchmarks: Fractions, Decimals, and Percents*

Book Description: Students use the fractions 1/2, 1/4, 3/4, and1/10; the decimals 0.1, 0.5, 0.25, and 0.75; and the percents 50%, 25%, 75%, 100%, and the multiples of 10% as benchmarks to describe and compare all part-whole relationships.		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/Topics Covered:**
Opening the Unit	Using Benchmarks	• Fraction knowledge assessed • 1/2 examples demonstrated
Lesson 1:	More Than, Less Than, or Equal to One-Half	• Part/whole relationship highlighted • Fractional amounts compared to 1/2 • Multiple representations for 1/2 – decimal and percent • A 'whole' stated as a fraction
Lesson 2:	Half of a Half	• 1/4 determined with various strategies • Multiple representations for 1/4 – decimal and percent • 3/4 and the whole detemined by knowing 1/4
Lesson 3:	Three Out of Four	• 3/4 determined • Multiple representations for 3/4 – decimal and percent • Multiplying and dividing to find 3/4
Lesson 4:	Fraction Stations	• Fractions compared and described in relation to benchmarks for assessment purposes
Lesson 5:	One-Tenth *	• 1/10 determined • Multiple representations for 1/10 described
Lesson 6:	More About One-Tenth	• Multiple representations for 1/10 matched • 1/10 and 9/10 connected
Closing the Unit	Benchmarks Revisited	• Multiple representations – fractions, decimals, and percents – used in problem solving for assessment purposes • Benchmark fraction/decimal/percent information interpreted for assessment purposes

T E R C

Professional Development is an important element of the EMPower Program. TERC offers a variety of professional development institutes designed to give teachers “hands-on” experience to better understand the EMPower approach.

To find out more about Professional Development for EMPower visit http://empower.terc.edu

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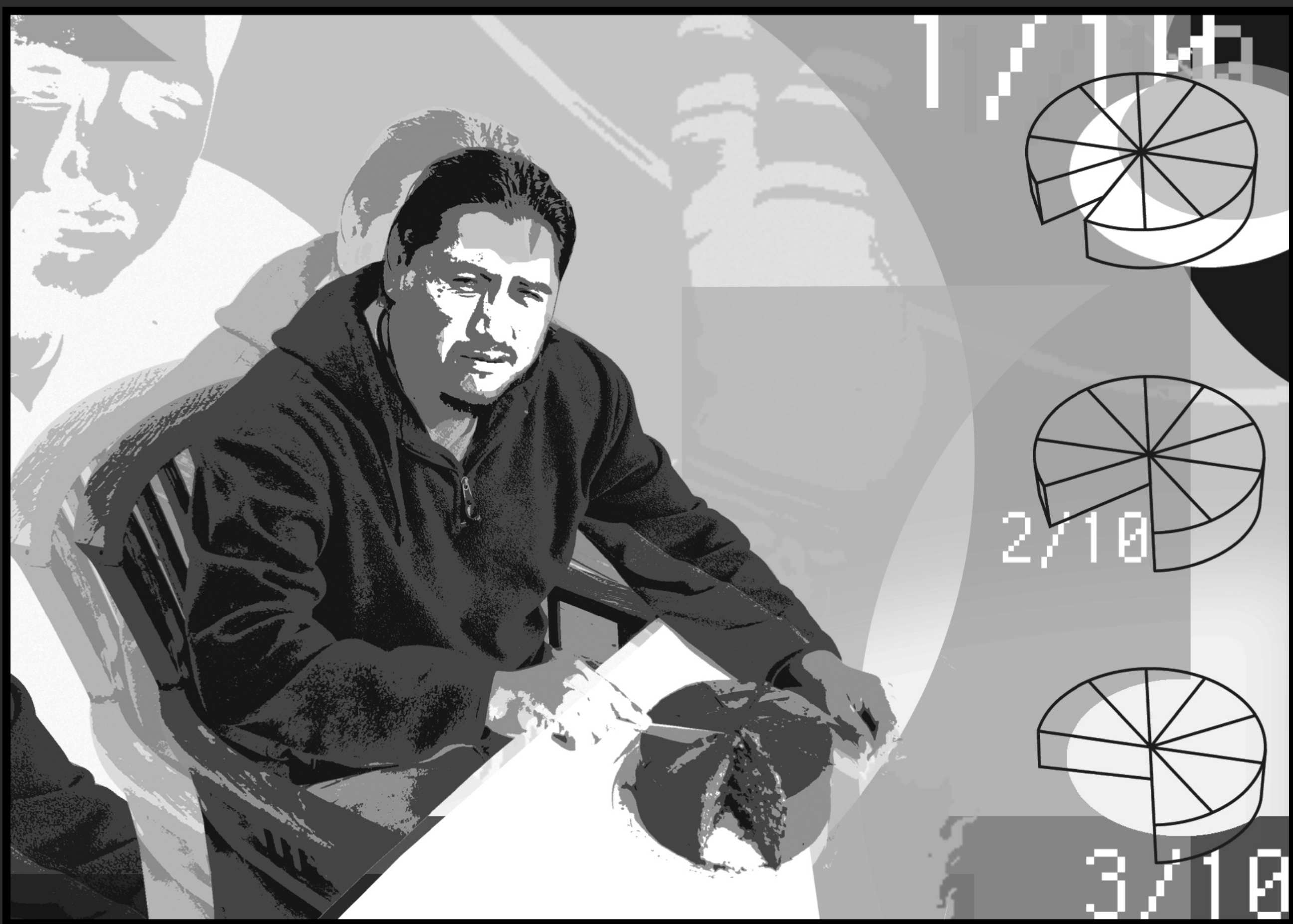
Split It Up

More Fractions, Decimals, and Percents



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Split It Up: More Fractions, Decimals, and Percents

Introduced in *Using Benchmarks*, 10% becomes the launch and a central math concept throughout *Split It Up*. This unit expands students' repertoire of fractions, decimals, and percents to include multiples of 10%, 1% or 1/100 and its multiples, as well as 1/8 and 1/3 and their multiples while maintaining the focus on fractions as representations of part/whole relationships and as models of the portion of an amount. Students puzzle over situations involving newspaper statistics, space allocations, taxes, material purchases, and nutrition labels, as they hone their rational-number mental math skills. The emphasis on mental math strategies allows students to reason about ways to combine and break apart amounts. For instance, they can regard 42% as the combination of four 10% amounts and two 1% amounts, or reason about 3/8 as 1/2 (50%) less 1/8 (12.5%), arriving at 37.5%.

Continued use of diagrams, manipulatives, and other visual models works to support reasoning. Students determine portions and determine the whole given a part. They calculate the percent of increase or decrease of whole numbers, compare fraction, decimal and percent amounts to benchmarks, and consider which form—fraction, decimal, or percent—seems best to use when solving problems. In *Split It Up*. Students learn these skills in ways that continue to serve them well in the world beyond the classroom. They come to rely on reasoning, not memorization, when solving mathematical problems that involve fractions, decimals, and percents.

Correlations for *Split It Up: More Fractions, Decimals, and Percents*

Book Description: Building upon their command of common benchmark fractions, students add 1/3's, 1/8's, and 1/100's, and their decimal and percent equivalents, to their repertoire of part-whole relationships.		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/ Topics Covered:**
Opening the Unit	Split It Up	• Prior knowledge of fractions, decimals, and percents assessed • Fractions, decimals, and percents vocabulary list started • Situation described using fractions and percents
Lesson 1:	Numbers in the News	• Total amount determined from 10% • 1/10 and 10% of an amount explored
Lesson 2:	What Is Your Plan?	• Multiples of 10% of an amount determined • Decisions involving percents based on the whole being 100% • Arrays of 50 and 100 used to show percents • Multiples of 10% with equivalent fractions named
Lesson 3:	One Percent of What?	• 1% and its multiples of three- and four-digit numbers found • 10% of an amount and 1% of another are compared to show effect of the size of the percent and the whole on the size of the answer
Lesson 4:	Taxes, Taxes, Taxes	• Multiples of 1% used to find single-digit percents • Multiples of 10% and 1% combined to find two-digit percents
Midpoint Assesment	Meeting Your Goals	• Knowledge of everyday percent and fractions assessed • 10%, 1%, and other benchmark percents used to solve problems • Three fraction and percent situations compared
Lesson 5:	Fold and Figure	• One-eighth of an amount determined • Eighths related to corresponding percents • 12.5% calculated using multiples of 10% and 1% • One-eighth and 125/1,000 demonstrated to be equal
Lesson 6:	Give Me a Third	• Thirds and their percent equivalents demonstrated with circle graphs, number lines, and countable objects • Thirds compared to other benchmark fractions • Given one-third or two-thirds, the whole is found
Lesson 7:	Order, Choose, and Change	• Fractions, decimals, and percents placed in order • Fraction amounts compared • Fraction, decimal, or percent chosen to solve problems • Given a part, the whole is found

Lesson 8:	Where's the Fat? *	• Percent compared with a benchmark fraction • Percents determined by a part and a whole • Two or more quantities combined and percent found
Lesson 9:	Put It Together	• Idea of finding half of a fraction generalized • Unit reviewed • Unit reviewed

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Keeping Things in Proportion

Reasoning with Ratios

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Keeping Things in Proportion: Reasoning with Ratios

Proportional reasoning is an essential skill. Adults call upon this type of reasoning in everyday situations as well as in many areas of mathematics study. Traditionally, mathematics classes rush to cross-multiplication as the tool of choice for solving proportion problems. However, *Keeping Things in Proportion* begins by building on students' intuitive knowledge and the multiplicative relationships that are at the heart of proportionality. The hands-on lessons in this unit connect the central ideas of proportion across the spectrum of mathematics. Students work with rates and ratios in shopping, graphic design, and sampling situations that draw upon data and geometry knowledge while laying the groundwork for algebra study.

As students progress from concrete experiences with ratios to more challenging situations, they develop a bank of tools and strategies to solve proportional problems, and to examine the relationships *within* and *between ratios*. Tools include the rule of equal fractions, tables, graphics, unit rates, and cross-multiplication. Always, students are asked to use two solution methods to arrive at an answer. Non-proportional situations are considered as well. To facilitate conceptual development, numbers start out 'friendly' and turn 'messier' as the unit progresses. The numbers, however, prove less daunting to students as they apply their secure knowledge about proportion. Formal proportional reasoning evolves over time, and the lessons in this unit ensure that students are able to make proportional predictions and adjustments using a variety of tools effectively.

Correlations for *Keeping Things in Proportion: Reasoning with Ratios*

Book Description: Students use various tools—objects, diagrams, tables, graphs, and equations—to understand proportional and non-proportional relationships.		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/ Topics Covered:**
Opening the Unit	Comparing and Predicting	• Additive and multiplicative ways to compare amounts demonstrated • Ability to solve proportional and non-proportional problems assessed • Experiences with rate, ratio, and proportion shared
Lesson 1:	A Close Look at Supermarket Ads	• Ratios in everyday consumer advertisements identified • Equal ratios determined and equality demonstrated with diagrams • Mathematical rule for establishing equal ratios developed • Problems solved using equal ratio diagram or mathematical rule
Lesson 2:	It's a Lot of Work!	• Sample of work conducted and described over a period of time • Sample used to make a prediction for a larger amount by reasoning with equal ratios
Lesson 3:	Tasty Ratios *	• Ratios used to describe taste and visual comparisons • Ingredients adjusted so that proportions are correct
Lesson 4:	Another Way to Say It	• Two amounts compared using alternate but equivalent methods • Percents as ratios used to compare the part to the whole amount • Whole numbers rounded to make comparisons more manageable
Lesson 5:	Mona Lisa, Is That You?	• Reproductions in various sizes judged by eye to determine if proportional to an original • Measurements used to determine whether reproductions are proportional • Graph used as a tool to test for good reproductions (equal ratios) • Graph points connected with number pairs
Lesson 6:	Redesigning Your Calculator	• Rectangular shapes that are similar to one another drawn and measured • Fractions of a centimeter expressed as decimals • Area and perimeter changes are contrasted when length and width are doubled and halved
Interim Assessment	Checking In	• Sets of equal ratios created • Comparisons, predictions, and decisions made with ratios • Ratios in various formats written and interpreted

Lesson 7	Comparing Walks	• Speed described and quantified as relationship of distance and time • Strategy to find unit rate for *a/b* developed
Lesson 8	Playing with the Numbers	• Cross-product property introduced as another tool to check that two ratios are equal • True and false proportion equations examined • Estimation used to predict for more complicated numbers in proportions • Missing number in a proportion determined
Lesson 9	The Asian Tsunami	• Proportional reasoning concepts applied to international currency conversion • Estimation used to predict for difficult numbers in proportion problems • Exact (or nearly exact) answer determined for a missing number in proportion problem
Lesson 10:	As If It Were 100	• Percents used to make comparisons between data sets of different sizes, some with very large numbers • 1,000 used as a base for comparison between data sets of different sizes
Closing the Unit	Reasoning with Ratios	• Various tools used to address different proportionality situations assessed • Comparisons, predictions, and decisions made with ratios

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Operation Sense

Even More Fractions, Decimals, and Percents

TEACHER BOOK

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Operation Sense: Even More Fractions, Decimals, and Percents

Some strategies that work when operating with whole numbers hold up when working with decimals or fractions; some do not. In *Operation Sense: Even More Fractions, Decimals, and Percents*, students use mostly mental math and estimation skills to reason about operations with rational numbers. They use diagrams, number lines, and models to visualize situations in which rational numbers are added, subtracted, multiplied, and divided, and they begin to formulate a sense of how results differ when, for instance, they multiply by a fraction rather than a whole number. The unit stresses decimal-fraction equivalencies throughout, so students work with the form that best fits a problem or their own preferred strategies.

As students solve problems taken from every-day life contexts, they connect operations with fractions, decimals, and percents to related math content, in particular geometry and data. The work begins with consideration of operations with benchmark rational numbers then moves tower 'messier' numbers, like 7/16. Always students are asked to consider: Does that answer make sense? Algorithms emerge from students' own reasoning and practices, so they remain robust long after students leave the classroom. This is not a book about copying steps outlined at the top of the page; it is a book that helps students understand phenomena that fly in the face of what people have come to expect when operation on whole numbers. For example, one can multiply to get an answer smaller than the factors (e.g., $1/2 \times 1/2 \approx 1/4$), or one can divide by a decimal and get a quotient *larger* than the dividend. Students learn to calculate; by they also learn to recognize reasonable answers, so if they hit the wrong calculator buttons or the right buttons in the wrong sequence, they know it.

Correlations for *Operation Sense: Even More Fractions, Decimals, and Percents*

Book Description: Students extend their understanding of the four operations with whole numbers as they puzzle over such questions as, "How is it possible that two fractions multiplied might yield a smaller amount?" and "What does it mean to divide one-half by six?"		
Lesson Number:	**Lesson Name:**	**Mathematical Concepts/ Topics Covered:**
Opening the Unit	Operation Sense	• Fractions, decimals, and percents in print materials identified • Problems involving benchmark numbers (1/2, 1/10) solved • Operations involving fractions or decimals assessed
Lesson 1:	Equivalents	• Equivalence of fractions, decimals, and percents used in reasoning • Fractions, decimals, and percents compared and placed in order • a/b interpreted either as a fraction (part/whole) or as a division problem ($a \div b$)
Lesson 2:	Addition—Combining	• Reasonableness of answers to addition problems involving fractions, decimals, and percents judged • Addition of fractions, decimals, and percents connected to combining quantities • Pictures and situations connected with math symbols • Place value in addition of fractions and decimals emphasized
Lesson 3:	Subtraction—Take Away, Comparison, and Difference	• Subtraction problems interpreted in three ways: take-away, distance between two numbers, and absolute comparison of two amounts • Number line used to determine distance between two numbers and absolute comparison
Lesson 4:	Multiplication—Repeated Addition and Portions of Amounts	• Multiplication understanding assessed with pictures and stories • Whole number multiplication as repeated addition connected to fraction and decimal multiplication • Differences distinguished in multiplication by numbers less than 1 and by numbers greater than 1 • Idea that multiplication is commutative for all numbers recognized • Idea that multiplying by $1/a$ is equivalent to dividing by a understood • Idea that multiplying a number by its reciprocal results in 1 understood

Lesson 5	Division—Splitting and Sharing	• Model for integer division of splitting or dealing out an amount extended to include fraction and decimal amounts • Verb language and symbolic notation matched for division as splitting to a concrete model • *a/b* and *b/a* compared and contrasted
Lesson 6	Division—How Many__in__? *	• Quotitive model of division extended to the domain of fractions and decimals • Mathematical symbols and diagrams used to express and visualize the action of division • Division related to multiplication
Lesson 7	Mixing It Up	• Four operations compared with fractions, decimals, and percents • Operations related to one another
Closing the Unit	Putting It Together	• Unit reviewed • Assessment completed

EMPower

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A Program Sampler for Adult Basic Education

EMPower is a unique program developed by authors from TERC, a not-for-profit education research and development organization in Cambridge, Massachusetts. For the first time, nontraditional adult learners can study the mathematics that people need to become more successful in their personal, professional, and civic lives. Unlike more traditional math books that require students to memorize procedures and formulas, EMPower helps students develop useful math skills through engaging exercises that are relevant to real-life situations.

This exceptional set of materials is ideal for adult learners in a variety of educational settings where students prepare for taking GED tests or simply want to enhance their math skills. This pre-GED program promotes a learning community in which students extend their understanding of mathematics through open-ended investigations, working collaboratively, sharing ideas orally and in writing, and discovering multiple ways for solving problems.

EMPower Teacher Books also offer extensive support, including discussion suggestions and examples from pilot classes. **Professional Development**, teacher support, and community sharing is available through **empower.terc.edu**. With EMPower, students and teachers expand their understanding of what it means to do mathematics.

The full curriculum comprises eight nonsequential units that emphasize whole numbers; fractions, decimals, and percents; proportions; geometry and measurement; algebra; and data and graphs:

EVERYDAY NUMBER SENSE
OVER, AROUND, AND WITHIN
USING BENCHMARKS
SPLIT IT UP
MANY POINTS MAKE A POINT
KEEPING THINGS IN PROPORTION
SEEKING PATTERNS, BUILDING RULES
OPERATION SENSE

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