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ALGEBRA 2

McGraw Hill Education

SAMPLER

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Our lead authors ensure that the Macmillan/McGraw-Hill and Glencoe/McGraw-Hill mathematics programs are truly vertically aligned by beginning with the end in mind — success in Algebra 1 and beyond. By "backmapping" the content from the high school programs, all of our mathematics programs are well articulated in their scope and sequence.

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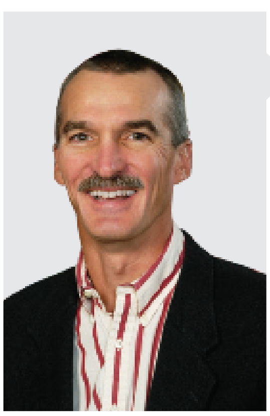

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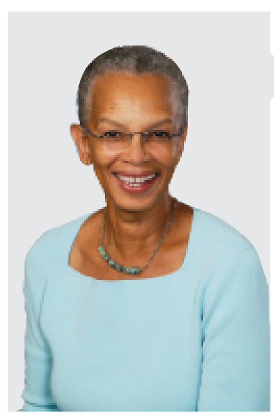

In Memoriam
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Dr. Carol Malloy was a fervent supporter of mathematics education. She was a Professor at the University of North Carolina, Chapel Hill, NCTM Board of Directors member, President of the Benjamin Banneker Association (BBA), and 2013 BBA Lifetime Achievement Award for Mathematics winner. She joined McGraw-Hill in 1996. Her influence significantly improved our programs' focus on real-world problem solving and equity. We will miss her inspiration and passion for education.

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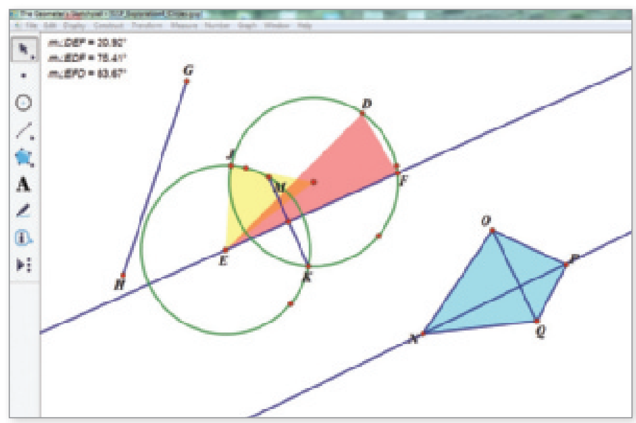

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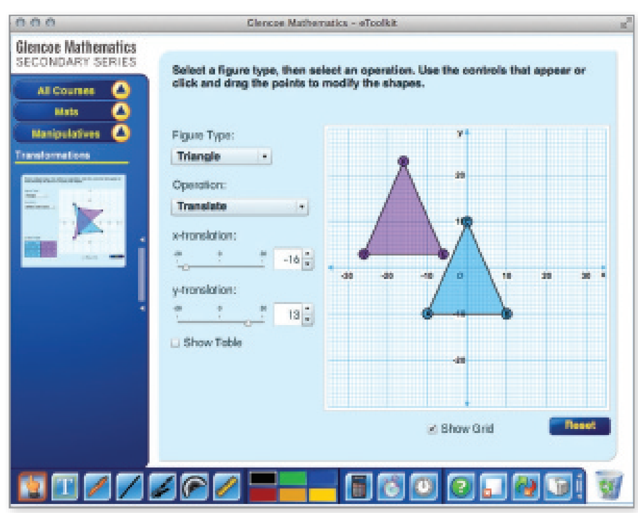

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LearnSmart Topic based online assessment

Animations illustrate key concepts through step-by-step tutorials and videos.

Tutors See and hear a teacher explain how to solve problems.

Calculator Resources provides other calculator keystrokes for each Graphing Technology Lab.

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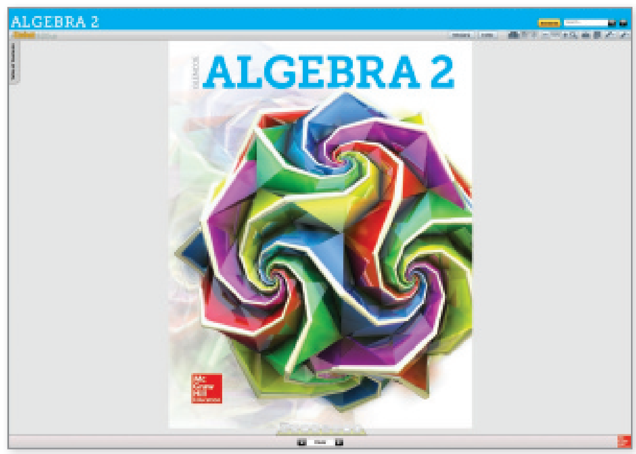

Interactive Student Guide

Interactive Student Guide is a dynamic resource to help you meet the challenges of content standards.

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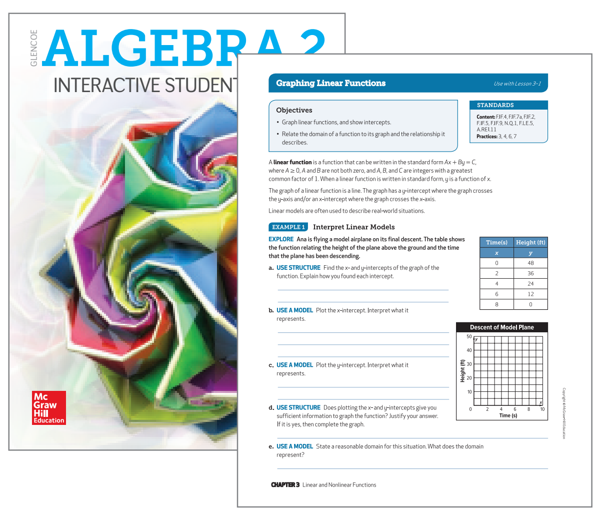

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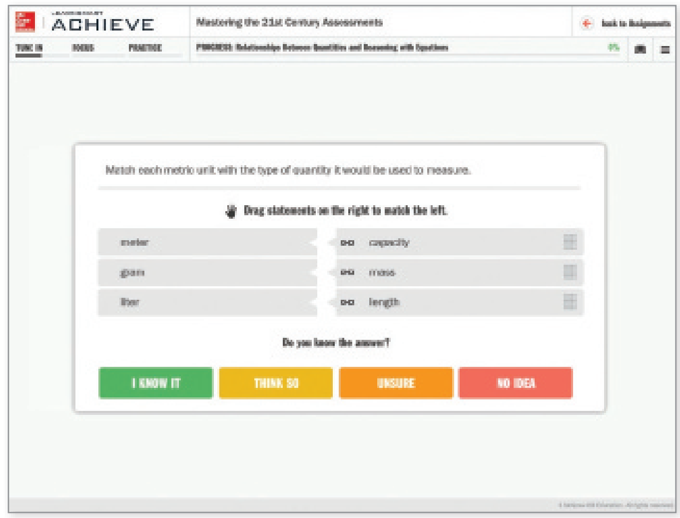
LearnSmart can be used as a self-study tool or it can be assigned to you by your teacher through your ConnectED platform.



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Through a cycle of assessment and learning, *ALEKS determines the topics that you are ready to learn next and develops a personalized learning path for you. ALEKS provides you with real-time, actionable data that informs you what you need to learn. It will help you review and master the skills needed to be successful in your math class. Use your ALEKS Pie to see a snapshot of your current progress of the course. Click on the Pie slice to see the number of topics mastered and ready to learn next.

Use the timeline to watch your progress toward your learning goals, and if needed you can toggle between English and Spanish translations of the content and interface.

*Ask your teacher if you have access to ALEKS.

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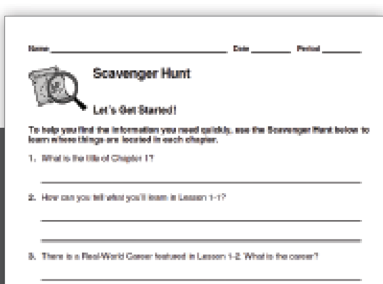

Worksheets help to explain key concepts, let you practice your skills, and offer opportunities for extending the lessons. Find them in the Resources in ConnectED.

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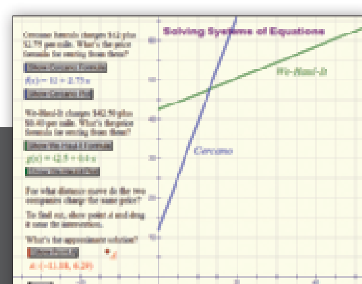
Geometer's Sketchpad® allows you to interact with functions in a visual way. Investigate systems of equations and inequalities with sketches in ConnectED.



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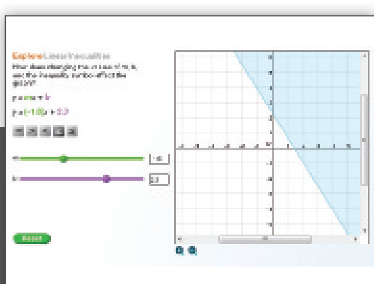

With the **Graphing Tools** in ConnectED, you can explore how changing parameters affects the graph of a function or an inequality.

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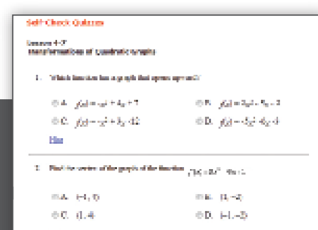

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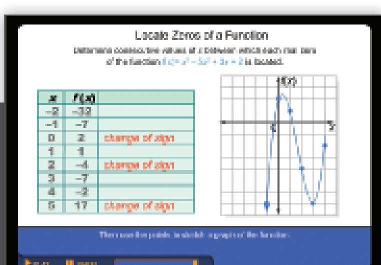

Animations demonstrate Key Concepts and topics from the chapter. Click to watch animations in ConnectED.

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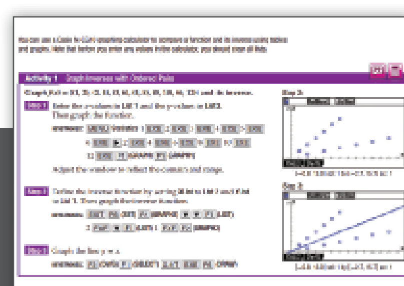
A graphing calculator is a powerful tool for studying algebra. **Graphing Calculator Keystrokes** help you harness that power. Find the keystrokes for your calculator in the Resources in ConnectED.



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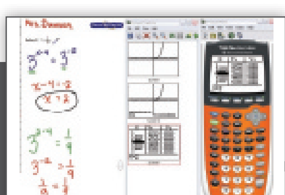

Personal Tutors that use graphing calculator technology show you every step to solving problems with this powerful tool. Find them in the Resources in ConnectED.

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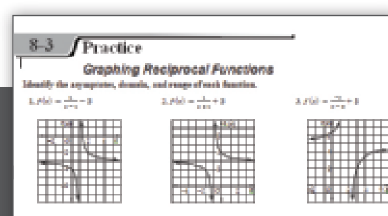
8-3 Practice
Graphing Reciprocal Functions
Identify the asymptotes, domain, and range of each function.

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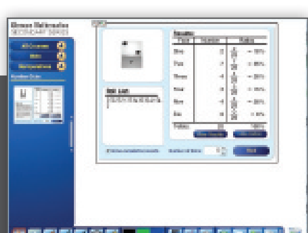

The number cube and coin toss tools can be helpful as you study this chapter. Find them in the **eToolkit** in ConnectED.

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Vocabulary is important to learning the Key Concepts in this chapter. Find all the terms with animations, English pronunciations, and translations to 13 languages in the eGlossary in ConnectED.

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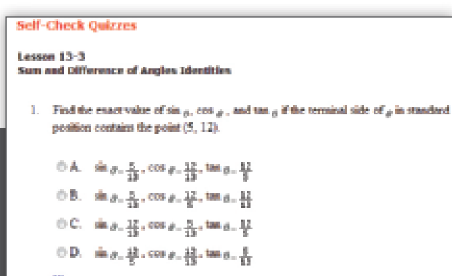

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Built-In Workbook

Extra Practice R1

Reference

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Index R118
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Symbols and Properties . . . . Inside Back Cover

Standards for Mathematical Practice

Glencoe Algebra 2 exhibits these practices throughout the entire program. All of the Standards for Mathematical Practice will be covered in each chapter. The MP icon notes specific areas of coverage.

Mathematical Practices	What does it mean?
1. Make sense of problems and persevere in solving them.	Solving a mathematical problem takes time. Use a logical process to make sense of problems, understand that there may be more than one way to solve a problem, and alter the process if needed.
2. Reason abstractly and quantitatively.	You can start with a concrete or real-world context and then represent it with abstract numbers or symbols (decontextualize), find a solution, then refer back to the context to check that the solution makes sense (contextualize).
3. Construct viable arguments and critique the reasoning of others.	Sound mathematical arguments require a logical progression of statements and reasons. Mathematically proficient students can clearly communicate their thoughts and defend them.
4. Model with mathematics.	Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. High school students at this level are expected to apply key takeaways from earlier grades to high-school level problems.
5. Use appropriate tools strategically.	Certain tools, including estimation and virtual tools are more appropriate than others. You should understand the benefits and limitations of each tool.
6. Attend to precision.	Precision in mathematics is more than accurate calculations. It is also the ability to communicate with the language of mathematics. In high school mathematics, precise language makes for effective communication and serves as a tool for understanding and solving problems.
7. Look for and make use of structure.	Mathematics is based on a well-defined structure. Mathematically proficient students look for that structure to find easier ways to solve problems.
8. Look for and express regularity in repeated reasoning.	Mathematics has been described as the study of patterns. Recognizing a pattern can lead to results more quickly and efficiently.

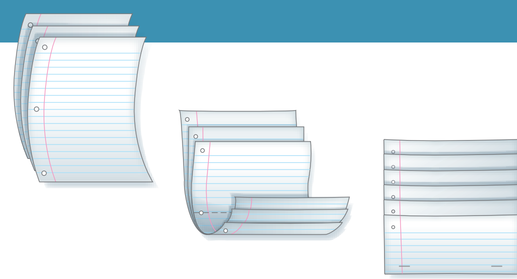
FOLDABLES® by Dinah Zike

Folding Instructions

The following pages offer step-by-step instructions to make the Foldables® study guides.

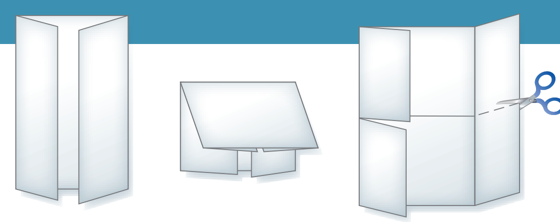
Layered-Look Book

1. Collect three sheets of paper and layer them about 1 cm apart vertically. Keep the edges level.
2. Fold up the bottom edges of the paper to form 6 equal tabs.
3. Fold the papers and crease well to hold the tabs in place. Staple along the fold. Label each tab.



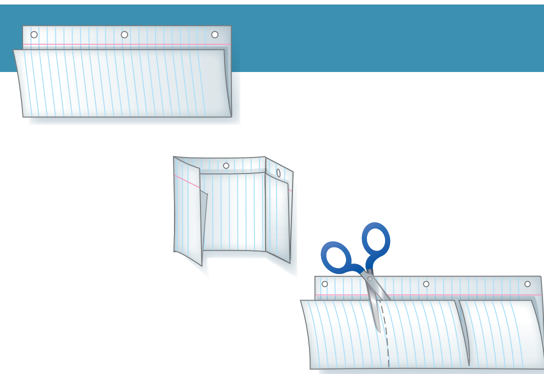
Shutter-Fold and Four-Door Books

1. Find the middle of a horizontal sheet of paper. Fold both edges to the middle and crease the folds. Stop here if making a shutter-fold book. For a four-door book, complete the steps below.
2. Fold the folded paper in half, from top to bottom.
3. Unfold and cut along the fold lines to make four tabs. Label each tab.



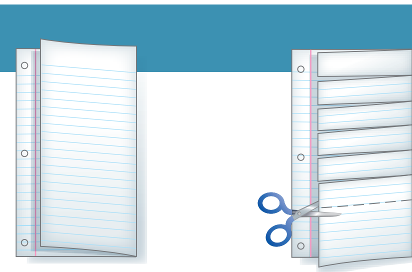
Concept-Map Book

1. Fold a horizontal sheet of paper from top to bottom. Make the top edge about 2 cm shorter than the bottom edge.
2. Fold width-wise into thirds.
3. Unfold and cut only the top layer along both folds to make three tabs. Label the top and each tab.



Vocabulary Book

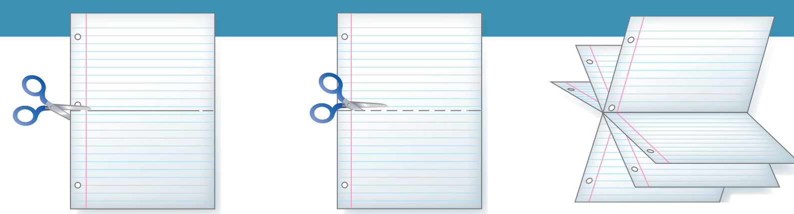
1. Fold a vertical sheet of notebook paper in half.
2. Cut along every third line of only the top layer to form tabs. Label each tab.



Pocket Book

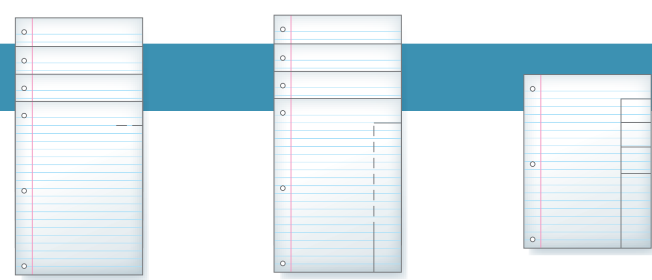
1. Fold the bottom of a horizontal sheet of paper up about 3 cm.
2. If making a two-pocket book, fold in half. If making a three-pocket book, fold in thirds.
3. Unfold once and dot with glue or staple to make pockets. Label each pocket.

Bound Book



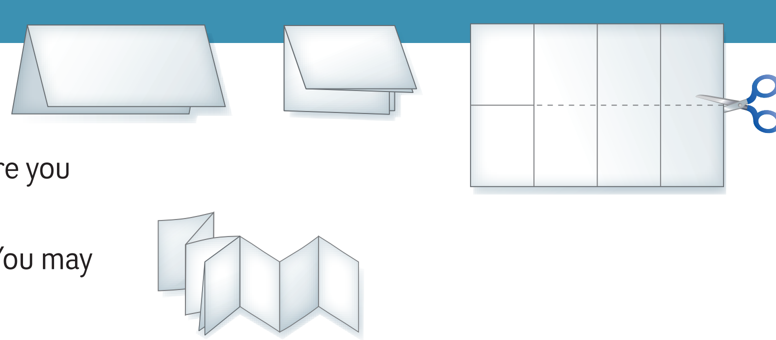
1. Fold several sheets of paper in half to find the middle. Hold all but one sheet together and make a 3-cm cut at the fold line on each side of the paper.
2. On the final page, cut along the fold line to within 3-cm of each edge.
3. Slip the first few sheets through the cut in the final sheet to make a multi-page book.

Top-Tab Book



1. Layer multiple sheets of paper so that about 2–3 cm of each can be seen.
2. Make a 2–3-cm horizontal cut through all pages a short distance (3 cm) from the top edge of the top sheet.
3. Make a vertical cut up from the bottom to meet the horizontal cut.
4. Place the sheets on top of an uncut sheet and align the tops and sides of all sheets. Label each tab.

Accordion Book



1. Fold a sheet of paper in half. Fold in half and in half again to form eight sections.
2. Cut along the long fold line, stopping before you reach the last two sections.
3. Refold the paper into an accordion book. You may want to glue the double pages together.

CHAPTER 5

Inverses and Radical Functions

THEN

You simplified polynomial expressions.

NOW

You will:

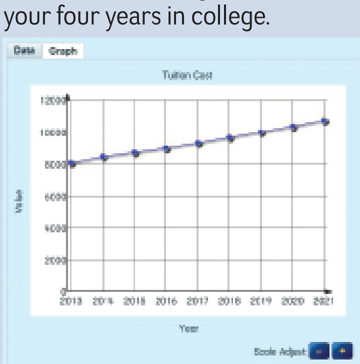
- Find compositions and inverses of functions.
- Graph and analyze square root functions and inequalities.
- Simplify and solve equations involving roots, radicals, and rational exponents.

WHY

FINANCES Connecting finances to mathematics is a skill that, once mastered, you will use your entire life. Learning to manage your finances includes anticipating costs, such as college, and preparing for them.

Use the Mathematical Practices to complete the activity.

1. **Using Tools** Use the Internet to find the annual tuition for the college you plan to attend. Then, use the Internet to find the tuition costs over the last four years.
2. **Applying Math** Calculate the average annual rate of increase in tuition. Then, create a Line Graph that plots the tuition costs over the last four years and projects the tuition costs through the end of your four years in college.



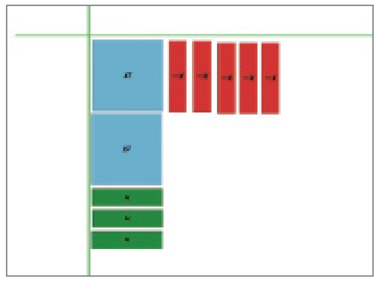
Peathegee Inc/Blend Images/Getty Images

Go Online to Guide Your Learning

Explore & Explain

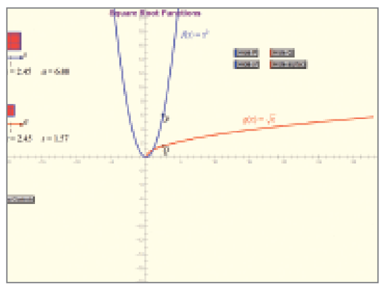
Product Mat and Algebra Tiles

Use the **Product Mat** and the **Algebra Tiles** to practice multiplying polynomials.



The Geometer's Sketchpad

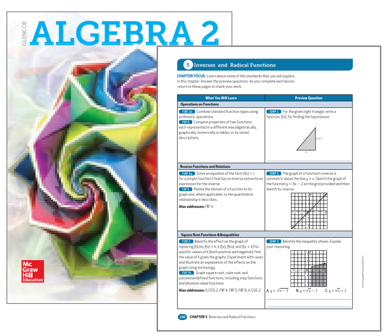
Use **The Geometer's Sketchpad** to perform operations with functions in Lesson 5-1 and Lesson 5-2, to illustrate inverses of relations and functions in Lesson 5-3, and to explore square root functions in Lesson 5-4.



eBook

Interactive Student Guide

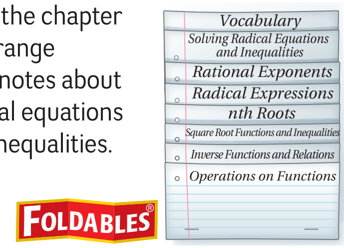
Before starting the chapter, answer the **Chapter Focus** preview questions. Check your answers as you complete each lesson. At the end of the chapter, try the **Performance Task**.



Organize

Foldables

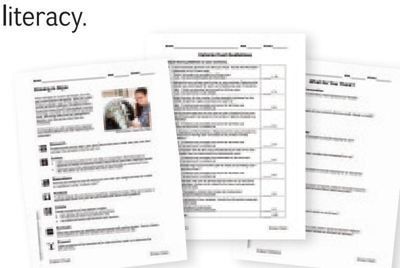
Get organized! Create an **Inverses and Radical Functions Foldable** before you start the chapter to arrange your notes about radical equations and inequalities.



Collaborate

Chapter Project

In the **Driving in Style** project, you will use what you have learned about inverse functions to complete a project that addresses financial literacy.



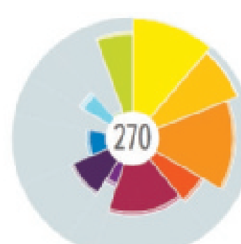
Focus

LEARNSMART

Need help studying? Complete the **Modeling with Functions** domain in LearnSmart to review for the chapter test.

ALEKS

You can use the **Radicals and Advanced Functions** topic in ALEKS to explore what you know about relations and functions and what you are ready to learn.*



* Ask your teacher if this is part of your program.

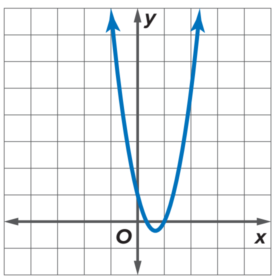
Get Ready for the Chapter

Go Online! for Vocabulary Review Games and key vocabulary in 13 languages.

Connecting Concepts

Concept Check

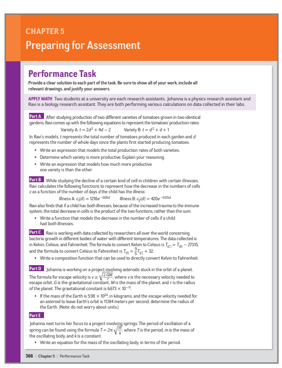
Review the concepts used in this chapter by answering the questions below.



1. In a graph such as the one shown, how do you define the "roots" of the graph?
2. If the exact roots of a graph cannot be found, what is typically stated?
3. What are the roots of the graph shown?
4. What type of division would you use to simplify $(5x^2 - 22x - 15) \div (x - 5)$?
5. What property would you use to rewrite $-\frac{1}{2}(2m - 5)$ without parentheses?
6. Given $= 3x^2 + 3x - 5x - 5$, what property can you apply to begin to simplify the equation?
7. Given $= 3x(x + 1) - 5(x + 1)$, what property can you apply to simplify the equation?
8. How would $-1(3b^2 + 2b - 1)$ be written so that it does not contain parentheses?
9. Given $4x^2 + 7x + 3$, what are the values for a, b, and c when applying the Quadratic Formula?
10. Given $\frac{3m + 5n}{p}$, where $m = -5$, $n = 2$, and $p = -1$, would you have a positive or negative number?

Performance Task Preview

You can use the concepts and skills in the chapter to perform various calculations on data collected in a university lab. Understanding inverses and radical functions will help you finish the Performance Task at the end of the chapter.



MP In this Performance Task you will:

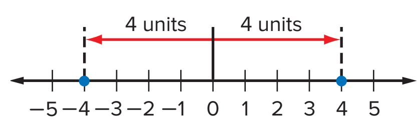
- make sense of problems and persevere in solving them
- reason abstractly and quantitatively
- attend to precision

New Vocabulary

English		Español
composition of functions	p. 322	composición de funciones
inverse relation	p. 329	relaciones inversas
inverse function	p. 329	función inversa
square root function	p. 338	función raíz cuadrada
radical function	p. 338	función radical
cube root function	p. 345	función ráiz cúbica
inflection point	p. 345	punto de inflexión
radical equation	p. 352	ecuación radical
extraneous solution	p. 352	solución extraña
radical inequality	p. 354	desigualdad radical

Review Vocabulary

absolute value valor absoluto a number's distance from zero on the number line, represented by $|x|$



rational number número racional any number $\frac{m}{n}$, where m and n are integers and n is not zero; the decimal form is either a terminating or repeating decimal.

relation relación a set of ordered pairs

LESSON 1

Operations with Functions

Then

- You performed operations on polynomials.

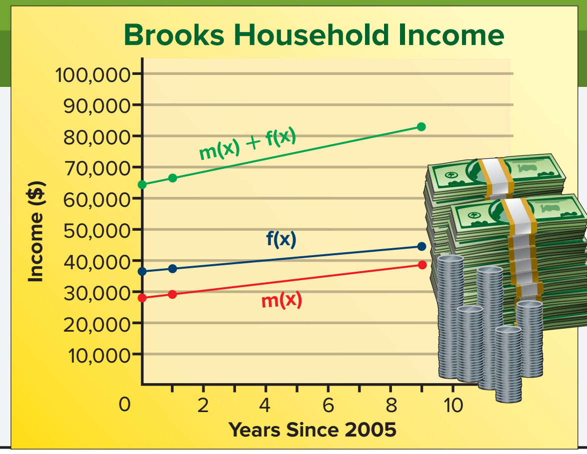
Now

1. Perform arithmetic operations with functions.
2. Apply arithmetic operations with functions.

Why?

- The graphs model the income for the Brooks family since 2005, where $m(x)$ represents Mr. Brooks' income and $f(x)$ represents Mrs. Brooks' income.

  The total household income for the Brooks household can be represented by $f(x) + m(x)$.



New Vocabulary
composition of functions

Mathematical Practices
4 Model with mathematics.
7 Look for and make use of structure.

1 Perform Operations with Functions

You have performed arithmetic operations with polynomials. You can also use addition, subtraction, multiplication, and division with functions.

You can perform arithmetic operations according to the following rules.

Key Concept Operations on Functions

Operation	Definition	Example Let $f(x) = 2x$ and $g(x) = -x + 5$.
Addition	$(f + g)(x) = f(x) + g(x)$	$2x + (-x + 5) = x + 5$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$2x - (-x + 5) = 3x - 5$
Multiplication	$(f \cdot g)(x) = f(x) \cdot g(x)$	$2x(-x + 5) = -2x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$	$\frac{2x}{-x + 5}, x \neq 5$

Example 1 Add and Subtract Functions

Given $f(x) = x^2 - 4$ and $g(x) = 2x + 1$, find each function.

a. $(f + g)(x)$

$(f + g)(x) = f(x) + g(x)$ — Addition of functions
$= (x^2 - 4) + (2x + 1)$ — $f(x) = x^2 - 4$ and $g(x) = 2x + 1$
$= x^2 + 2x - 3$ — Simplify.

b. $(f - g)(x)$

$(f - g)(x) = f(x) - g(x)$ — Subtraction of functions
$= (x^2 - 4) - (2x + 1)$ — $f(x) = x^2 - 4$ and $g(x) = 2x + 1$
$= x^2 - 2x - 5$ — Simplify.

Guided Practice

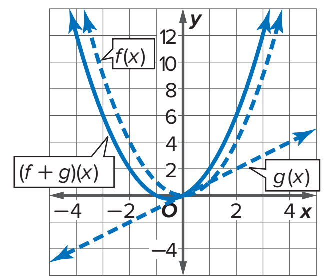
Given $f(x) = x^2 + 5x - 2$ and $g(x) = 3x - 2$, find each function.

1A. $(f + g)(x)$ **1B.** $(f - g)(x)$

You can graph sum and difference functions by graphing each function involved separately, then adding their corresponding functional values. Let $f(x) = x^2$ and $g(x) = x$. Examine the graphs of $f(x)$, $g(x)$, and their sum and difference.

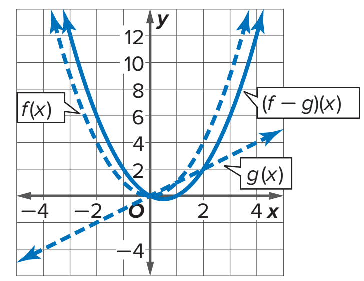
Find $(f + g)(x)$.

x	$f(x) = x^2$	$g(x) = x$	$(f + g)(x) = x^2 + x$
−3	9	−3	$9 + (-3) = 6$
−2	4	−2	$4 + (-2) = 2$
−1	1	−1	$1 + (-1) = 0$
0	0	0	$0 + 0 = 0$
1	1	1	$1 + 1 = 2$
2	4	2	$4 + 2 = 6$
3	9	3	$9 + 3 = 12$



Find $(f - g)(x)$.

x	$f(x) = x^2$	$g(x) = x$	$(f - g)(x) = x^2 - x$
−3	9	−3	$9 - (-3) = 12$
−2	4	−2	$4 - (-2) = 6$
−1	1	−1	$1 - (-1) = 2$
0	0	0	$0 - 0 = 0$
1	1	1	$1 - 1 = 0$
2	4	2	$4 - 2 = 2$
3	9	3	$9 - 3 = 6$



Reading Math Tip

intersection Everyday use—the intersection of two roads is where the two roads meet; Math meaning—the intersection of two sets is the set of elements common to them.

In Example 1, the functions $f(x)$ and $g(x)$ have the same domain of all real numbers. The functions $(f + g)(x)$ and $(f - g)(x)$ also have domains that include all real numbers. For each new function, the domain consists of the intersection of the domains of $f(x)$ and $g(x)$. Under division, the domain of the new function is restricted by excluded values that cause the denominator to equal zero.

Example 2 Multiply and Divide Functions

Given $f(x) = x^2 + 7x + 12$ and $g(x) = 3x - 4$, find each function. Indicate any restrictions in the domain.

a. $(f \cdot g)(x)$

$(f \cdot g)(x) = f(x) \cdot g(x)$ — Multiplication of functions

$= (x^2 + 7x + 12)(3x - 4)$ — Substitution

$= 3x^3 + 21x^2 + 36x - 4x^2 - 28x - 48$ — Distributive Property

$= 3x^3 + 17x^2 + 8x - 48$ — Simplify.

b. $\left(\frac{f}{g}\right)(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ — Division of functions

$= \frac{x^2 + 7x + 12}{3x - 4}, x \neq \frac{4}{3}$ — Substitution

Because $x = \frac{4}{3}$ makes the denominator $3x - 4 = 0$, $\frac{4}{3}$ is excluded from the domain of $\left(\frac{f}{g}\right)(x)$.

Guided Practice **Given $f(x) = x^2 - 7x + 2$ and $g(x) = x + 4$, find each function.**

2A. $(f \cdot g)(x)$

2B. $\left(\frac{f}{g}\right)(x)$

Real-World Link

In a robotics competition, teams compete to build robots that can accomplish a given task. In 2016, about 29,000 students from 40 countries participated in a robotics competition in St. Louis.

Source: U.S. News & World Report

Watch Out!

Evaluating the Function Remember that x is the total number of team members, so evaluate the function for $x = 15$, not $x = 13$.

2 Apply Operations with Functions

In many real-world modeling situations you write multiple functions that each represent one aspect of the problem. You can combine the functions to build a new function that models a different aspect of the problem.

Real-World Example 3 Build a New Function

ROBOTICS **A robotics competition team is preparing for a tournament. The team's captain is ordering custom T-shirts for each of the members. The T-shirts cost \$8 each, plus a one-time set-up fee of \$25. Sales tax on the order is 8%. The team decides that the members will split the cost of the T-shirts equally, and that the team's captain and vice-captain will not have to pay for their shirts.**

a. Write a function $C(x)$ that represents the total cost of the T-shirts, where x is the number of team members.

$8x + 25$ represents the cost of T-shirts before sales tax. Multiply by 1.08 to find the total cost after the 8% sales tax is applied.

$C(x) = (1.08)(8x + 25)$ — Multiply $8x + 25$ by 1.08.

$= (1.08)(8x) + (1.08)(25)$ — Distributive Property

$= 8.64x + 27$ — Simplify.

So, $C(x) = 8.64x + 27$ represents the total cost of the T-shirts for x team members.

b. Write a function $N(x)$ to represent the number of team members who pay for the T-shirts.

All of the team members except the captain and vice-captain pay for T-shirts.

So, $N(x) = x - 2$.

c. Find $\left(\frac{C}{N}\right)(x)$ and explain what this function represents.

$\left(\frac{C}{N}\right)(x) = \frac{C(x)}{N(x)}$ — Division of functions

$= \frac{8.64x + 27}{x - 2}$ — Substitution

So, $\left(\frac{C}{N}\right)(x) = \frac{8.64x + 27}{x - 2}$, $x \neq 2$. This function represents the dollar amount that each paying team member will contribute to the cost of the T-shirts.

d. If the team has 15 members, how much does each paying team member contribute to the cost of the T-shirts?

$\left(\frac{C}{N}\right)(15) = \frac{8.64(15) + 27}{15 - 2}$ — Evaluate the function for $x = 15$.

≈ 12.05 — Simplify.

Each paying member of the team contributes \$12.05.

Guided Practice

3. **CHEMISTRY** A chemist has 500 grams of a 15% saline solution. She adds x grams of salt to the solution. Write a function $S(x)$ that represents the number of grams of salt in the new solution, a function $T(x)$ that represents the total number of grams of the new solution, and then find $\left(\frac{S}{T}\right)(x)$ and explain what this function represents.

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Examples 1–2 **Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Indicate any restrictions in the domain.**

1. $f(x) = x + 2$
$g(x) = 3x - 1$

2. $f(x) = x^2 - 5$
$g(x) = -x + 8$

Example 3

3. PHOTOGRAPHY A group of photographers is planning an exhibit of their work. Each photographer will contribute 5 prints to the exhibit and the cost of framing each print is $7.85. There is also a flat fee of $200 to rent the room for the exhibit. The photographers plan to split the cost of the exhibit equally.

a. Write a function $C(x)$ that represents the total cost of the exhibit, where x is the number of photographers.

b. In addition to the photographers, 3 family members offer to participate in sharing the cost of the exhibit. Write a function $P(x)$ to represent the number of people who pay for the exhibit.

c. Find $\left(\frac{C}{P}\right)(x)$ and explain what this function represents.

d. If there are 8 photographers, how much does each photographer contribute to the cost of the exhibit?

Practice and Problem Solving

Extra Practice is on page R5.

Examples 1–2 **Find $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$ for each $f(x)$ and $g(x)$. Indicate any restrictions in the domain.**

4. $f(x) = 2x$
$g(x) = -4x + 5$

5. $f(x) = x - 1$
$g(x) = 5x - 2$

6. $f(x) = x^2$
$g(x) = -x + 1$

7 ● $f(x) = 3x$
$g(x) = -2x + 6$

8. $f(x) = x - 2$
$g(x) = 2x - 7$

9. $f(x) = x^2$
$g(x) = x - 5$

10. $f(x) = -x^2 + 6$
$g(x) = 2x^2 + 3x - 5$

11. $f(x) = 3x^2 - 4$
$g(x) = x^2 - 8x + 4$

Examples 3

12. BASKETBALL During a practice session, Dimitri makes 20 free-throw attempts and makes 60% of the free throws. Then, on his next attempt, he begins a streak in which he makes x free throws in a row.

a. What do the functions $f(x) = 12 + x$ and $g(x) = 20 + x$ represent?

b. Find $\left(\frac{f}{g}\right)(x)$ and explain what this function represents.

b. Find $\left(\frac{f}{g}\right)(7)$ and explain what this value represents.

13. POPULATION In a particular county, the population of the two largest cities can be modeled by $f(x) = 200x + 25$ and $g(x) = 175x - 15$, where x is the number of years since 2010 and the population is in thousands.

a. What is the population of the two cities combined after any number of years?

b. What is the difference in the populations of the two cities?

Perform each operation if $f(x) = x^2 + x - 12$ and $g(x) = x - 3$. State the domain of the resulting function.

14. $(f - g)(x)$

15. $2(g \cdot f)(x)$

16. $\left(\frac{f}{g}\right)(x)$

17. **MULTIPLE REPRESENTATIONS** Let $f(x) = x^2$ and $g(x) = x$.

a. Tabular Make a table showing values for $f(x)$, $g(x)$, $(f + g)(x)$, and $(f - g)(x)$.

b. Graphical Graph $f(x)$, $g(x)$, and $(f + g)(x)$ on the same coordinate grid.

c. Graphical Graph $f(x)$, $g(x)$, and $(f - g)(x)$ on the same coordinate grid.

d. Verbal Describe the relationship among the graphs of $f(x)$, $g(x)$, $(f + g)(x)$, and $(f - g)(x)$.

Use the table to find each value.

x	$f(x)$	$g(x)$	$h(x)$
−5	−8	8	2
−2	4	5	−10
−1	−2	−4	0
0	3	−5	−5
3	2	0	8
4	0	−1	7

18. $(f + g)(-5)$

19. $(g - f)(-1)$

20. $(f \cdot g)(3)$

21. $(h \cdot f)(0)$

22. $\left(\frac{f}{g}\right)(-1)$

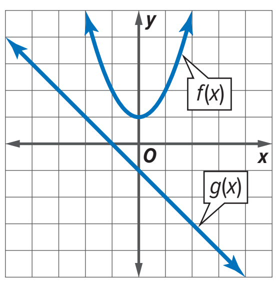
23. $\left(\frac{h}{g}\right)(0)$

24. $\left(\frac{g}{f}\right)(4)$

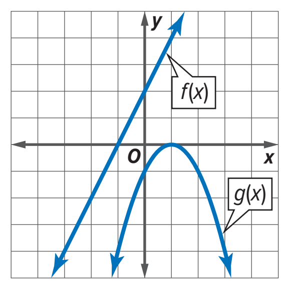
25. $\left(\frac{g}{h}\right)(-5)$

Use the graph of $f(x)$ and $g(x)$ to find each value.

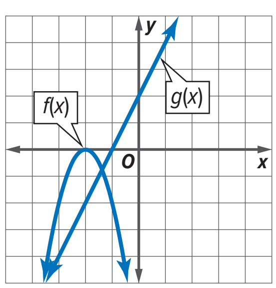
26. $(f - g)(1)$



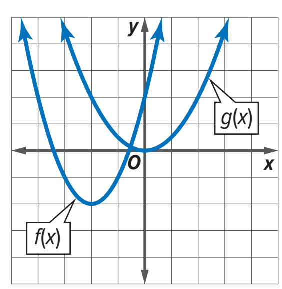
27 $(f \cdot g)(0)$



28. $\left(\frac{g}{f}\right)(-3)$



29. $\left(\frac{f}{g}\right)(-2)$



If $f(x) = -x + 1$, $g(x) = 4x + 2$, and $h(x) = x^2 - 1$, find each value.

30. $(2f + g)(1)$

31. $(3f + 2h)(0)$

32. $(-f + 2g)(3)$

33. $(5f \cdot h)(-1)$

34. $\left(\frac{3f}{g}\right)(2)$

35. $\left(\frac{g}{2h}\right)(0)$

36. $(h - 2f)(5)$

37. $(-f - h)(1)$

38. $(5h - 0.1g)(2)$

39 **EMPLOYMENT** The number of women and men age 16 and over employed each year in the United States can be modeled by the following equations, where x is the number of years since 2000 and y is the number of people in thousands.

women: $y = 548.6x + 66{,}527$ men: $y = 2090.7x + 62{,}243$

a. Write a function that models the total number of men and women employed in the United States during this time.

b. If f is the function for the number of men, and g is the function for the number of women, what does $(f - g)(x)$ represent?

If $f(x) = x + 2$, $g(x) = -4x + 3$, and $h(x) = x^2 - 2x + 1$, find each value.

40. $(f \cdot g \cdot h)(3)$ **41.** $[(f + g) \cdot h](1)$ **42.** $\left(\frac{h}{fg}\right)(-6)$

43. **MULTIPLE REPRESENTATIONS** You will explore $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$, if $f(x) = x^2 + 1$ and $g(x) = x - 3$.

a. **Tabular** Make a table showing values for $(f \cdot g)(x)$ and $\left(\frac{f}{g}\right)(x)$.

b. **Graphical** Use a graphing calculator to graph $(f \cdot g)(x)$ and $\left(\frac{f}{g}\right)(x)$ on the same coordinate plane.

c. **Verbal** Explain the relationship between $(f \cdot g)(x)$ and $\left(\frac{f}{g}\right)(x)$.

44. **MULTI-STEP** Ice cream cones are one of many treats sold at Sam's Desserts. They sell 60 scoops for every gallon of ice cream. They pay \$6 per gallon of ice cream, \$2 for every box of 24 cones, and allocate a fixed monthly cost of \$400 to ice cream. Their sales reports for the past 6 months are shown.

Month	January	February	March	April	May	June
Price	\$3.50	\$3.70	\$3.90	\$3.75	\$3.55	\$3.80
Scoops Sold	224	208	188	205	219	199

a. What is their maximum monthly profit from ice cream sales?

b. Describe your solution process.

H.O.T. Problems Use Higher-Order Thinking Skills

45. **OPEN-ENDED** Write two functions $f(x)$ and $g(x)$ such that $(f \cdot g)(x) = 2x^2 - 2$.

46. **CHALLENGE** Given that $(f + g)(4) = 8$ and $(f - g)(4) = -6$, find $f(4)$ and $g(4)$.

47. MP **REASONING** State whether each statement is *sometimes*, *always*, or *never* true. Explain.

a. If $f(x)$ and $g(x)$ are linear functions, then there is one value that is excluded from the domain of $(f + g)(x)$.

b. If $f(x)$ and $g(x)$ are linear functions, then there is one value that is excluded from the domain of $\left(\frac{f}{g}\right)(x)$.

48. MP **STRUCTURE** Suppose $f(x) = ax^2 + bx + c$ and $g(x) = mx^2 + nx + p$, for constants a, b, c, m, n, and p, with $a \neq 0$ and $m \neq 0$. What can you conclude about the constants if the domain of $\left(\frac{f}{g}\right)(x)$ is all real numbers? Explain.

49. **WRITING IN MATH** If $f(x)$ and $g(x)$ are polynomials, what can you say about the domains of $(f + g)(x)$, $(f - g)(x)$, $(f \cdot g)(x)$, and $\left(\frac{f}{g}\right)(x)$?

50. Let $f(x) = x^2 - 4$ and $g(x) = x^2 - 1$. What is the domain of the function $\left(\frac{f}{g}\right)(x)$? MP 7

- ◯ **A** all real numbers
- ◯ **B** all real numbers except $x = 0$
- ◯ **C** all real numbers except $x = \pm 1$
- ◯ **D** all real numbers except $x = \pm 2$

51. Let $f(m) = p$ where m and p are both nonzero integers. Which statement(s) must be true? MP 7

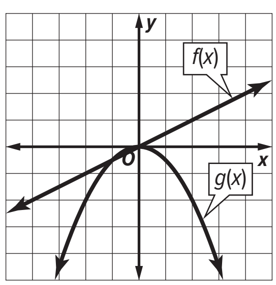
I. $f\left(\frac{m}{2}\right) = \frac{p}{2}$

II. $2f(m) = 2p$

III. $f(2m) = 2p$

- ◯ **A** I only
- ◯ **B** II only
- ◯ **C** III only
- ◯ **D** I and II only

52. The graphs of $f(x)$ and $g(x)$ are shown. MP 7



What is $\left(\frac{f}{g}\right)(-2)$?

- ◯ **A** -2
- ◯ **B** $-\frac{1}{2}$
- ◯ **C** $\frac{1}{2}$
- ◯ **D** 2

53. If $f(x) = 2x - 10$ and $g(x) = x^2 + 3x + 1$, what is $(f \cdot g)(3)$? MP 6

$(f \cdot g)(3) =$ ______

54. Find $(f + g)(x)$ for the following functions. MP 1, 6

$f(x) = -5x^2 + 4x - 7$

$g(x) = 6x^2 - 4x + 12$

55. Find $(f \cdot g)(-2)$ for the following functions. MP 1, 6

$f(x) = -x^2 + 2x - 2$

$g(x) = 2x + 3$

- ◯ **A** -20
- ◯ **B** 8
- ◯ **C** 120
- ◯ **D** 215

56. For which pair(s) of functions is the domain of $\left(\frac{f}{g}\right)(x)$ all real numbers? MP 7

- ☐ **A** $f(x) = x$ and $g(x) = x^2 + 4$
- ☐ **B** $f(x) = x$ and $g(x) = x^2 - 4$
- ☐ **C** $f(x) = x^2 - 4$ and $g(x) = 4$
- ☐ **D** $f(x) = x + 4$ and $g(x) = x - 4$
- ☐ **E** $f(x) = 4$ and $g(x) = x^2 + 4$
- ☐ **F** $f(x) = x^2 + 4$ and $g(x) = x^2 - 4$

57. If $f(-2) = a$ and $(f \cdot g)(-2) = 2a^2$, which of the following is g(−2)? MP 2

- ◯ **A** $2a$
- ◯ **B** $-4a^2$
- ◯ **C** $-2a$
- ◯ **D** $2a^3$

58. **MULTI-STEP** Jordan is ordering books online for the members of his book club. Each member of the club will receive a copy of the book and each book costs \$8.95. Because Jordan is ordering a large quantity of books, one of them is free. The shipping fee for the order is a flat rate of \$4.50 and there is no sales tax. MP 1, 4

a. Let x represent the number of members of the club. Write a function $B(x)$ that represents the total cost of the books.

b. The club members decide to split the cost evenly and they decide that Jordan should not have to pay anything since he placed the order. Write a function $N(x)$ that represents the number of club members who pay for books.

c. Find $\left(\frac{B}{N}\right)(x)$ and explain what it represents.

d. The book club has 12 members. How much does each member pay?

LESSON 2

Composition of Functions

Then

- You performed arithmetic operations on functions.

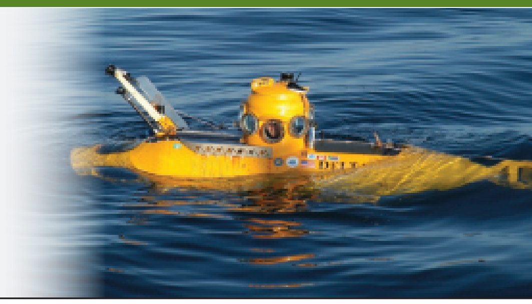
Now

1 Perform compositions of functions.

2 Apply compositions of functions.

Why?

- Submersibles can descend several miles below the surface of the ocean. You can write a function $d(t)$ that gives the depth of the submersible after t minutes and a function $p(d)$ that gives the pressure at depth d. The composition of the functions $p[d(t)]$ gives the pressure on the submersible after t minutes.



New Vocabulary

composition of functions

Mathematical Practices

4 Model with mathematics.

7 Look for and make use of structure.

1 Perform Compositions of Functions

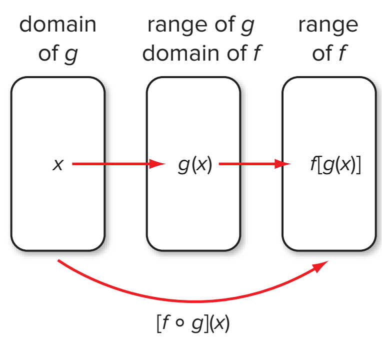
You have already combined functions with arithmetic operations. Another method used to combine functions is a composition of functions. In a **composition of functions**, the results of one function are used to evaluate a second function.

Key Concept Composition of Functions

Words Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composition function $f \circ g$ can be described by

$$[f \circ g](x) = f[g(x)].$$

Model



Example 1 Evaluate Compositions of Functions

Given $f(x) = x^2 - 4$ and $g(x) = 2x + 1$, find each value.

a. $[f \circ g](3)$

$[f \circ g](3) = f[g(3)]$	Composition of functions
$= f(7)$	$g(3) = 2(3) + 1 = 7$
$= 45$	$f(7) = 7^2 - 4 = 45$

So, $[f \circ g](3) = 45$.

b. $[g \circ f](3)$

$[g \circ f](3) = g[f(3)]$	Composition of functions
$= g(5)$	$f(3) = 3^2 - 4 = 5$
$= 11$	$g(5) = 2(5) + 1 = 11$

So, $[g \circ f](3) = 11$.

Guided Practice

Given $f(x) = 3x - 6$ and $g(x) = x^3 + 1$, find each value.

1A. $[f \circ g](5)$

1B. $[g \circ f](5)$

Go Online!

The composition of f and g, denoted by $f \circ g$ or $f[g(x)]$, is read f of g. To hear more pronunciations of expressions, log into your **eStudent Edition**. Ask your teacher or a partner for clarification as you need it.

The composition of two functions may not exist. Given two functions f and g, $[f \circ g](x)$ is defined only if the range of $g(x)$ is a subset of the domain of f. Likewise, $[g \circ f](x)$ is defined only if the range of $f(x)$ is a subset of the domain of g.

Study Tip

Composition Be careful not to confuse a composition $f[g(x)]$ with multiplication of functions $(f \cdot g)(x)$.

Example 2 Perform Compositions of Functions

For each pair of functions, find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

a. $f = \{(1, 8), (0, 13), (15, 11), (14, 9)\}$, $g = \{(8, 15), (5, 1), (10, 14), (9, 0)\}$

To find $f \circ g$, evaluate $g(x)$ first. Then use the range to evaluate $f(x)$.

$f[g(8)] = f(15)$ or 11 — $g(8) = 15$

$f[g(5)] = f(1)$ or 8 — $g(5) = 1$

$f[g(10)] = f(14)$ or 9 — $g(10) = 14$

$f[g(9)] = f(0)$ or 13 — $g(9) = 0$

$f \circ g = \{(8, 11), (5, 8), (10, 9), (9, 13)\}$

$D = \{5, 8, 9, 10\}$, $R = \{8, 9, 11, 13\}$

To find $g \circ f$, evaluate $f(x)$ first. Then use the range to evaluate $g(x)$.

$g[f(1)] = g(8)$ or 15 — $f(1) = 8$

$g[f(0)] = g(13)$ — $g(13)$ is undefined.

$g[f(15)] = g(11)$ — $g(11)$ is undefined.

$g[f(14)] = g(9)$ or 0 — $f(14) = 0$

Because 11 and 13 are not in the domain of g, $g \circ f$ is undefined for $x = 0$ and $x = 15$. However, $g[f(1)] = 15$ and $g[f(14)] = 0$.

So, $g \circ f = \{(1, 15), (14, 0)\}$.

$D = \{1, 14\}$, $R = \{0, 15\}$

b. $f(x) = 2x - 5$, $g(x) = 4x$

$[f \circ g](x)$			$[g \circ f](x)$
$[f \circ g](x)$	$= f[g(x)]$	Composition of functions	$[g \circ f](x) = g[f(x)]$
	$= f(4x)$	Substitute.	$= g(2x - 5)$
	$= 2(4x) - 5$	Substitute again.	$= 4(2x - 5)$
	$= 8x - 5$	Simplify.	$= 8x - 20$

So, $[f \circ g](x) = 8x - 5$ and $[g \circ f](x) = 8x - 20$.

For $[f \circ g](x)$, D = {all real numbers} and R = {all real numbers}, and for $[g \circ f](x)$, D = {all real numbers} and R = {all real numbers}.

Guided Practice

2A. $f(x) = \{(3, -2), (-1, -5), (4, 7), (10, 8)\}$, $g(x) = \{(4, 3), (2, -1), (9, 4), (3, 10)\}$

2B. $f(x) = x^2 + 2$ and $g(x) = x - 6$

Notice that in most cases, $f \circ g \neq g \circ f$. Therefore, the order in which two functions are composed is important.

Real-World Career

Adjusted for inflation, the average price of a new car increased from $22,013 in 2005 to about $30,000 in 2013.

Source: Federal Trade Commission Consumer Information

2 Apply Compositions of Functions

In some real-world situations, functions are applied in sequence, one after the other. In such cases, you can write a model for the situation using a composition of functions.

Real-World Example 3 Apply Compositions of Functions

SHOPPING A new car dealer is discounting all new cars by 12%. At the same time, the manufacturer is offering a $1500 rebate on all new cars. Mr. Navarro is buying a car that is priced $24,500. Will the final price be lower if the discount is applied before the rebate or if the rebate is applied before the discount?

Understand Let x represent the original price of a new car, $d(x)$ represent the price of a car after the discount, and $r(x)$ the price of the car after the rebate.

Plan Write equations for $d(x)$ and $r(x)$.

The original price is discounted by 12%. $d(x) = x - 0.12x$

There is a $1500 rebate on all new cars. $r(x) = x - 1500$

Solve If the discount is applied *before* the rebate, then the final price of Mr. Navarro's new car is represented by $[r \circ d](24{,}500)$.

$[r \circ d](x) = r[d(x)]$

$$\begin{aligned}[r \circ d](24{,}500) &= r[24{,}500 - 0.12(24{,}500)] \\ &= r(24{,}500 - 2940) \\ &= r(21{,}560) \\ &= 21{,}560 - 1500 \\ &= 20{,}060\end{aligned}$$

If the rebate is given *before* the discount is applied, then the final price of Mr. Navarro's car is represented by $[d \circ r](24{,}500)$.

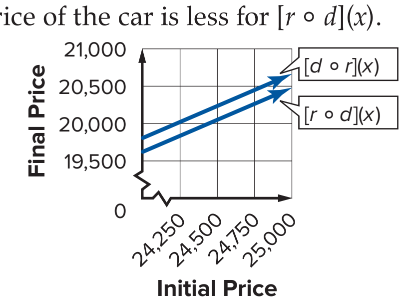
$[d \circ r](x) = d[r(x)]$

$$\begin{aligned}[d \circ r](24{,}500) &= d(24{,}500 - 1500) \\ &= d(23{,}000) \\ &= 23{,}000 - 0.12(23{,}000) \\ &= 23{,}000 - 2760 \\ &= 20{,}240\end{aligned}$$

$[r \circ d](24{,}500) = 20{,}060$ and $[d \circ r](24{,}500) = 20{,}240$. So, the final price of the car is less when the discount is applied before the rebate.

Check Graph $[d \circ r](x)$ and $[r \circ d](x)$. The final price of the car is less for $[r \circ d](x)$.

The answer seems reasonable because the 12% discount is being applied to a greater amount. Thus, the dollar amount of the discount is greater.



Guided Practice

3. **SHOPPING** Gadgets-to-Go offers both an in-store $35 rebate and a 15% discount on a tablet that normally sells for $300. Which provides the better price: taking the discount before the rebate or taking the discount after the rebate?

Kali9/iStock/Getty Images

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Example 1 **Given $f(x) = x^2 + 3$ and $g(x) = 2x - 10$, find each value.**

1. $[f \circ g](4)$
2. $[g \circ f](4)$
3. $[f \circ g](0)$
4. $[g \circ f](0)$
5. $[f \circ g](-1)$
6. $[g \circ f](-1)$
7. $[f \circ g](5)$
8. $[f \circ g](-4)$
9. $[g \circ f](2)$

Example 2 **For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.**

10. $f = \{(2, 5), (6, 10), (12, 9), (7, 6)\}$
 $g = \{(9, 11), (6, 15), (10, 13), (5, 8)\}$
11. $f = \{(-5, 4), (14, 8), (12, 1), (0, -3)\}$
 $g = \{(-2, -4), (-3, 2), (-1, 4), (5, -6)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

12. $f(x) = -3x$
 $g(x) = 5x - 6$
13. $f(x) = x + 4$
 $g(x) = x^2 + 3x - 10$

Example 3

14. **MP MODELING** Dora has 8% of her earnings deducted from her paycheck for a college savings plan. She can choose to take the deduction either before taxes are withheld, which reduces her taxable income, or after taxes are withheld. Dora's tax rate is 17.5%. If her pay before taxes and deductions is \$950, will she save more money if the deductions are taken before or after taxes are withheld? Explain.

Practice and Problem Solving

Extra Practice is on page R5.

Example 1 **Given $f(x) = x + 4$, $g(x) = -2x^2$, and $h(x) = 5$, find each value.**

15. $[f \circ g](10)$
16. $[g \circ f](10)$
17. $[f \circ h](0)$
18. $[h \circ f](3)$
19. $[g \circ h](-2)$
20. $[h \circ g](-2)$
21. $[f \circ g](-4)$
22. $[g \circ f](-4)$
23. $[g \circ f](-12)$

Example 2 **For each pair of functions, find $f \circ g$ and $g \circ f$, if they exist. State the domain and range for each composed function.**

24. $f = \{(-8, -4), (0, 4), (2, 6), (-6, -2)\}$
 $g = \{(4, -4), (-2, -1), (-4, 0), (6, -5)\}$
25. $f = \{(-7, 0), (4, 5), (8, 12), (-3, 6)\}$
 $g = \{(6, 8), (-12, -5), (0, 5), (5, 1)\}$
26. $f = \{(5, 13), (-4, -2), (-8, -11), (3, 1)\}$
 $g = \{(-8, 2), (-4, 1), (3, -3), (5, 7)\}$
27. $f = \{(-4, -14), (0, -6), (-6, -18), (2, -2)\}$
 $g = \{(-6, 1), (-18, 13), (-14, 9), (-2, -3)\}$
28. $f = \{(-15, -5), (-4, 12), (1, 7), (3, 9)\}$
 $g = \{(3, -9), (7, 2), (8, -6), (12, 0)\}$
29. $f = \{(-1, 11), (2, -2), (5, -7), (4, -4)\}$
 $g = \{(5, -4), (4, -3), (-1, 2), (2, 3)\}$
30. $f = \{(7, -3), (-10, -3), (-7, -8), (-3, 6)\}$
 $g = \{(4, -3), (3, -7), (9, 8), (-4, -4)\}$
31. $f = \{(1, -1), (2, -2), (3, -3), (4, -4)\}$
 $g = \{(1, -4), (2, -3), (3, -2), (4, -1)\}$
32. $f = \{(-4, -1), (-2, 6), (-1, 10), (4, 11)\}$
 $g = \{(-1, 5), (3, -4), (6, 4), (10, 8)\}$
33. $f = \{(12, -3), (9, -2), (8, -1), (6, 3)\}$
 $g = \{(-1, 5), (-2, 6), (-3, -1), (-4, 8)\}$

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function.

34. $f(x) = 2x$
$g(x) = x + 5$

35. $f(x) = -3x$
$g(x) = -x + 8$

36. $f(x) = x + 5$
$g(x) = 3x - 7$

37. $f(x) = x - 4$
$g(x) = x^2 - 10$

38. $f(x) = x^2 + 6x - 2$
$g(x) = x - 6$

39. $f(x) = 2x^2 - x + 1$
$g(x) = 4x + 3$

40. $f(x) = 4x - 1$
$g(x) = x^3 + 2$

41. $f(x) = x^2 + 3x + 1$
$g(x) = x^2$

42. $f(x) = 2x^2$
$g(x) = 8x^2 + 3x$

Example 3

43. MP **SENSE-MAKING** Ms. Smith wants to buy a home theater system, which is on sale for 35% off the original price of $2299. The sales tax is 6.25%.

a. Write two functions representing the price after the discount $p(x)$ and the price after sales tax $t(x)$.

b. Which composition of functions represents the price of the home theater system, $[p \circ t](x)$ or $[t \circ p](x)$? Explain your reasoning.

c. How much will Ms. Smith pay for the home theater system?

If $f(x) = 5x$, $g(x) = -2x + 1$, and $h(x) = x^2 + 6x + 8$, find each value.

44. $f[g(3a)]$

45. $f[h(a + 4)]$

46. $g[f(a^2 - a)]$

Use the table to find each value.

47. $[f \circ g](-2)$

48. $[g \circ f](-2)$

49. $[f \circ g](-1)$

50. $[g \circ f](1)$

51. $[h \circ g](2)$

52. $[g \circ h](2)$

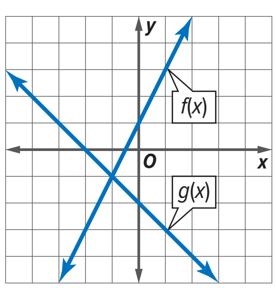
53. $[f \circ h](0)$

54. $[f \circ g](-1)$

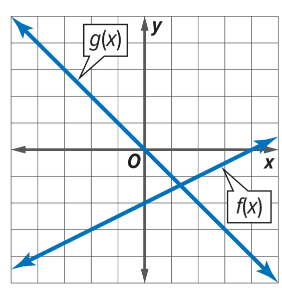
x	$f(x)$	$g(x)$	$h(x)$
−2	−2	1	0
−1	−4	1	0
0	−2	−2	−2
1	2	−1	−2
2	−1	0	5

Use the graph of $f(x)$ and $g(x)$ to find each value.

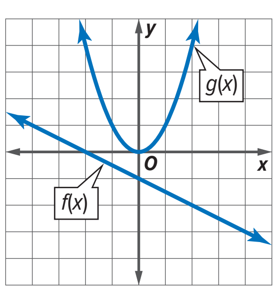
55. $[g \circ f](1)$



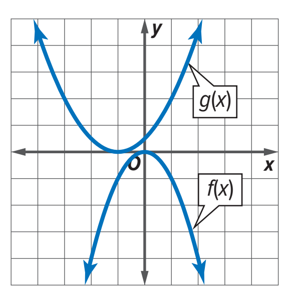
56. $[f \circ g](-2)$



57. $[f \circ g](2)$



58. $[g \circ f](1)$



If $f(x) = x + 2$, $g(x) = -4x + 3$, and $h(x) = x^2 - 2x + 1$, find each value.

59. $[f \circ (g \circ h)](2)$

60. $[g \circ (h \circ f)](-4)$

61. $[h \circ (f \circ g)](5)$

62. **MULTIPLE REPRESENTATIONS** You will explore $[f \circ g](x)$ and $[g \circ f](x)$ if $f(x) = x^2 + 1$ and $g(x) = x - 3$.
    - **a. Tabular** Make a table showing values for practices $[f \circ g](x)$ and $[g \circ f](x)$.
    - **b. Graphical** Use a graphing calculator to graph $[f \circ g](x)$, and $[g \circ f](x)$ on the same coordinate plane.
    - **c. Verbal** Explain the relationship between $[f \circ g](x)$, and $[g \circ f](x)$.

63. MP **REASONING** Copy and complete the table. Use the following clues and logical reasoning to help you.
    - $f(x)$ and $g(x)$ are linear functions.
    - $[f \circ g](2) = 6$
    - $[g \circ f](3) = 10$

x	$f(x)$	$g(x)$
1		
2		
3		
4	6	10
5		

Given that $f(x) = mx + d$ and $g(x) = ax^2 + bx + c$, find each composition.

64. $(f \circ g)(x)$

65. $(g \circ f)(x)$

66. $(f \circ f)(x)$

67. Suppose $f(x) = x^p$ and $g(x) = x^q$, where p and q are positive integers. What can you say about the power of $(f \circ g)(x)$ and $(g \circ f)(x)$? Explain.

H.O.T. Problems Use Higher-Order Thinking Skills

68. **OPEN-ENDED** Write two functions $f(x)$ and $g(x)$ such that $(f \circ g)(4) = 0$.

69. **ERROR ANALYSIS** Denise and Keiko were asked to find $[f \circ g](x)$ given that $f(x) = 6x + 5$ and $g(x) = 2x - 1$. Is either of them correct? Explain your reasoning. If neither student is correct, provide the correct answer.

Denise

$[f \circ g](x) = (6x + 5)(2x - 1)$
$= (6x)(2x) + 6x(-1) + 5(2x) + 5(-1)$
$= 12x^2 - 6x + 10x - 5$
$= 12x^2 + 4x - 5$

Keiko

$[f \circ g](x) = f[g(x)]$
$= f(2x - 1)$
$= 6(2x - 1) + 5$
$= 12x - 1 + 5$
$= 12x + 4$

70. **CHALLENGE** Given that $f(x) = 3x + 4$, find $[f \circ f \circ f](2)$.

71. MP **REASONING** State whether each statement is *sometimes*, *always*, or *never* true. Explain.
    - **a.** The domain of two functions $f(x)$ and $g(x)$ that are composed $g[f(x)]$ is restricted by the domain of $f(x)$.
    - **b.** The domain of two functions $f(x)$ and $g(x)$ that are composed $g[f(x)]$ is restricted by the domain of $g(x)$.

72. **WRITING IN MATH** In the real world, why would you ever perform a composition of functions?

73. What is the value of $f[g(5)]$ if $f(x) = \frac{x+2}{2}$ and $g(x) = x^2 - 2$? MP 1

- ◯ **A** $3\frac{1}{2}$
- ◯ **B** $10\frac{1}{4}$
- ◯ **C** $12\frac{1}{2}$
- ◯ **D** 23

74. Let $f(x) = x^2 + 1$. What is the value of $f(f(3))$? MP 1

- ◯ **A** 2
- ◯ **B** 10
- ◯ **C** 82
- ◯ **D** 101

75. For which pair(s) of functions is $[f \circ g](x)$ a quadratic function? MP 7

- ☐ **A** $f(x) = x^2$ and $g(x) = 3x + 7$
- ☐ **B** $f(x) = x - 4$ and $g(x) = x^2 - 1$
- ☐ **C** $f(x) = x + 9$ and $g(x) = 2x - 3$
- ☐ **D** $f(x) = x$ and $g(x) = 6x$
- ☐ **E** $f(x) = x^2 - 1$ and $g(x) = x^2 + 1$
- ☐ **F** $f(x) = x^2 - 4$ and $g(x) = 3x$

76. If $f(x) = 2x + 1$ and $g(x) = 4x - 5$, which of the following is $[f \circ g](x)$? MP 1

- ◯ **A** $[f \circ g](x) = 8x - 9$
- ◯ **B** $[f \circ g](x) = 8x - 4$
- ◯ **C** $[f \circ g](x) = 8x - 1$
- ◯ **D** $[f \circ g](x) = 8x - 10$

77. Find $(f \circ g)(4)$ for the following functions. MP 1, 6

$f(x) = 2x + 5$

$g(x) = x^2 - 2x + 3$

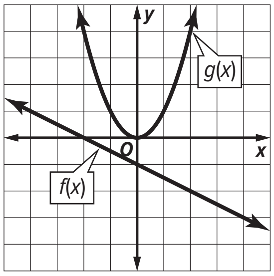
78. Find $(g \circ f)(-2)$ for the following functions. MP 1, 6

$f(x) = -x^2 + 6x - 1$

$g(x) = 2x^2 - 3x$

- ◯ **A** -113
- ◯ **C** 629
- ◯ **B** 476
- ◯ **D** 1084

79. The graphs of $f(x)$ and $g(x)$ are shown.



What is $[f \circ g](2)$? MP 7

- ◯ **A** -8
- ◯ **B** -3
- ◯ **C** 2
- ◯ **D** 4

80. If $f(x) = x^2 - 8$ and $g(x) = 3x + 1$, what is the value of $[g \circ f](-4)$? MP 6

$[g \circ f](-4) =$ ______

81. **MULTI-STEP** Amelia is shopping at an online store. Shipping for each order costs a flat fee of \$6.75. Sales tax is 8%. MP 1, 4

a. Let x represent the cost of an order before shipping or sales tax. Write a function $s(x)$ that represents the cost of an order with shipping, and a function $t(x)$ that represents the cost of an order with sales tax.

b. Find $[s \circ t](x)$ and explain what this function represents.

c. Find $[t \circ s](x)$ and explain what this function represents.

d. Will Amelia get a better deal if the shipping fee is applied to the order before sales tax, or after sales tax? Explain.

LESSON 3

Inverse Functions and Relations

Then

- You transformed and solved equations for a specific variable.

Now

1. Find the inverse of a function or relation.
2. Determine whether two functions or relations are inverses.

Why?

- The table shows the value of \$1 (U.S.) compared to Canadian dollars and Mexican pesos.

The equation $p = 12.45d$ represents the number of pesos p you can receive for every U.S. dollar d. To determine how many U.S. dollars you can receive for one Mexican peso, solve the equation $p = 12.45d$ for d. The result, $d \approx 0.08p$, is the inverse function.

	U.S.	Canada	Mexico
U.S.		1.05	12.45
Canada	0.95		11.97
Mexico	0.08	0.08	

New Vocabulary

inverse relation
inverse function

Mathematical Practices

7 Look for and make use of structure.

8 Look for and express regularity in repeated reasoning.

1 Find Inverses

Recall that a relation is a set of ordered pairs. The **inverse relation** is the set of ordered pairs obtained by exchanging the coordinates of each ordered pair. The domain of a relation becomes the range of its inverse, and the range of the relation becomes the domain of its inverse.

Key Concept Inverse Relations

Words Two relations are inverse relations if and only if whenever one relation contains the element (a, b), the other relation contains the element (b, a).

Example A and B are inverse relations.

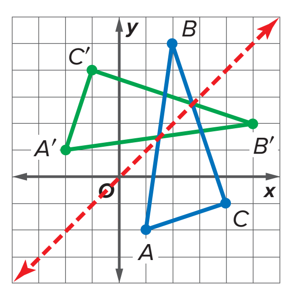
$A = \{(1, 5), (2, 6), (3, 7)\}$ $B = \{(5, 1), (6, 2), (7, 3)\}$

Example 1 Find an Inverse Relation

GEOMETRY The vertices of $\triangle ABC$ can be represented by the relation $\{(1, -2), (2, 5), (4, -1)\}$. Find the inverse of this relation. Describe the graph of the inverse.

Graph the relation. To find the inverse, exchange the coordinates of the ordered pairs. The inverse of the relation is $\{(-2, 1), (5, 2), (-1, 4)\}$.

Plotting these points shows that the ordered pairs describe the vertices of $\triangle A'B'C'$ as a reflection of $\triangle ABC$ in the line $y = x$.



Guided Practice

1. **GEOMETRY** The ordered pairs of the relation $\{(-8, -3), (-8, -6), (-3, -6)\}$ are the coordinates of the vertices of a right triangle. Find the inverse of this relation. Describe the graph of the inverse.

As with relations, the ordered pairs of **inverse functions** are also related. We can write the inverse of the function $f(x)$ as $f^{-1}(x)$.

Dave Moyer

Reading Math

MP Sense-Making f^{-1} is read *f inverse* or *the inverse of f*. Note that -1 is *not* an exponent.

Key Concept Property of Inverses

Words	If f and f^{-1} are inverses, then $f(a) = b$ if and only if $f^{-1}(b) = a$.
Example	Let $f(x) = x - 4$ and represent its inverse as $f^{-1}(x) = x + 4$.

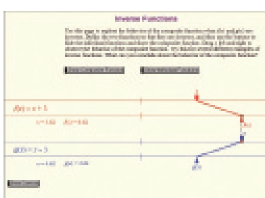
Evaluate $f(6)$.	Evaluate $f^{-1}(2)$.
$f(x) = x - 4$	$f^{-1}(x) = x + 4$
$f(6) = 6 - 4$ or 2	$f^{-1}(2) = 2 + 4$ or 6

Because $f(x)$ and $f^{-1}(x)$ are inverses, $f(6) = 2$ and $f^{-1}(2) = 6$.

The inverse of a function can be found by exchanging the domain and the range.

Go Online!

Explore the inverses of several functions and figure out when inverse functions exist and when they don't using **The Geometer's Sketchpad®** activity in ConnectED.



Example 2 Find and Graph an Inverse

Find the inverse of each function. Then graph the function and its inverse.

a. $f(x) = 2x - 5$

Step 1 Rewrite the function as an equation relating x and y.

$f(x) = 2x - 5 \rightarrow y = 2x - 5$

Step 2 Exchange x and y in the equation. $x = 2y - 5$

Step 3 Solve the equation for y.

$x = 2y - 5$ Inverse of $y = 2x - 5$

$x + 5 = 2y$ Add 5 to each side.

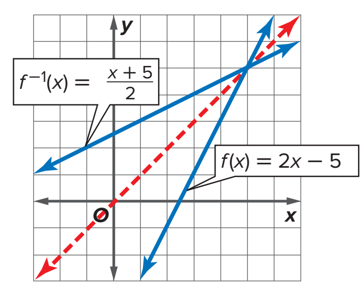
$\frac{x+5}{2} = y$ Divide each side by 2.

Step 4 Replace y with $f^{-1}(x)$.

$y = \frac{x+5}{2} \rightarrow f^{-1}(x) = \frac{x+5}{2}$

The inverse of $f(x) = 2x - 5$ is $f^{-1}(x) = \frac{x+5}{2}$.

The graph of $f^{-1}(x) = \frac{x+5}{2}$ is the reflection of the graph of $f(x) = 2x - 5$ in the line $y = x$.



b. $f(x) = x^2 + 1$

Step 1 $f(x) = x^2 + 1 \rightarrow y = x^2 + 1$

Step 2 $x = y^2 + 1$

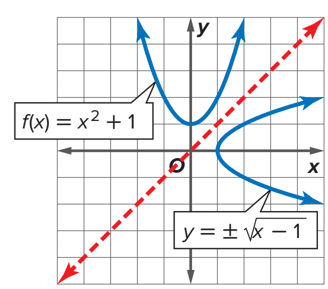
Step 3 $x = y^2 + 1$

$x - 1 = y^2$

$\pm\sqrt{x-1} = y$ Take the square root of each side.

Step 4 $y = \pm\sqrt{x-1}$

Graph $y = \pm\sqrt{x-1}$ by reflecting the graph of $f(x) = x^2 + 1$ in the line $y = x$.



Study Tip

Functions The inverse of the function in part **b** is not a function since it does not pass the vertical line test.

Guided Practice

Find the inverse of each function. Then graph the function and its inverse.

2A. $f(x) = \frac{x-3}{5}$

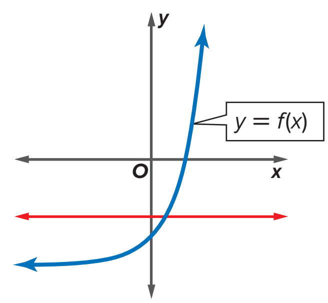
2B. $f(x) = 3x^2$

Not all functions have an inverse function. The graph of the initial relation in Example 2b is a function because it passes the vertical line test. However, its inverse relation fails this test, so it is not a function. The reflection relationship between the graph of a function and its inverse relation leads to the horizontal line test for determining whether an inverse of a function is itself a function.

Key Concept Horizontal Line Test

Words A function f has an inverse function f^{-1} if and only if each horizontal line intersects the graph of the function in at most one point.

Example Because no horizontal line intersects the graph of f more than once, the inverse function f^{-1} exists.



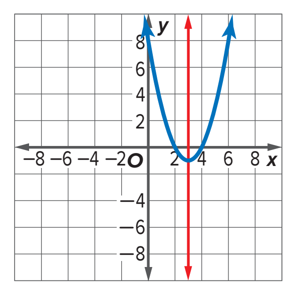
Sometimes it is necessary to restrict the domain of a function in order for its inverse to be a function.

Example 3 Inverses with Restricted Domains

Find the inverse of $f(x) = x^2 - 6x + 8$. Then graph the function and its inverse. If necessary, restrict the domain of $f(x)$ so that the inverse is a function.

Step 1 Use a graph to determine whether $f(x)$ and $f^{-1}(x)$ are functions.

$f(x)$ is a function because it passes the vertical line test. However, $f(x)$ does not pass the horizontal line text, which indicates that $f^{-1}(x)$ is not a function.



Step 2 Identify the axis of symmetry.

The axis of symmetry is $x = 3$.

Step 3 Find $f^{-1}(x)$.

$f(x) = x^2 - 6x + 8$	Original function
$y = x^2 - 6x + 8$	Replace $f(x)$ with y.
$x = y^2 - 6y + 8$	Exchange x and y.
$x - 8 + 9 = y - 6y + 9$	Complete the square.
$x + 1 = (y - 3)^2$	Simplify.
$\pm\sqrt{x+1} = y - 3$	Take the square root of each side.
$3 \pm \sqrt{x+1} = y$	Add 3 to each side.
$f^{-1}(x) = 3 \pm \sqrt{x+1}$	Replace y with $f^{-1}(x)$.

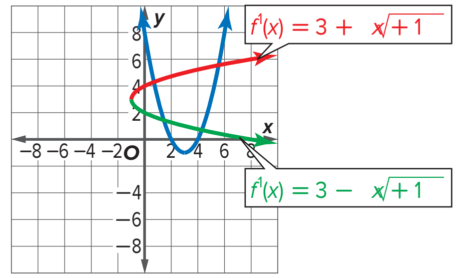
Step 4 Find a restricted domain of $f(x)$ so that $f^{-1}(x)$ will be a function.

If the domain is restricted to $(-\infty, 3]$, then the inverse is $f^{-1}(x) = 3 - \sqrt{x+1}$.

If the domain is restricted to $[3, \infty)$, then the inverse is $f^{-1}(x) = 3 + \sqrt{x+1}$.

Step 5 Graph.

Notice that in each case, the range of $f^{-1}(x)$ is restricted so that the graph passes the vertical line test.



Guided Practice

3. Find the inverse of $f(x) = x^2 + 7x + 12$. Then graph the function and its inverse. If necessary, restrict the domain of $f(x)$ so that the inverse is a function.

2 Verifying Inverses

You can determine whether two functions are inverses by finding both of their compositions. If both compositions equal the identity function $I(x) = x$, then the functions are inverse functions.

Key Concept Inverse Functions

Words	Two functions f and g are inverse functions if and only if both of their compositions are the identity function.
Symbols	$f(x)$ and $g(x)$ are inverses if and only if $[f \circ g](x) = x$ and $[g \circ f](x) = x$.

Watch Out!

Inverse Functions Be sure to check both $[f \circ g](x)$ and $[g \circ f](x)$ to verify that functions are inverses. By definition, both compositions must be the identity function.

Example 4 Verify that Two Functions are Inverses

Determine whether each pair of functions are inverse functions. Explain your reasoning.

a. $f(x) = 3x + 9$ **and** $g(x) = \frac{1}{3}x - 3$

Verify that the compositions of $f(x)$ and $g(x)$ are identity functions.

$$[f \circ g](x) = f[g(x)] = f\left(\frac{1}{3}x - 3\right) = 3\left(\frac{1}{3}x - 3\right) + 9 = x - 9 + 9 \text{ or } x$$

$$[g \circ f](x) = g[f(x)] = g(3x + 9) = \frac{1}{3}(3x + 9) - 3 = x + 3 - 3 \text{ or } x$$

The functions are inverses because $[f \circ g](x) = [g \circ f](x) = x$.

b. $f(x) = 4x^2$ **and** $g(x) = 2\sqrt{x}$

$$[f \circ g](x) = f\left(2\sqrt{x}\right) = 4\left(2\sqrt{x}\right)^2 = 4(4x) \text{ or } 16x$$

Because $[f \circ g](x) \neq x$, $f(x)$ and $g(x)$ are not inverses.

Guided Practice

4A. $f(x) = 3x - 3$, $g(x) = \frac{1}{3}x + 4$

4B. $f(x) = 2x^2 - 1$, $g(x) = \sqrt{\frac{x + 1}{2}}$

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Example 1 Find the inverse of each relation.

1. $\{(-9, 10), (1, -3), (8, -5)\}$
2. $\{(-2, 9), (4, -1), (-7, 9), (7, 0)\}$

Example 2 Find the inverse of each function. Then graph the function and its inverse.

3. $f(x) = -3x$
4. $g(x) = 4x - 6$
5. $h(x) = x^2 - 3$

Example 3 Find the inverse for each function. Then graph the function and its inverse. If necessary, restrict the domain of $f(x)$ so that the inverse is a function.

6. $f(x) = x^2 + 4x - 5$
7. $f(x) = x^2 - 16x + 63$

Example 4 Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

8. $f(x) = x - 7$
   $g(x) = x + 7$
9. $f(x) = \frac{1}{2}x + \frac{3}{4}$
   $g(x) = 2x - \frac{4}{3}$
10. $f(x) = 2x^3$
    $g(x) = \frac{1}{3}\sqrt{x}$

Practice and Problem Solving

Extra Practice is on page R5.

Example 1 Find the inverse of each relation.

11. $\{(-8, 6), (6, -2), (7, -3)\}$
12. $\{(7, 7), (4, 9), (3, -7)\}$
13. $\{(8, -1), (-8, -1), (-2, -8), (2, 8)\}$
14. $\{(4, 3), (-4, -4), (-3, -5), (5, 2)\}$

Example 2 MP **SENSE-MAKING** Find the inverse of each function. Then graph the function and its inverse.

15. $f(x) = x + 2$
16. $g(x) = 5x$
17. $f(x) = -2x + 1$
18. $h(x) = \frac{x - 4}{3}$
19. $f(x) = -\frac{5}{3}x - 8$
20. $g(x) = x + 4$
21. $h(x) = x^2 + 4$
22. $f(x) = \frac{1}{2}x^2 - 1$
23. $f(x) = (x + 1)^2 + 3$

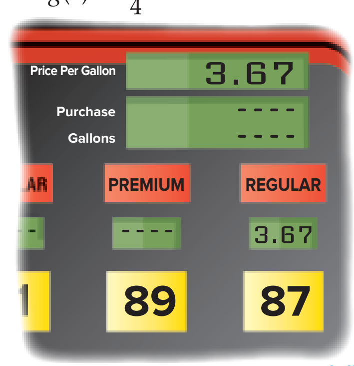
Example 3 Find the inverse for each function. Then graph the function and its inverse. If necessary, restrict the domain of $f(x)$ so that the inverse is a function.

24. $f(x) = 4x$
25. $f(x) = -8x + 9$
26. $f(x) = 5x^2$
27. $f(x) = x^2 + 12x + 32$
28. $f(x) = x^2 - 22x + 120$
29. $f(x) = x^2 - 36x - 160$

Example 4 Determine whether each pair of functions are inverse functions. Write *yes* or *no*.

30. $f(x) = 2x + 3$
    $g(x) = 2x - 3$
31. $f(x) = 4x + 6$
    $g(x) = \frac{x - 6}{4}$
32. $f(x) = -\frac{1}{3}x + 3$
    $g(x) = -3x + 9$
33. $f(x) = -6x$
    $g(x) = \frac{1}{6}x$
34. $f(x) = \frac{1}{2}x + 5$
    $g(x) = 2x - 10$
35. $f(x) = \frac{x + 10}{8}$
    $g(x) = 8x - 10$
36. $f(x) = \frac{2}{3}x^3$
    $g(x) = \sqrt{\frac{2}{3}x}$
37. ● $f(x) = (x + 6)^2$
    $g(x) = \sqrt{x} - 6$
38. $f(x) = 2\sqrt{x - 5}$
    $g(x) = \frac{1}{4}x^2 - 5$

39. **FUEL** The average miles traveled for every gallon g of gas consumed by Leroy's car is represented by the function $m(g) = 28g$.
    a. Find a function $c(g)$ to represent the cost per gallon of gasoline.
    b. Use inverses to determine the function used to represent the cost per mile traveled in Leroy's car.



40. **MULTI-STEP** Carlos is looking to trade in his old car for a new one. He has 6 payments remaining on his old car. The dealer is offering 0% financing and a \$4000 trade-in with the purchase of a new car. Carlos plans to take out a 5-year loan on the new car.

a. If his current monthly payment is \$280 and he doesn't want to pay more than \$300 per month on a car, what is the most expensive new car that he can afford?

b. Describe your solution process.

41. **GEOMETRY** The formula for the area of a circle is $A = \pi r^2$.

a. Find the inverse of the function.

b. Use the inverse to find the radius of a circle with an area of 36 square centimeters.

Use the horizontal line test to determine whether the inverse of each function is also a function.

42. $f(x) = 2x^2$

43. $f(x) = x^3 - 8$

44. $g(x) = x^4 - 6x^2 + 1$

45. $h(x) = -2x^4 - x - 2$

46. $g(x) = x^5 + x^2 - 4x$

47. $h(x) = x^3 + x^2 - 6x + 12$

48. **SHOPPING** Felipe bought a used car. The sales tax rate was 7.25% of the selling price, and he paid \$350 in processing and registration fees. Find the selling price if Felipe paid a total of \$8395.75.

49. **TEMPERATURE** A formula for converting degrees Celsius to Fahrenheit is $F(x) = \frac{9}{5}x + 32$.

a. Find the inverse $F^{-1}(x)$. Show that $F(x)$ and $F^{-1}(x)$ are inverses.

b. Explain what purpose $F^{-1}(x)$ serves.

50. **MEASUREMENT** There are approximately 1.852 kilometers in a nautical mile.

a. Write a function that converts nautical miles to kilometers.

b. Find the inverse of the function that converts kilometers back to nautical miles.

c. Using composition of functions, verify that these two functions are inverses.

51. **MULTIPLE REPRESENTATIONS** Consider the functions $y = x^n$ for $n = 0, 1, 2, \dots$.

a. **Graphing** Use a graphing calculator to graph $y = x^n$ for $n = 0, 1, 2, 3,$ and 4.

b. **Tabular** For which values of n is the inverse a function? Record your results in a table.

c. **Analytical** Make a conjecture about the values of n for which the inverse of $f(x) = x^n$ is a function. Assume that n is a whole number.

H.O.T. Problems — Use Higher-Order Thinking Skills

52. MP **REASONING** If a relation is *not* a function, then its inverse is *sometimes*, *always*, or *never* a function. Explain your reasoning.

53. **OPEN-ENDED** Give an example of a function and its inverse. Verify that the two functions are inverses.

54. **CHALLENGE** Give an example of a function that is its own inverse.

55. MP **CONSTRUCT ARGUMENTS** Show that the inverse of a linear function $y = mx + b$, where $m \neq 0$ and $x \neq b$, is also a linear function.

56. **WRITING IN MATH** Suppose you have a composition of two functions that are inverses. When you put in a value of 5 for x, why is the result always 5?

Preparing for Assessment

57. The inverse relation of which function is *not* a function? MP 3

- A $f(x) = x^2 - 2$
- B $f(x) = 2x^3 + 3$
- C $f(x) = \sqrt{x - 3}$
- D $f(x) = \sqrt[3]{2x + 1}$

58. Find the inverse of $f(x) = \frac{1}{x^2}$ MP 1, 6

59. Let $f(n) = 2n$ where $n \neq 0$. If $n = m$, which statement must be true? MP 3

- A $f^{-1}(m) = m$
- B $f^{-1}(m) = 2m$
- C $f^{-1}\left(\frac{1}{2}m\right) = m$
- D $f^{-1}(2m) = m$
- E $f^{-1}(2m) = 4m$

60. **MULTI-STEP** Consider the function $f(x) = \frac{x + 2}{2}$.

a. Work is shown below to find the inverse function $f^1(x)$. Which lines contain errors if the intention is to find the inverse function? Choose all that apply. MP 1

- A $f(x) = \frac{x + 2}{2}$
- B $y = \frac{x + 2}{2}$
- C $x = \frac{y + 2}{2}$
- D $2y = y + 2$
- E $2x - 2 = y$
- F $f^{-1}(x) = 2y - 2$

b. Write the correct inverse function for $f(x) = \frac{x + 2}{2}$.

c. What results when $f(f^{-1}(x))$ is calculated?

d. What results when $f^{-1}(f(x))$ is calculated?

61. Which pairs of functions below are inverse functions? Choose all that apply. MP 1

- A $f(x) = \frac{x + 2}{2}$ and $f(x) = \frac{y + 2}{2}$
- B $f(x) = 3x + 9$ and $f(x) = \frac{1}{3}x - 3$
- C $x = \frac{\sqrt{y}}{2}$ and $2x^2 = y$
- D $x = \frac{\sqrt{y}}{2}$ and $2y^2 = x$
- E $3x - 3 = y$ and $y = \frac{x + 3}{3}$
- F $3x - 3 = y$ and $y = \frac{x - 3}{3}$

62. Does the inverse of each function pass the vertical line test? Choose all for which the answer is yes. MP 3

- A $f(x) = \frac{x + 2}{2}$
- B $f(x) = 3\sqrt{x} + 9$
- C $2y^2 = x$
- D $x = \frac{\sqrt{y}}{2}$
- E $y = \frac{x + 3}{3}$
- F $y = \frac{x^4 - 3}{3}$

63. Find the inverse of $f(x) = \frac{\sqrt{x}}{2}$ and then graph the function and its inverse. MP 1, 4, 6

64. Given that $f(2) = 4$, $f(4) = 6$, and $f(6) = 8$, what is the value of $f^{-1}(f^{-1}(6))$? MP 2

- A 2
- B 4
- C 6
- D 8

EXTEND 5-3

Graphing Technology Lab

Inverse Functions and Relations

You can use a TI-83/84 Plus graphing calculator to compare a function and its inverse using tables and graphs. Note that before you enter any values in the calculator, you should clear all lists.

Activity 1 Graph Inverses with Ordered Pairs

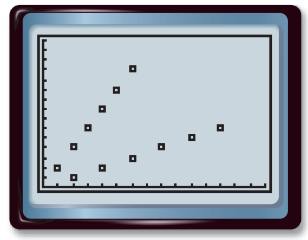
Graph $f(x) = \{(1, 2), (2, 4), (3, 6), (4, 8), (5, 10), (6, 12)\}$ and its inverse.

Step 1 Enter the x-values in **L1** and the y-values in **L2**. Then graph the function.

KEYSTROKES: [STAT] [ENTER] 1 [ENTER] 2 [ENTER] 3 [ENTER] 4 [ENTER] 5 [ENTER] 6 [ENTER] [▶] 2 [ENTER] 4 [ENTER] 6 [ENTER] 8 [ENTER] 10 [ENTER] 12 [ENTER] [2nd] [STAT PLOT] [ENTER] [ENTER] [GRAPH]

Adjust the window to reflect the domain and range.

Step 2:



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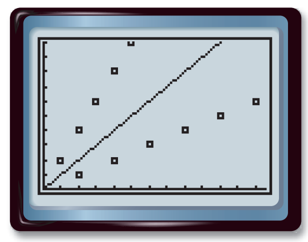
Step 2 Define the inverse function by setting **Xlist** to **L2** and **Ylist** to **L1**. Then graph the inverse function.

KEYSTROKES: [2nd] [STAT PLOT] [▼] [ENTER] [ENTER] [▼] [▼] [2nd] [L2] [▼] [2nd] [L1] [GRAPH]

Step 3:



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Step 3 Graph the line $y = x$.

KEYSTROKES: [Y=] [X,T,θ,n] [GRAPH]

Activity 2 Graph Inverses with Function Notation

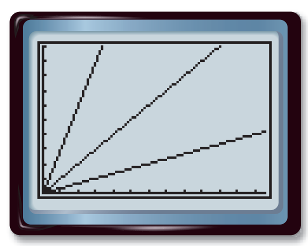
Graph $f(x) = 3x$ and its inverse $g(x) = \frac{x}{3}$.

Step 1 Clear the data from Activity 1.

KEYSTROKES: [2nd] [STAT PLOT] [ENTER] [▶] [ENTER] [▲] [▶] [ENTER] [▶] [ENTER] [2nd] [QUIT]

Step 2 Enter $f(x)$ as **Y1**, $g(x)$ as **Y2**, and $y = x$ as **Y3**. Then graph.

KEYSTROKES: [Y=] 3 [X,T,θ,n] [ENTER] [X,T,θ,n] [÷] 3 [ENTER] [X,T,θ,n] [GRAPH]



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Exercises

Graph each function $f(x)$ and its inverse $g(x)$. Then graph $(f \circ g)(x)$.

1. $f(x) = 5x$
2. $f(x) = x - 3$
3. $f(x) = 2x + 1$
4. $f(x) = \frac{1}{2}x + 3$
5. $f(x) = x^2$
6. $f(x) = x^2 - 3$
7. What is the relationship between the graphs of a function and its inverse?
8. **MAKE A CONJECTURE** For any function $f(x)$ and its inverse $g(x)$, what is $(f \circ g)(x)$?

Mid-Chapter Quiz

Lessons 5-1 through 5-3

Given $f(x) = 2x^2 + 4x - 3$ and $g(x) = 5x - 2$, find each function. (Lesson 5-1)

1. $(f + g)(x)$

2. $(f - g)(x)$

3. $(f \cdot g)(x)$

4. $\left(\frac{f}{g}\right)(x)$

5. **FINANCE** A small company is producing a new product. The revenue $r(x)$ from the sale of x units of the new product is expected to be $r(x) = 10x$. The cost of manufacturing x units is $c(x) = 2.25x + 2000$. (Lesson 5-1)

 a. Write the profit function.

 b. Find the profit on 1000 units of the product.

 c. MP What mathematical practice did you use to solve this problem?

Given $f(x) = 2x^2 + 4x - 3$ and $g(x) = 5x - 2$, find each function. (Lesson 5-2)

6. $[f \circ g](x)$

7. $[g \circ f](x)$

8. **PRODUCTION** The cost in dollars of producing p cell phones in a factory is represented by $C(p) = 5p + 60$. The number of cell phones produced in h hours is represented by $P(h) = 40h$. (Lesson 5-2)

 a. Find the composition function.

 b. Determine the cost of producing cell phones for 8 hours.

Find $[f \circ g](x)$ and $[g \circ f](x)$, if they exist. State the domain and range for each composed function. (Lesson 5-2)

9. $f(x) = 4x$
 $g(x) = x - 8$

10. $f(x) = 3x - 1$
 $g(x) = 5x + 1$

11. $f(x) = -2x$
 $g(x) = x^2 - 8$

12. **SHOPPING** Mrs. Ross is shopping for her children's school clothes. She has a coupon for 25% off her total. The sales tax of 6% is added to the total after the coupon is applied. (Lesson 5-2)

 a. Express the total price after the discount and the total price after the tax using function notation. Let x represent the price of the clothing, $p(x)$ represent the price after the 25% discount, and $g(x)$ represent the price after the tax is added.

 b. Which composition of functions represents the final price, $p[g(x)]$ or $g[p(x)]$? Explain your reasoning.

Determine whether each pair of functions are inverse functions. Write *yes* or *no*. (Lesson 5-3)

13. $f(x) = 2x + 16$
 $g(x) = \frac{1}{2}x - 8$

14. $g(x) = 4x + 15$
 $h(x) = \frac{1}{4}x - 15$

15. $f(x) = x^2 - 5$
 $g(x) = 5 + x^{-2}$

16. $g(x) = -6x + 8$
 $h(x) = \frac{8 - x}{6}$

Find the inverse of each function, if it exists. (Lesson 5-3)

17. $h(x) = \frac{2}{5}x + 8$

18. $f(x) = \frac{4}{9}(x - 3)$

19. $h(x) = -\frac{10}{3}(x + 5)$

20. $f(x) = \frac{x + 12}{7}$

Use the horizontal line test to determine whether the inverse of each function is also a function. (Lesson 5-3)

21. $f(x) = 4x - 1$

22. $f(x) = 10x^2$

23. $f(x) = 3x^3 - 8$

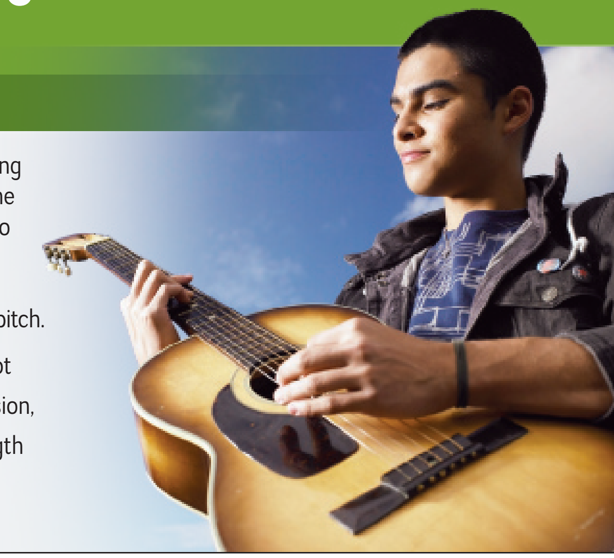
24. **JOBS** Louise runs a lawn care service. She charges \$25 for supplies plus \$15 per hour. The function $f(h) = 15h + 25$ gives the cost $f(h)$ for h hours of work. (Lesson 5-3)

 a. Find $f^{-1}(h)$. What is the significance of $f^{-1}(h)$?

 b. If Louise charges a customer \$85, how many hours did she work?

LESSON 4

Graphing Square Root Functions



Then

- You simplified expressions with square roots.

Now

1. Graph square root functions.
2. Analyze square root functions.

Why?

- With guitars, pitch is dependent on string length and string tension. The longer the string is, the higher the tension needs to be to produce a desired pitch. Likewise, the heavier the string is, the higher the tension needs to be to reach a desired pitch.

  This can be modeled by the square root function $f = \frac{1}{2L}\sqrt{\frac{T}{P}}$, where T is the tension, P is the mass of the string, L is the length of the string, and f is the pitch.

New Vocabulary

square root function
radical function

Mathematical Practices

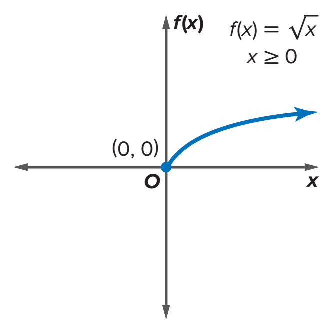
1 Make sense of problems and persevere in solving them.
2 Reason abstractly and quantitatively.
4 Model with mathematics.

1 Square Root Functions

If a function contains the square root of a variable, it is called a **square root function**. The square root function is a type of **radical function**.

Key Concept Parent Function of Square Root Functions

Parent function:	$f(x) = \sqrt{x}$
Domain:	$\{x \mid x \geq 0\}$ or $[0, +\infty)$
Range:	$\{f(x) \mid f(x) \geq 0\}$ or $[0, +\infty)$
Intercepts:	$x = 0, f(x) = 0$
Symmetry:	none
Not defined:	$x < 0$
End behavior:	$x \to 0, f(x) \to 0$; $x \to +\infty, f(x) \to +\infty$
Extrema:	minimum at (0, 0)



The domain of a square root function is limited to values for which the square root function is defined.

Example 1 Identify Domain and Range

Identify the domain and range of $f(x) = \sqrt{x + 4}$.

The domain only includes values for which the radicand is nonnegative.

$x + 4 \geq 0$ Write an inequality.
$x \geq -4$ Subtract 4 from each side.

D = $[-4, +\infty)$, $\{x \mid x \geq -4\}$, or $\{-4 \leq x < \infty\}$.

Find $f(-4)$ to determine the lower limit of the range.

$f(-4) = \sqrt{-4 + 4}$ or 0

R = $[0, +\infty)$, $\{f(x) \mid f(x) \geq 0\}$, or $\{0 \leq x < \infty\}$

Guided Practice

Identify the domain and range of each function.

1A. $f(x) = \sqrt{x - 3}$

1B. $f(x) = \sqrt{x + 6} + 2$

The same techniques used to transform the graph of other functions you have studied can be applied to the graphs of square root functions.

Key Concept Transformations of Square Root Functions

$$f(x) = a\sqrt{x - h} + k$$

h—Horizontal Translation	*k*—Vertical Translation
h units right if *h* is positive $\lvert h \rvert$ units left if *h* is negative The domain is $\{x \mid x \geq h\}$.	*k* units up if *k* is positive $\lvert k \rvert$ units down if *k* is negative If $a > 0$, then the range is $\{f(x) \mid f(x) \geq k\}$. If $a < 0$, then the range is $\{f(x) \mid f(x) \leq k\}$.

a—Orientation and Shape

- If $a < 0$, the graph is reflected across the *x*-axis.
- If $\lvert a \rvert > 1$, the graph is stretched vertically.
- If $0 < \lvert a \rvert < 1$, the graph is compressed vertically.

Study Tip

Domain and Range The limits on the domain and range also represent the initial point of the graph of a square root function.

Example 2 Graph Square Root Functions

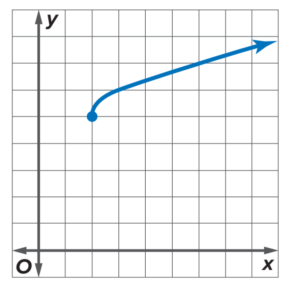
Graph each function. State the domain and range.

a. $y = \sqrt{x - 2} + 5$

The minimum point is at $(h, k) = (2, 5)$. Make a table of values for $x \geq 2$, and graph the function. The graph is the same shape as $f(x) = \sqrt{x}$, but is translated 2 units right and 5 units up. Notice the end behavior. As *x* increases, *y* increases.

D = $[2, +\infty)$, $\{x \mid x \geq 2\}$, or $\{2 \leq x < +\infty\}$
R = $[5, +\infty)$, $\{y \mid y \geq 5\}$, or $\{5 \leq x < +\infty\}$

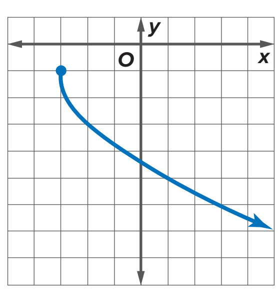
x	y
2	5
3	6
4	6.4
5	6.7
6	7
7	7.2
8	7.4



b. $y = -2\sqrt{x + 3} - 1$

The minimum domain value is at *h* or −3. Make a table of values for $x \geq -3$, and graph the function. Because *a* is negative, the graph is similar to the graph of $f(x) = \sqrt{x}$, but is reflected in the *x*-axis. Because $\lvert a \rvert > 1$, the graph is vertically stretched. It is also translated 3 units left and 1 unit down.

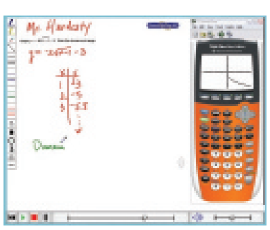
x	y
−3	−1
−2	−3
−1	−3.8
0	−4.5
1	−5
2	−5.5
3	−5.9



D = $[-3, +\infty)$, $\{x \mid x \geq -3\}$, or $\{-3 \leq x < +\infty\}$
R = $(-\infty, -1]$, $\{y \mid y \leq -1\}$, or $\{-\infty < x \leq -1\}$

Go Online!

Follow along with your graphing calculator as you watch a **Personal Tutor** graph a square root function.



Guided Practice

2A. $f(x) = 2\sqrt{x + 4}$

2B. $f(x) = \frac{1}{4}\sqrt{x - 5} + 3$

2 Analyze Square Root Functions

Previously you learned that an inverse relation interchanges the x- and y-coordinates of the original relation. For the power function $f(x) = x^2$, if the domain of x is restricted to nonnegative values, then the inverse of f is the function $f^{-1}(x) = \sqrt{x}, x \geq 0$.

Real-World Link

On every string, the guitar player has an option of decreasing the length of the string in about 24 different ways. This will produce 24 different frequencies on each string.

Source: *Guitar World*

Real-World Example 3 Use Graphs to Analyze Square Root Functions

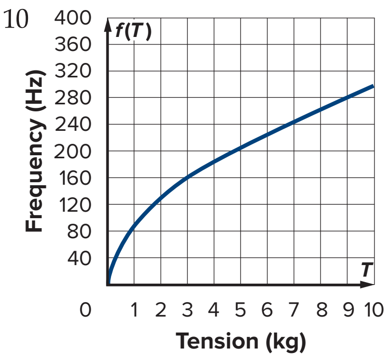
MUSIC Refer to the application at the beginning of the lesson. The pitch, or frequency, measured in hertz (Hz) of a certain string can be modeled by $f(T) = \frac{1}{1.28}\sqrt{\frac{T}{0.0000708}}$, where T is tension in kilograms.

a. Graph the function for tension in the domain $\{T \mid 0 \leq T \leq 10\}$.

Make a table of values for $0 \leq T \leq 10$ and graph.

T	$y(T)$
0	0
1	92.8
2	131.3
3	160.8
4	185.7
5	207.6

T	$f(T)$
6	227.4
7	245.7
8	262.6
9	278.5
10	293.6



b. How much tension is needed for a pitch of over 200 Hz?

According to the graph and the table, more than 4.5 kilograms of tension is needed for a pitch of more than 200 hertz.

Guided Practice

3. **MUSIC** The frequency of vibrations for a certain guitar string when it is plucked can be determined by $F = 200\sqrt{T}$, where F is the number of vibrations per second and T is the tension measured in pounds. Graph the function for $0 \leq T \leq 10$. Then determine the frequency for $T = 3$, 6, and 9 pounds.

Problem-Solving Tip

MP Modeling Making a table is a good way to organize ordered pairs in order to see the general behavior of a graph.

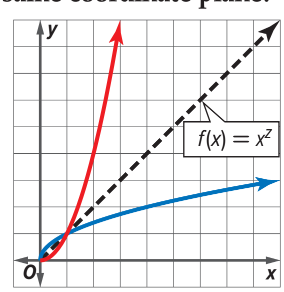
Problem-Solving Tip

MP Reasoning Point out that if the domain of $f(x) = x^2$ is restricted to $x \leq 0$, its inverse would be the function $f^{-1}(x) = -\sqrt{x}$.

Example 4 Find the Inverse of Power Function $f(x) = x^2$

Find the inverse of $f(x) = x^2, x \geq 0$. Graph $f(x)$ and $f^{-1}(x)$ on the same coordinate plane.

$f(x) = x^2$	Write the original function.
$y = x^2$	Replace $f(x)$ with y.
$x = y^2$	Interchange x and y.
$\pm\sqrt{x} = y$	Take the square root of both sides.



Because the domain of f is restricted to nonnegative values of x, the range of f^{-1} must also be restricted to nonnegative values. So, the inverse of f is $f^{-1}(x) = \sqrt{x}, x \geq 0$.

The graph of $f^{-1}(x) = \text{R}x$ is a reflection of the graph of $f(x) = x^2$, $x \geq 0$, in the line $y = x$, shown as a dashed line on the graph.

Guided Practice

4. Find the inverse of $f(x) = x^2 + 1, x \geq 0$ and graph $f(x)$ and $f^{-1}(x)$ on the same coordinate plane.

Check Your Understanding

(green circle) = Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Example 1 **Identify the domain and range of each function.**

1. $f(x) = \sqrt{4x}$
2. $f(x) = \sqrt{x - 5}$
3. $f(x) = \sqrt{x + 8} - 2$

Example 2 **Graph each function. State the domain and range.**

4. $f(x) = \sqrt{x} - 2$
5. $f(x) = 3\sqrt{x - 1}$
6. $f(x) = \frac{1}{2}\sqrt{x + 4} - 1$
7. $f(x) = -\sqrt{3x - 5} + 5$

Example 3

8. **OCEAN** The speed that a tsunami, or tidal wave, can travel is modeled by the equation $v = 356\sqrt{d}$, where v is the speed in kilometers per hour and d is the average depth of the water in kilometers. A tsunami in the ocean is found to be traveling at 145 kilometers per hour. What is the average depth of the water rounded to the nearest hundredth of a kilometer?

Example 4 **Find the inverse of each function. Then graph the function and its inverse on the same coordinate plane.**

9. $f(x) = 3x^2, x \geq 0$
10. $f(x) = x^2 + 2, x \geq 0$
11. $f(x) = -6x^2, x \geq 0$
12. $f(x) = \frac{1}{2}x^2, x \geq 0$

Practice and Problem Solving

Extra Practice is on page R5.

Example 1 **Identify the domain and range of each function.**

13. $f(x) = -\sqrt{2x} + 2$
14. $f(x) = \sqrt{x} - 6$
15. $f(x) = 4\sqrt{x - 2} - 8$
16. $f(x) = \sqrt{x + 2} + 5$
17. $f(x) = \sqrt{x - 4} - 6$
18. $f(x) = -\sqrt{x - 6} + 5$

Example 2 **Graph each function. State the domain and range.**

19. $f(x) = \sqrt{6x}$
20. $f(x) = -\sqrt{5x}$
21. $f(x) = \sqrt{x - 8}$
22. $f(x) = \sqrt{x + 1}$
23. $f(x) = \sqrt{x + 3} + 2$
24. $f(x) = \sqrt{x - 4} - 10$
25. $f(x) = 2\sqrt{x - 5} - 6$
26. $f(x) = \frac{3}{4}\sqrt{x + 12} + 3$
27. $f(x) = -\frac{1}{5}\sqrt{x - 1} - 4$
28. $f(x) = -3\sqrt{x + 7} + 9$

Example 3

29. **SKYDIVING** The approximate time t in seconds that it takes an object to fall a distance of d feet is given by $t = \sqrt{\frac{d}{16}}$. Suppose a parachutist falls 11 seconds before the parachute opens. How far does the parachutist fall during this time?

30. **MP MODELING** The velocity of a roller coaster as it moves down a hill is $V = \sqrt{v^2 + 64h}$, where v is the initial velocity in feet per second and h is the vertical drop in feet. The designer wants the coaster to have a velocity of 90 feet per second when it reaches the bottom of the hill.

    a. If the initial velocity of the coaster at the top of the hill is 10 feet per second, write an equation that models the situation.

    b. How high should the designer make the hill?

Example 4 **Find the inverse of the function. Then graph the function and its inverse on the same coordinate plane.**

31. $f(x) = 2x^2, x \geq 0$

32. $f(x) = x^2 + 1, x \geq 0$

33. $f(x) = -4x^2, x \geq 0$

34. $f(x) = \frac{1}{4}x^2, x \geq 0$

35. $f(x) = -\frac{1}{2}x^2, x \geq 0$

36. $f(x) = 4x^2 + 2, x \geq 0$

37. $f(x) = 9x^2 - 4, x \geq 0$

38. $f(x) = \frac{3}{4}x^2 + 8, x \geq 0$

39. **PHYSICS** The kinetic energy of an object is the energy produced due to its motion and mass. The formula for kinetic energy, measured in joules j, is $E = 0.5mv^2$, where m is the mass in kilograms and v is the velocity of the object in meters per second.

a. Solve the above formula for v.

b. If a 1500-kilogram vehicle is generating 1 million joules of kinetic energy, how fast is it traveling?

c. *Escape velocity* is the minimum velocity at which an object must travel to escape the gravitational field of a planet or other object. Suppose a ship that weighs 100,000 kilograms must have a kinetic energy of 3.624×10^{14} joules to escape the gravitational field of Jupiter. Estimate the escape velocity of Jupiter.

40. MP **REASONING** After an accident, police can determine how fast a car was traveling before the driver put on his or her brakes by using the equation $v = \sqrt{30fd}$. In this equation, v represents the speed in miles per hour, f represents the coefficient of friction, and d represents the length of the skid marks in feet. The coefficient of friction varies depending on road conditions. Assume that $f = 0.6$.

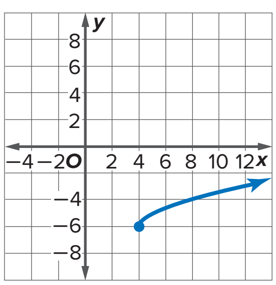
a. Find the speed of a car that skids 25 feet.

b. If your car is going 35 miles per hour, how many feet would it take you to stop?

c. If the speed of a car is doubled, will the skid be twice as long? Explain.

Write the square root function represented by each graph.

41.



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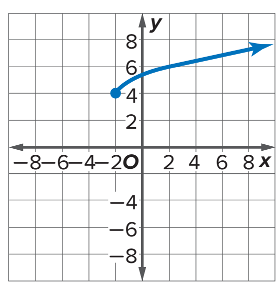

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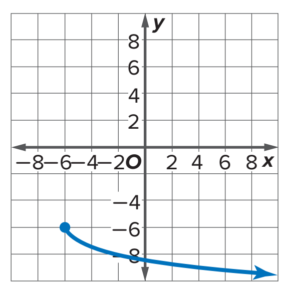


44. **REASONING** In this problem, you will use the following functions to investigate transformations of square root functions.

$f(x) = 4\sqrt{x - 6} + 3$ | $g(x) = \sqrt{16x + 1} - 6$ | $h(x) = \sqrt{x + 3} + 2$

a. **Graphical** Graph each function on the same set of axes.

b. **Analytical** Identify the transformation on the graph of the parent function. What values caused each transformation?

c. **Analytical** Which functions appear to be stretched or compressed vertically? Explain your reasoning.

d. **Verbal** The two functions that are stretched appear to be stretched by the same magnitude. How is this possible?

e. **Tabular** Make a table of the rate of change for all three functions between 8 and 12 as compared to 12 and 16. What generalization about rate of change in square root functions can be made as a result of your findings?

45 **PENDULUMS** The period of a pendulum can be represented by $T = 2\pi\sqrt{\frac{L}{g}}$, where T is the time in seconds, L is the length in feet, and g is gravity, 32 feet per second squared.

a. Graph the function for $0 \le L \le 10$.

b. What is the period for lengths of 2, 5, and 8 feet?

46. **PHYSICS** Using the function $m = \frac{m_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$, Einstein's theory of relativity states that the apparent mass m of a particle depends on its velocity v. An object that is traveling extremely fast, close to the speed of light c, will appear to have more mass compared to its mass at rest, m_0.

a. Use a graphing calculator to graph the function for a ship that weighs 10,000 kilograms for the domain $0 \le v \le 300{,}000{,}000$. Use 300 million meters per second for the speed of light.

b. What viewing window did you use to view the graph?

c. Determine the apparent mass m of the ship for speeds of 100 million, 200 million, and 299 million meters per second.

H.O.T. Problems Use Higher-Order Thinking Skills

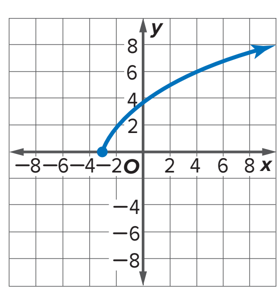
47. **CHALLENGE** Write an equation for a square root function with a domain of $\{x \mid x \ge -4\}$, a range of $\{y \mid y \le 6\}$, and that passes through (5, 3).

48. MP **REASONING** For what positive values of a are the domain and range of $f(x) = \sqrt[a]{x}$ the set of real numbers?

49. **OPEN-ENDED** Write a square root function for which the domain is $\{x \mid x \ge 8\}$ and the range is $\{y \mid y \le 14\}$.

50. **WRITING IN MATH** Explain why there are limitations on the domain and range of functions that have inverses that are functions.

51. **ERROR ANALYSIS** Cleveland thinks that the graph and the equation represent the same function. Molly disagrees. Who is correct? Explain your reasoning.



$y = \sqrt{5x + 10}$

52. **WRITING IN MATH** Explain why $y = \pm\sqrt{x}$ is not a function.

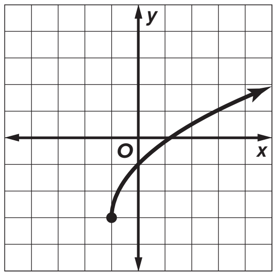
53. **OPEN-ENDED** Write an equation of a relation that contains a radical and its inverse such that

a. the original relation is a function, and its inverse is not a function.

b. the original relation is not a function, and its inverse is a function.

54a. Which function is represented by the graph? MP 1

- A $f(x) = 2\sqrt{x+1} + 3$
- B $f(x) = 2\sqrt{x-1} + 3$
- C $f(x) = 2\sqrt{x+1} - 3$
- D $f(x) = 2\sqrt{x-1} - 3$



b. What are the domain and range of the function shown in the graph? Use set notation. MP 2

____

c. Which of the following statements are true? Select all true solutions. MP 2

- A An x-intercept is 1.
- B The domain is all real numbers.
- C The range is all real numbers.
- D The graph is of the inverse of $f(x) = \frac{(x-1)^2}{4} + 3$.
- E The graph has a y-intercept of -1.

55a. What is the domain of the function? MP 2

$$f(x) = 3\sqrt{x-6} - 2$$

- A $\{x \mid x \geq -6\}$
- B $\{x \mid x \geq -2\}$
- C $\{x \mid x \geq 2\}$
- D $\{x \mid x \geq 6\}$

b. What is the range of the function? Use set notation. MP 1, 6

$f(x) = 3\sqrt{x-6} - 2$

56. What is the range of the function? MP 2

$$f(x) = -4\sqrt{x-5} + 3$$

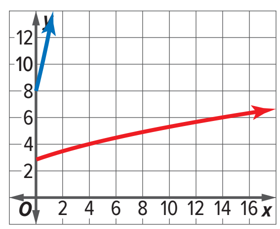
- A $\{f(x) \mid f(x) \leq -12\}$
- B $\{f(x) \mid f(x) \geq -12\}$
- C $\{f(x) \mid f(x) \leq 3\}$
- D $\{f(x) \mid f(x) \geq 3\}$

57. Let $f(x) = \sqrt{x-3} + 1$ and $g(x) = 4f(x)$. How do the domain and range of $f(x)$ and $g(x)$ compare? MP 2

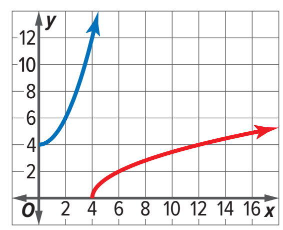
- A They have the same domain and range.
- B They have the same domain, but the graph of $f(x)$ has lower y-values for each value of x than $g(x)$ does.
- C They have the same domain, but the graph of $g(x)$ has lower y-values for each value of x than $f(x)$ does.
- D They have the same range, but the domain of $f(x)$ has lower x-values for each value of y than $g(x)$ does.
- E They have the same range, but the domain of $g(x)$ has lower x-values for each value of y than $f(x)$ does.

58. Which of the following is the graph of $f(x) = \frac{1}{2}x^2 - 4$ and its inverse? MP 1

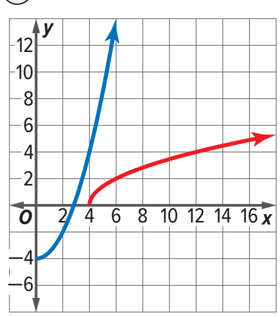
A



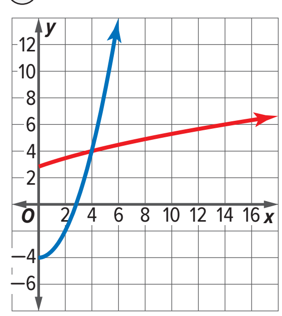
B



C



D



59. For which value(s) of x would $f(x) = \sqrt{2x-4}$ be undefined? Select all solutions. MP 2

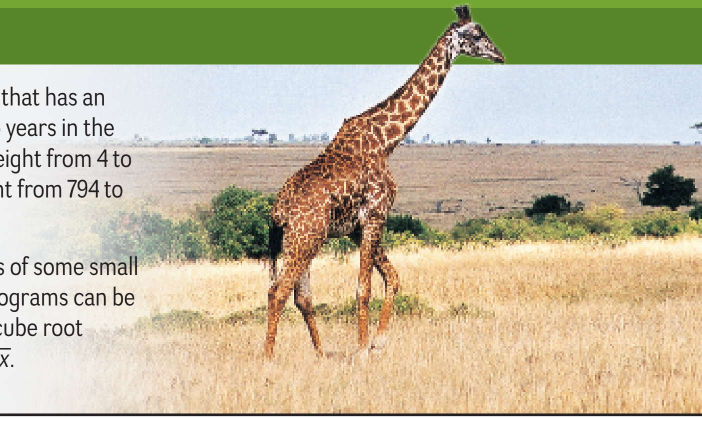
- A $x = -4$
- B $x = 0$
- C $x = 2$
- D $x = 4$

60. Write the equation of a square root function that has been vertically stretched by a factor of 5, translated 3 units to the left, and translated 2 units up from the parent function, $f(x) = \sqrt{x}$. MP 2

____

LESSON 5

Graphing Cube Root Functions



Then

- You graphed square root functions.

Now

1. Graph cube root functions.
2. Analyze cube root functions.

Why?

- A giraffe is a mammal that has an average life span of 25 years in the wild. It can range in height from 4 to 6 meters, and in weight from 794 to 1270 kilograms.

The height h in meters of some small giraffes weighing x kilograms can be approximated by the cube root function $h(x) = 0.45\sqrt[3]{x}$.

New Vocabulary

cube root function

inflection point

Mathematical Practices

1 Make sense of problems.

2 Reason abstractly and quantitatively.

4 Model with mathematics.

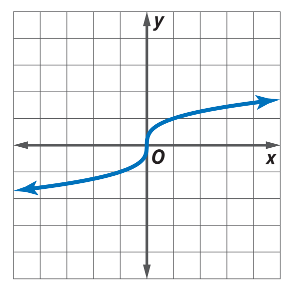
7 Make use of structure.

1 Graph Cube Root Functions

A radical function that contains the cube root of a variable is called a **cube root function.** The domain and range of a cube root function are both all real numbers, and the graph of a cube root function has an **inflection point**, a point on the curve where the curvature changes direction.

Key Concept Parent Function of Cube Root Functions

Parent function:	$f(x) = \sqrt[3]{x}$
Domain:	$\{x \mid -\infty < x < +\infty\}$ or $(-\infty, +\infty)$
Range:	$\{f(x) \mid -\infty < f(x) < +\infty\}$ or $(-\infty, +\infty)$
Intercepts:	$x = 0, y = 0$
Symmetry:	Point symmetry about the origin
End behavior:	$f(x) \to +\infty$ as $x \to +\infty$, $f(x) \to -\infty$ as $x \to -\infty$
Extrema	None
Inflection point	(0, 0)



Example 1 Identify Attributes of Cube Root Functions

Identify the domain and range of $f(x) = -\sqrt[3]{x-2}$. Describe the attributes of the graph.

Domain and Range

Domain: all numbers for which the radicand is defined, all real numbers
$D = \{x \mid -\infty < x < +\infty\}$ or $(-\infty, +\infty)$

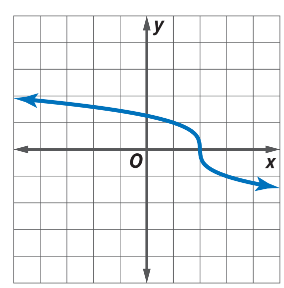
Range: all output values for the function, all real numbers
$R = \{f(x) \mid -\infty < f(x) < +\infty\}$ or $(-\infty, +\infty)$

Attributes

End behavior: The values of y decrease as the values of x increase.

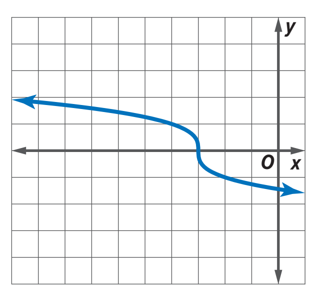
$f(x) \to -\infty$ as $x \to +\infty$
$f(x) \to +\infty$ as $x \to -\infty$

Inflection point: (2, 0)



Guided Practice

1. $f(x) = -\sqrt[3]{x+3}$



The same techniques used to transform the graph of other functions you have studied can be applied to the graphs of cube root functions.

Key Concept Transformations of Cube Root Functions

$f(x) = a\sqrt[3]{x-h} + k$	Inflection Point (h, k)
***h*—Horizontal Translation** *h* units right if *h* is positive $\lvert h \rvert$ units left if *h* is negative The domain is all real numbers.	***k*—Vertical Translation** *k* units up if *k* is positive $\lvert k \rvert$ units down if *k* is negative The range is all real numbers.

***a*—Orientation and Shape**

- If $a < 0$, the graph is reflected across the *x*-axis.
- If $\lvert a \rvert > 1$, the graph is stretched vertically.
- If $0 < \lvert a \rvert < 1$, the graph is compressed vertically.

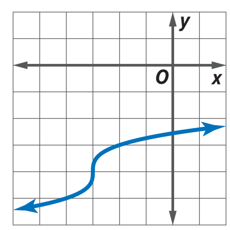
Example 2 Graph Cube Root Functions

Graph each function. State key features of the graph.

a. $y = \sqrt[3]{x+3} - 4$

Make a table of values and graph the function. The graph is the same shape as $f(x) = \sqrt[3]{x}$, but is translated 3 units to the left and 4 units down.

x	*y*
−4	−5
−3	−4
−2	−3
0	−2.6
2	−2.3
5	−2



Domain and Range:

D = $\{x \mid -\infty < x < +\infty\}$ or $(-\infty, +\infty)$

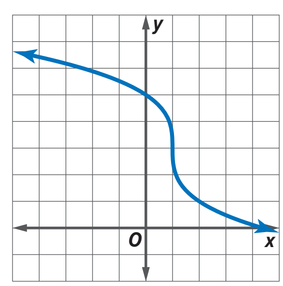
R = $\{f(x) \mid -\infty < f(x) < +\infty\}$ or $(-\infty, +\infty)$

End behavior: The value of *y* increases as the value of *x* increases. Inflection point: $(h, k) = (-3, 4)$

b. $y = -2\sqrt[3]{x-1} + 3$

Make a table of values and graph the function. The graph of $f(x) = \sqrt[3]{x}$ is stretched by a factor of 2, translated 1 unit to the right, 3 units up, and reflected in the line $x = 1$.

x	*y*
−7	7
−1	5.5
0	5
1	3
2	1
3	0.5



Domain and Range:

D = $\{x \mid -\infty < x < +\infty\}$ or $(-\infty, +\infty)$

R = $\{f(x) \mid -\infty < f(x) < +\infty\}$ or $(-\infty, +\infty)$

End behavior: The value of *y* decreases as the value of *x* increases. $(h, k) = (1, 3)$

Guided Practice

2A. $y = \sqrt[3]{x-3} + 2$

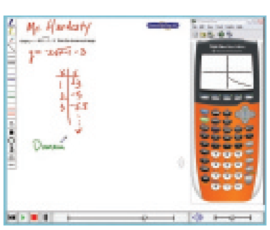
2B. $y = -3\sqrt[3]{x+1} - 2$

Teaching Tip

MP Reason Abstractly Encourage students to compare the transformations of a cube root function with transformations of a square root function. Have them explain to each other the similarities and differences.

Go Online!

Follow along with your graphing calculator as you watch a **Personal Tutor** graph a square root function.



2 Analyze Cube Root Functions

Previously you learned that an inverse relation interchanges the *x*- and *y*-coordinates of the original relation. For the power function $f(x) = x^3$, the inverse of *f* is the function $f^{-1}(x) = \sqrt[3]{x}$ for all real numbers.

Real-World Link

As the world's tallest mammals, giraffes use their height for reaching the leaves of tall trees, and their long legs to help them run as fast as 35 miles per hour for short distances. For longer distances, they run 10 miles per hour.

Source: http://animals.nationalgeographic.c om/animals/mammals/giraffe/

Real-World Example 3 Use Graphs to Analyze Cube Root Functions

NATURE Refer to the application at the beginning of the lesson. A zookeeper found that the height of some small giraffes weighing x kilograms can be modeled by the cube root function $h(x) = 0.45\sqrt[3]{x}$, where h is the height in meters.

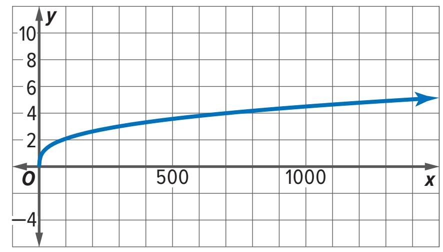
a. Make a table and graph the function in the domain $\{x \mid 0 \leq x \leq 1500\}$.

Round output values to the nearest tenth.

x	h
0	0
200	2.6
300	3.0
400	3.3
500	3.6

x	h
600	3.8
700	4.0
800	4.2
900	4.3
1000	4.5

x	h
1100	4.6
1200	4.8
1300	4.9
1400	5.0
1500	5.2



b. Analyze the graph. How should the domain be restricted?

The domain should be restricted to be between about 700 to 1300 kilograms, because that is the approximate weight range for giraffes.

c. What are key features of the graph?

The graph is in the first quadrant only. The values of y increase as the values of x increase.

d. What is the approximate height of a giraffe that weighs 850 kilograms?

According to the graph and the table, the height is about 4.26 meters.

Problem-Solving Tip

MP Modeling Making a table is a good way to organize ordered pairs in order to see the general behavior of a graph.

Guided Practice

3. The height of some larger giraffes weighing x kilograms is better modeled by the cube root function $h(x) = 0.55\sqrt[3]{x}$, where h is the height in meters. Graph the function in the domain $\{x \mid 0 \leq x \leq 1300\}$ and analyze the graph. Then determine the approximate height of a giraffe that weighs 1150 kilograms.

Example 4 Find the Inverse of Power Function $f(x) = x^3$

Find the inverse of $f(x) = x^3$. Then graph $f(x)$ and $f^{-1}(x)$ on the same coordinate plane.

$f(x) = x^3$ Write the original function.

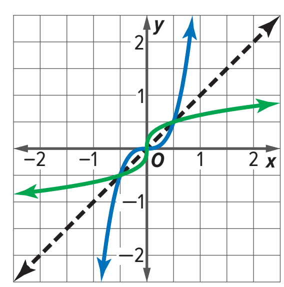
$y = x^3$ Replace $f(x)$ with y.

$x = x^3$ Interchange x and y.

$\sqrt[3]{x} = y$ Take the cube root of both sides.

Because the domain of f is all real numbers, the range of f^{-1} is also all real numbers. So, the inverse of f is $f^{-1}(x) = \sqrt[3]{x}$ with domain and range all real numbers.

The graph of $f^{-1}(x) = \sqrt[3]{x}$ is a reflection of the graph of $f(x) = x^3$ in the line $y = x$, shown as a dashed line on the graph.



Guided Practice

4. Find the inverse of $f(x) = x^3 - 1$. Then graph $f(x)$ and $f^{-1}(x)$ on the same coordinate plane.

Check Your Understanding

= Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Example 1 **Identify the domain and range of each function. Describe the attributes of the graph.**

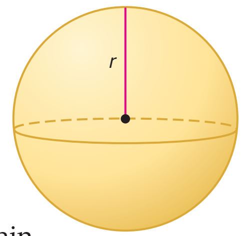
1. $f(x) = \sqrt[3]{x - 4}$
2. $f(x) = \sqrt[3]{x + 2} - 3$
3. $f(x) = 2\sqrt[3]{x} - 6$

Example 2 **Graph each function. State key features of the graph.**

4. $f(x) = \sqrt[3]{x} + 2$
5. $f(x) = 2\sqrt[3]{x - 1}$
6. $f(x) = \frac{1}{2}\sqrt[3]{x + 2} - 3$
7. $f(x) = -3\sqrt[3]{x - 4} + 4$

Example 3

8. **GEOMETRY** The radius r of a sphere with volume V can be found using the formula $r = \sqrt[3]{\frac{3V}{4\pi}}$.
   a. Graph the function.
   b. Use the graph to determine the approximate radius for volumes of 1000 cubic meters, 8000 cubic meters, and 64,000 cubic meters.
   c. How does the volume of the sphere change if the radius is doubled? Explain.



Example 4 **Find the inverse of the function. Then graph the function and its inverse on the same coordinate plane.**

9. $f(x) = 2x^3$
10. $f(x) = x^3 + 1$
11. $f(x) = -4x^3$
12. $f(x) = \frac{1}{2}x^3$

Practice and Problem Solving

Extra Practice is on page R5.

Example 1 **Identify the domain and range of each function. Describe the attributes of the graph.**

13. $f(x) = 2\sqrt[3]{x + 4}$
14. $f(x) = -\frac{1}{4}\sqrt[3]{x} + 1$
15. $f(x) = \frac{1}{3}\sqrt[3]{x - 2} + 1$

Example 2 **Graph each function. State key features of the graph.**

16. $f(x) = \sqrt[3]{x} - 3$
17. $f(x) = 3\sqrt[3]{x + 1}$
18. $f(x) = \frac{1}{4}\sqrt[3]{x - 1} - 4$
19. $f(x) = -\sqrt[3]{x + 1} - 2$
20. $f(x) = 2\sqrt[3]{x} + 3$
21. $f(x) = 3\sqrt[3]{x + 2} - 4$
22. $f(x) = \frac{1}{2}\sqrt[3]{x - 3} + 1$
23. $f(x) = -2\sqrt[3]{x - 4} - 2$
24. $f(x) = \frac{1}{3}\sqrt[3]{x} - 1$
25. $f(x) = \frac{1}{2}\sqrt[3]{x - 2} + 2$

26. MP **PERSERVERANCE** The radius r of the orbit of a communications satellite is given by $r = \sqrt[3]{\frac{GMt^2}{4\pi^2}}$, where G is the universal gravitational constant, M is the mass of Earth, and t is the time it takes the satellite to complete one orbit. Find the radius of the satellite's orbit if G is $6.67 \times 10^{-11}\ N \cdot m^2/kg^2$, M is 5.98×10^{24} kilograms, and t is 2.6×10^6 seconds.

27. **MODELING** A fruit grower has found that the circumference C, in inches, of most of his fruit crop can be modeled by the cube root equation $C = \sqrt[3]{\frac{W}{0.009}}$, where W is the weight in ounces.
   a. Graph the function.
   b. What is the approximate circumference of a piece of fruit that weighs 8.5 ounces?
   c. What is the approximate weight of a piece of fruit with a circumference of 8 inches?
   d. Explain why it is difficult to give a function that will work on all fruit.

28. **PHYSICS** Johannes Kepler developed the formula $d = \sqrt[3]{6t^2}$, where d is the distance of a planet to the Sun in millions of miles and t is the number of Earth days that it takes for the planet to orbit the Sun.

a. Use a graphing calculator to graph the function. Explain why the graph of the function $f(t) = \sqrt[3]{6t^2}$ is not the graph of a standard cube root function.

b. If the length of a year on Venus is 224.7 Earth days, how far is the Sun from Venus?

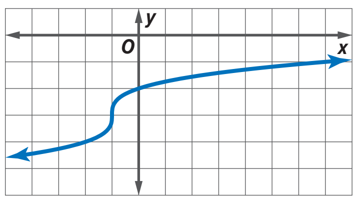
29. **MODELING** The surface area S of a sphere can be determined from the volume of the sphere using the formula $S = \sqrt[3]{36\pi V^2}$, where V is the volume.

a. Use a graphing calculator to graph the function. Explain why the graph of $S = \sqrt[3]{36\pi V^2}$ is not the graph of a standard cube root function.

b. Determine the surface area of a sphere with a volume of 200 cubic inches.

Write the cube root function represented by each graph. Points with integer coordinates are shown, with one point the inflection point.

30.



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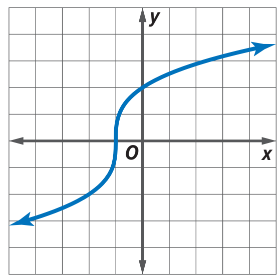

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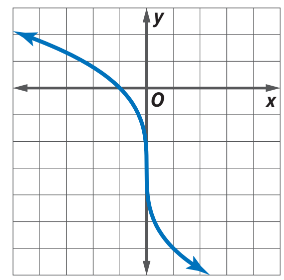


33. **GEOMETRY** The side length of a cube is determined by $r = \sqrt[3]{V}$, where V is the volume in cubic units.

a. Graph the function.

b. What is the domain and range of the function? Justify your reasoning.

c. What is the side length of a cube with a volume of 512 cubic centimeters?

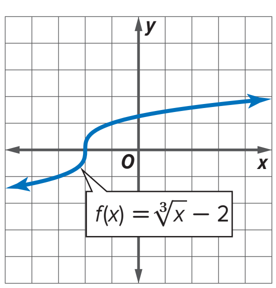
H.O.T. Problems Use Higher-Order Thinking Skills

34. **REASONING** Explain how to find an equation for a cube root function that passes through the points $(-8, -1)$ if the inflection point is $(0, 0)$.

35. **REASONING** Is the function $f(x) = \sqrt[3]{x}$ *odd*, *even*, or *neither*? Explain.

36. **WRITING IN MATH** Explain why there are no limitations on the domain and range of a cube root function.

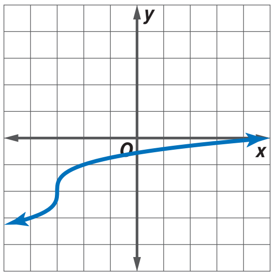
37. **ERROR ANALYSIS** Lin thinks that the graph and the equation $g(x) = \sqrt[3]{x}$, represent the same function. Milo disagrees. Who is correct? Explain your reasoning.



38. **CHALLENGE** Use a graphing calculator to explore the graphs of $f(x) = \sqrt{x}$, $g(x) = \sqrt[3]{x}$, $h(x) = \sqrt[4]{x}$, and $j(x) = \sqrt[5]{x}$. Describe the graph of $s(x) = \sqrt[n]{x}$, if n is even and if n is odd?

Preparing for Assessment

39. Use the graph to answer the following. MP 1



a. Which function is represented by the graph? MP 1

- ◯ **A** $y = \sqrt[3]{x - 3} - 2$
- ◯ **B** $y = \sqrt[3]{x - 3} + 2$
- ◯ **C** $y = \sqrt[3]{x + 3} - 2$
- ◯ **D** $y = \sqrt[3]{x + 2} - 3$

b. What are the domain and range of the function shown in the graph? Use set notation. MP 2

[ ]

c. Which statements are true for the function in the graph? Select all solutions. MP 2

- ☐ **A** An x-intercept is 5.
- ☐ **B** The domain is all real numbers.
- ☐ **C** A point of inflection is $(-3, -2)$.
- ☐ **D** The graph is the inverse of
- ☐ **E** The graph has a y-intercept of -1.

40. Which of the following will have a graph that is stretched by a factor of 2, translated 4 units right and 1 unit down? MP 2

- ◯ **A** $y = 2\sqrt[3]{x + 4} - 1$
- ◯ **B** $y = 2\sqrt[3]{x - 4} - 1$
- ◯ **C** $y = \sqrt[3]{2x - 4} - 1$
- ◯ **D** $y = 2\sqrt[3]{x - 1} + 4$

41. The formula $d = \sqrt[3]{6t^2}$ represents d, the distance in millions of miles a planet is from the Sun, in terms of t, the number of Earth days it takes for the planet to orbit the Sun. It takes Mercury 88 Earth days to complete one orbit. To the nearest hundredth, how many millions of miles is Mercury from the Sun? MP 2

[ ]

42. Write the equation of the inverse of $f(x) = \frac{x^3}{3} + 6$. MP 1

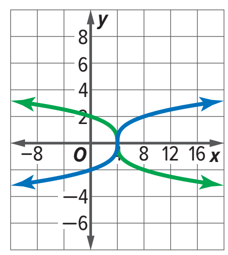
[ ]

43. Let $f(x) = \sqrt[3]{x}$ and $g(x) = 3\sqrt[3]{x + 2} - 5$. How do the domain and range of $f(x)$ and $g(x)$ compare? MP 2

- ◯ **A** They have the same domain and range.
- ◯ **B** They have the same domain, but the graph of $f(x)$ has higher y-values for each value of x.
- ◯ **C** They have the same domain, but the graph of $g(x)$ has higher y-values for each value of x.
- ◯ **D** They have the same range, but the domain of $f(x)$ has lower x-values for each value of y.

44. Which of the following is the graph of $f(x) = \frac{1}{2}x^3 + 4$ and its inverse? MP 1

A



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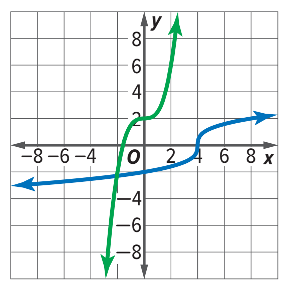
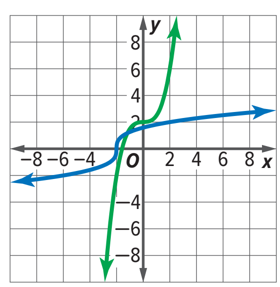
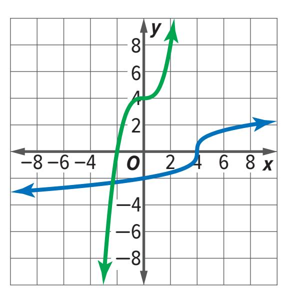


C



D



EXTEND 5

Graphing Technology Lab

Graphing *n*th Root Functions

In general, the graph of a cube root function is an s-shaped curve. The graphs are not symmetric with the *x*- or *y*-axis or the line $y = -x$ there are no asymptotes. The parent function $(x) = \sqrt[3]{x}$ can be transformed using by $af(x)$, $f(x) + d$, $f(bx)$, and $f(x - c)$ for positive and negative values of *a*, *b*, *c*, and *d*.

Mathematical Practices

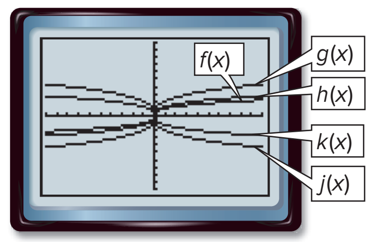
MP **4** Model with Mathematics

Activity 1 $af(x)$, $-af(x)$, $f(bx)$, and $f(-bx)$

Work cooperatively. Graph the set of functions on the same screen. Describe any similarities and differences among the graphs.

$f(x) = \sqrt[3]{x}$, $g(x) = 2\sqrt[3]{x}$, $h(x) = \sqrt[3]{2x}$,

$j(x) = -2\sqrt[3]{x}$, $k(x) = \sqrt[3]{-2x}$



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The graphs have the same general shape. The functions $f(x)$, $g(x)$, and $h(x)$ lie in the first and third quadrants. The graphs $j(x)$ and $k(x)$ are reflections and lie in the second and fourth quadrants. The graphs of $g(x)$ and $j(x)$ are more vertically stretched than $f(x)$. The graphs of $h(x)$ and $k(x)$ are more horizontally compressed than $f(x)$ but less than $g(x)$ and $j(x)$.

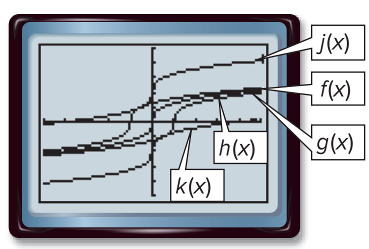
Some transformations cause the graph of a function to change location but do not change the shape from the parent function.

Activity 2 $f(x - c)$, $f(x + c)$, $f(x) + d$, and $f(x) - d$

Work cooperatively. Graph the set of functions on the same screen. Describe any similarities and differences among the graphs.

$f(x) = \sqrt[3]{x}$, $g(x) = \sqrt[3]{x + 2}$, $h(x) = \sqrt[3]{x - 2}$,

$j(x) = \sqrt[3]{x} + 2$, $k(x) = \sqrt[3]{x} - 2$



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The graphs have the same shape, but have been translated. The graph of $g(x)$ is shifted 2 units left and $h(x)$ is shifted 2 units right while the graph of $j(x)$ is shifted 2 units up and $k(x)$ shifted 2 units down.

Exercises

Work cooperatively. Examine each set of functions and predict the similarities and differences in their graphs. Use a graphing calculator to confirm your prediction. Write a sentence or two comparing the graphs.

1. $f(x) = \sqrt[3]{x}$, $g(x) = 3\sqrt[3]{x}$, $h(x) = -3\sqrt[3]{x}$
2. $f(x) = \sqrt[3]{x}$, $g(x) = \sqrt[3]{x} + 1$, $h(x) = \sqrt[3]{x} - 1$
3. $f(x) = \sqrt[3]{x}$, $g(x) = \sqrt[3]{8x}$, $h(x) = \sqrt[3]{-8x}$
4. $f(x) = \sqrt[3]{x}$, $g(x) = \sqrt[3]{x - 8}$, $h(x) = \sqrt[3]{x + 8}$

LESSON 6

Solving Radical Equations

Then

- You solved polynomial equations.

Now

1 Solve equations containing radicals.

2 Solve inequalities containing radicals.

Why?

- When you jump, the time that you are in the air is your hang time. Hang time can be calculated in seconds t if you know the height h of the jump in feet. The formula for hang time is $t = 0.5\sqrt{h}$.

  A volleyball player had a hang time of about 0.67 second. How would you calculate the height of her jump?

Christopher Futcher/iStockphoto.com

New Vocabulary

radical equation
extraneous solution
radical inequality

Mathematical Practices

4 Model with mathematics.

1 Solve Radical Equations

Radical equations include radical expressions. You can solve a radical equation by raising each side of the equation to a power.

Key Concept Solving Radical Equations

Step 1 Isolate the radical on one side of the equation.

Step 2 Raise each side of the equation to a power equal to the index of the radical to eliminate the radical.

Step 3 Solve the resulting polynomial equation. Check your results.

When solving radical equations, the result may be a number that does not satisfy the original equation. Such a number is called an **extraneous solution**.

Example 1 Solve Radical Equations

Solve each equation.

a. $\sqrt{x+2} + 4 = 7$

$\sqrt{x+2} + 4 = 7$	Original equation
$\sqrt{x+2} = 3$	Subtract 4 from each side to isolate the radical.
$\left(\sqrt{x+2}\right)^2 = 3^2$	Square each side to eliminate the radical.
$x + 2 = 9$	Find the squares.
$x = 7$	Subtract 2 from each side.

CHECK

$\sqrt{x+2} + 4 = 7$	Original equation
$\sqrt{7+2} + 4 \stackrel{?}{=} 7$	Replace x with 7.
$7 = 7$ ✓	Simplify.

b. $\sqrt{x-12} = 2 - \sqrt{x}$

$\sqrt{x-12} = 2 - \sqrt{x}$	Original equation
$\left(\sqrt{x-12}\right)^2 = \left(2 - \sqrt{x}\right)^2$	Square each side.
$x - 12 = 4 - 4\sqrt{x} + x$	Find the squares.
$-16 = -4\sqrt{x}$	Isolate the radical.
$4 = \sqrt{x}$	Divide each side by -4.
$16 = x$	Square each side.

Study Tip

MP Tools You can use a graphing calculator to check solutions of equations. Graph each side of the original equation, and examine the intersection.

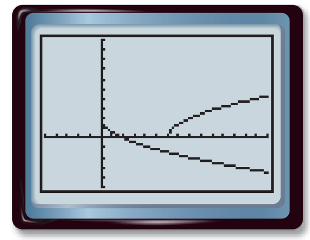
CHECK $\sqrt{x-12} = 2 - \sqrt{x}$

$\sqrt{16-12} \stackrel{?}{=} 2 - \sqrt{16}$

$\sqrt{4} \stackrel{?}{=} 2 - 4$

$2 \neq -2$ ✗

The solution does not check, so the equation has an extraneous solution. The graphs of $y = \sqrt{x-12}$ and $y = 2 - \sqrt{x}$ do not intersect, which confirms that there is no real solution.



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Guided Practice

1A. $5 = \sqrt{x-2} - 1$

1B. $\sqrt{x+15} = 5 + \sqrt{x}$

To undo a square root, you square the expression. To undo a cube root, you must raise the expression to the third power.

Example 2 Solve a Cube Root Equation

Solve $2(6x-3)^{\frac{1}{3}} - 4 = 0$.

In order to remove the $\frac{1}{3}$ power, or cube root, you must first isolate it and then raise each side of the equation to the third power.

$2(6x-3)^{\frac{1}{3}} - 4 = 0$ Original equation

$2(6x-3)^{\frac{1}{3}} = 4$ Add 4 to each side.

$(6x-3)^{\frac{1}{3}} = 2$ Divide each side by 2.

$\left[(6x-3)^{\frac{1}{3}}\right]^3 = 2^3$ Cube each side.

$6x - 3 = 8$ Evaluate the cubes.

$6x = 11$ Add 3 to each side.

$x = \frac{11}{6}$ Divide each side by 6.

CHECK $2(6x-3)^{\frac{1}{3}} - 4 = 0$ Original equation

$2\left(6 \cdot \frac{11}{6} - 3\right)^{\frac{1}{3}} - 4 \stackrel{?}{=} 0$ Replace x with $\frac{11}{6}$.

$2(8)^{\frac{1}{3}} - 4 \stackrel{?}{=} 0$ Simplify.

$2(2) - 4 \stackrel{?}{=} 0$ The cube root of 8 is 2.

$0 = 0$ ✓ Subtract.

Guided Practice

Solve each equation.

2A. $(3n+2)^{\frac{1}{3}} + 1 = 0$

2B. $3(5y-1)^{\frac{1}{3}} - 2 = 0$

You can apply the methods used to solve square and cube root equations to solving equations with roots of any index. To undo an nth root, raise to the nth power.

Study Tip

Substitute Values You could also solve the multiple-choice question by substituting each answer for n in the equation to see if the solution is correct.

Example 3 Solve a Radical Equation

What value of n is a solution to $3(\sqrt[4]{2n+6}) - 6 = 0$?

A -1 **B** 1 **C** 5 **D** 11

$3(\sqrt[4]{2n+6}) - 6 = 0$	Original equation
$3(\sqrt[4]{2n+6}) = 6$	Add 6 to each side.
$\sqrt[4]{2n+6} = 2$	Divide each side by 3.
$(\sqrt[4]{2n+6})^4 = 2^4$	Raise each side to the fourth power.
$2n + 6 = 16$	Evaluate each side.
$2n = 10$	Subtract 6 from each side.
$n = 5$	The answer is C.

Guided Practice

3. What value of x is a solution of $4(3x+6)^{\frac{1}{4}} - 12 = 0$?

A $x = 7$ **B** $x = 25$ **C** $x = 29$ **D** $x = 37$

2 Solve Radical Inequalities

A **radical inequality** has a variable in the radicand. To solve radical inequalities, complete the following steps.

Key Concept Solving Radical Inequalities

Step 1 If the index of the root is even, identify the values of the variable for which the radicand is nonnegative.

Step 2 Solve the inequality algebraically.

Step 3 Test values to check your solution.

Study Tip

Radical Inequalities Because a principal square root is never negative, inequalities that simplify to the form $\sqrt{ax+b} \le c$, where c is a negative number, have no solutions.

Example 4 Solve Radical Equations

Solve $3 + \sqrt{5x-10} \le 8$.

Step 1 Because the radicand of a square root must be greater than or equal to zero, first solve $5x - 10 \ge 0$ to identify the values of x for which the left side of the inequality is defined.

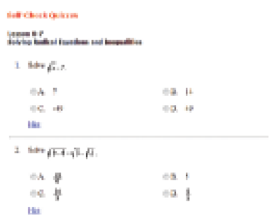
$5x - 10 \ge 0$	Set the radicand ≥ 0.
$5x \ge 10$	Add 10 to each side.
$x \ge 2$	Divide each side by 5.

Step 2 Solve $3 + \sqrt{5x-10} \le 8$.

$3 + \sqrt{5x-10} \le 8$	Original inequality
$\sqrt{5x-10} \le 5$	Isolate the radical.
$5x - 10 \le 25$	Eliminate the radical.
$5x \le 35$	Add 10 to each side.
$x \le 7$	Divide each side by 5.

Go Online!

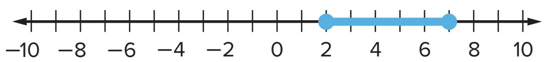
Check your mastery of solving radical inequalities by using the **Self-Check Quiz** in ConnectED.



Step 3 It appears that $2 \le x \le 7$. You can test some x-values to confirm the solution. Use three test values: one less than 2, one between 2 and 7, and one greater than 7. Organize the test values in a table.

$x = 0$	$x = 4$	$x = 9$
$3 + \sqrt{5(0) - 10} \overset{?}{\le} 8$ $3 + \sqrt{-10} \le 8$ ✗ Because $\sqrt{-10}$ is not a real number, the inequality is not satisfied.	$3 + \sqrt{5(4) - 10} \overset{?}{\le} 8$ $6.16 \le 8$ ✓ Because $6.16 \le 8$, the inequality is satisfied.	$3 + \sqrt{5(9) - 10} \overset{?}{\le} 8$ $8.92 \le 8$ ✗ Because $8.92 \not\le 8$, the inequality is not satisfied.

The solution checks. Only values in the interval $2 \le x \le 7$ satisfy the inequality. You can summarize the solution with a number line.



Guided Practice

Solve each inequality.

4A. $\sqrt{2x + 2} + 1 \ge 5$

4B. $\sqrt{4x - 4} - 2 < 4$

Check Your Understanding

● = Step-by-Step Solutions begin on page R14.

Go Online! for a Self-Check Quiz

Examples 1–2 **Solve each equation.**

1. $\sqrt{x - 4} + 6 = 10$
2. $\sqrt{x + 13} - 8 = -2$
3. $8 - \sqrt{x + 12} = 3$
4. $\sqrt{x - 8} + 5 = 7$
5. $\sqrt[3]{x - 2} = 3$
6. $(x - 5)^{\frac{1}{3}} - 4 = -2$
7. $(4y)^{\frac{1}{3}} + 3 = 5$
8. $\sqrt[3]{n + 8} - 6 = -3$
9. $\sqrt{y} - 7 = 0$
10. $2 + 4z^{\frac{1}{2}} = 0$
11. $5 + \sqrt{4y - 5} = 12$
12. $\sqrt{2t - 7} = \sqrt{t + 2}$

13. **MP REASONING** The time T in seconds that it takes a pendulum to make a complete swing back and forth is given by the formula $T = 2\pi\sqrt{\frac{L}{g}}$, where L is the length of the pendulum in feet and g is the acceleration due to gravity, 32 feet per second squared.
    a. In Tokyo, Japan, a huge pendulum in the Shinjuku building measures 73 feet 9.75 inches. How long does it take for the pendulum to make a complete swing?
    b. A clockmaker wants to build a pendulum that takes 20 seconds to swing back and forth. How long should the pendulum be?

Example 3

14. **MULTIPLE CHOICE** Solve $(2y + 6)^{\frac{1}{4}} - 2 = 0$.
    A $y = 1$ **B** $y = 5$ **C** $y = 11$ **D** $y = 15$

Example 4 **Solve each inequality.**

15. $\sqrt{3x + 4} - 5 \le 4$
16. $\sqrt{b - 7} + 6 \le 12$
17. $2 + \sqrt{4y - 4} \le 6$
18. $\sqrt{3a + 3} - 1 \le 2$
19. $1 + \sqrt{7x - 3} > 3$
20. $\sqrt{3x + 6} + 2 \le 5$
21. $-2 + \sqrt{9 - 5x} \ge 6$
22. $6 - \sqrt{2y + 1} < 3$

Practice and Problem Solving

Extra Practice is on page R5.

Example 1 **Solve each equation. Confirm by using a graphing calculator.**

23. $\sqrt{2x+5}-4=3$

24. $6+\sqrt{3x+1}=11$

25. $\sqrt{x+6}=5-\sqrt{x+1}$

26. $\sqrt{x-3}=\sqrt{x+4}-1$

27. $\sqrt{x-15}=3-\sqrt{x}$

28. $\sqrt{x-10}=1-\sqrt{x}$

29. $6+\sqrt{4x+8}=9$

30. $2+\sqrt{3y-5}=10$

31. $\sqrt{x-4}=\sqrt{2x-13}$

32. $\sqrt{7a-2}=\sqrt{a+3}$

33. $\sqrt{x-5}-\sqrt{x}=-2$

34. $\sqrt{b-6}+\sqrt{b}=3$

35. MP **SENSE-MAKING** Isabel accidentally dropped her keys from the top of a Ferris wheel. The formula $t=\frac{1}{4}\sqrt{d-h}$ describes the time t in seconds at which the keys are h meters above the ground and Isabel is d meters above the ground. If Isabel was 65 meters high when she dropped the keys, how many meters above the ground will the keys be after 2 seconds?

Example 2 **Solve each equation.**

36. $(5n-6)^{\frac{1}{3}}+3=4$

37. $(5p-7)^{\frac{1}{3}}+3=5$

38. $(6q+1)^{\frac{1}{4}}+2=5$

39. $(3x+7)^{\frac{1}{4}}-3=1$

40. $(3y-2)^{\frac{1}{5}}+5=6$

41. $(4z-1)^{\frac{1}{5}}-1=2$

42. $2(x-10)^{\frac{1}{3}}+4=0$

43. $3(x+5)^{\frac{1}{3}}-6=0$

44. $\sqrt[3]{5x+10}-5=0$

45. $\sqrt[3]{4n-8}-4=0$

46. $\frac{1}{7}(14a)^{\frac{1}{3}}=1$

47. $\frac{1}{4}(32b)^{\frac{1}{3}}=1$

Example 3

48. **MULTIPLE CHOICE** Solve $\sqrt[4]{y+2}+9=14$.

A 23 **B** 53 **C** 123 **D** 623

49. **MULTIPLE CHOICE** Solve $(2x-1)^{\frac{1}{4}}-2=1$.

F 41 **G** 28 **H** 13 **J** 1

Example 4 **Solve each inequality.**

50. $1+\sqrt{5x-2}>4$

51. $\sqrt{2x+14}-6\geq 4$

52. $10-\sqrt{2x+7}\leq 3$

53. $6+\sqrt{3y+4}<6$

54. $\sqrt{2x+5}-\sqrt{9+x}>0$

55. $\sqrt{d+3}+\sqrt{d+7}>4$

56. $\sqrt{3x+9}-2<7$

57. $\sqrt{2y+5}+3\leq 6$

58. $-2+\sqrt{8-4z}\geq 8$

59. $-3+\sqrt{6a+1}>4$

60. $\sqrt{2}-\sqrt{b+6}\leq -\sqrt{b}$

61. $\sqrt{c+9}-\sqrt{c}>\sqrt{3}$

62. **PENDULUMS** The formula $s=2\pi\sqrt{\frac{\ell}{32}}$ represents the swing of a pendulum, where s is the time in seconds to swing back and forth and ℓ is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 1.5 seconds.

63. **TURTLES** The relationship between the length and weight of certain turtles can be approximated by the equation $L=0.55\sqrt[3]{W}$, where L is the length in feet and W is the weight in pounds. Solve this equation for W.

64. **HANG TIME** Refer to the information at the beginning of the lesson regarding hang time. Describe how the height of a jump is related to the amount of time in the air. Write a step-by-step explanation of how to determine the height of the volleyball player's 0.98-second jump.

65. **CONCERTS** The organizers of a concert are preparing for the arrival of 50,000 people in the open field where the concert will take place. Each person is allotted 5 square feet of space, so the organizers rope off a circular area of 250,000 square feet. Using the formula $A = \pi r^2$, where A represents the area of the circular region and r represents the radius of the region, find the radius of this region.

66. **WEIGHTLIFTING** The formula $M = 512 - 146{,}230B^{-\frac{8}{5}}$ can be used to estimate the maximum total mass that a weightlifter of mass B kilograms can lift using the snatch and the clean and jerk. According to the formula, how much does a person weigh who can lift at most 470 kilograms?

H.O.T. Problems Use Higher-Order Thinking Skills

67. **ERROR ANALYSIS** Which equation does not have a solution?

$\sqrt{x-1} + 3 = 4$	$\sqrt{x+1} + 3 = 4$
$\sqrt{x-2} + 7 = 10$	$\sqrt{x+2} - 7 = -10$

68. **CHALLENGE** Lola is working to solve $(x + 5)^{\frac{1}{4}} = -4$. She said that she could tell there was no real solution without even working the problem. Is Lola correct? Explain your reasoning.

69. **REASONING** Determine whether $\frac{\sqrt{(x^2)^2}}{-x} = x$ is *sometimes*, *always*, or *never* true when x is a real number. Explain your reasoning.

70. **OPEN-ENDED** Select a whole number. Now work backward to write two radical equations that have that whole number as solutions. Write one square root equation and one cube root equation. You may need to experiment until you find a whole number you can easily use.

71. **WRITING IN MATH** Explain the relationship between the index of the root of a variable in an equation and the power to which you raise each side of the equation to solve the equation.

72. **OPEN-ENDED** Write an equation that can be solved by raising each side of the equation to the given power.

a. $\frac{3}{2}$ power **b.** $\frac{5}{4}$ power **c.** $\frac{7}{8}$ power

73. **CHALLENGE** Solve $7^{3x-1} = 49^{x+1}$ for x. (*Hint:* $b^x = b^y$ if and only if $x = y$.)

REASONING Determine whether the following statements are *sometimes*, *always*, or *never* true for $x^{\frac{1}{n}} = a$. Explain your reasoning.

74. If n is odd, there will be extraneous solutions.

75. If n is even, there will be extraneous solutions.

Preparing for Assessment

76. For what value of x is $\sqrt{x-8} = 1 - \sqrt{x}$? MP 1

- A 1.5
- B 4.5
- C 20.25
- D no real solution

77. The equation $v = \sqrt{64h}$ approximates the velocity v of a roller coaster car, in feet per second, at the bottom of a hill h feet high. If the car travels at a velocity of 55 feet per second at the bottom of a hill, how high is the hill? Round to the nearest tenth of a foot. MP 1

78. If $(2y+3)^{\frac{1}{3}} + 2 = 5$, what is the value of y? MP 6

- A 6
- B 12
- C 30
- D 170

79. If $\sqrt{p-1} = \frac{2}{w} + 3$, what is the value of p in terms of w? MP 5

- A $p = \frac{4}{w^2} + 16$
- B $p = \frac{4}{w^2} + 10$
- C $p = \frac{4}{w^2} + \frac{6}{w} + 10$
- D $p = \frac{4}{w^2} + \frac{12}{w} + 10$
- E $p = \frac{4}{w^2} + \frac{12}{w} + 16$

80. Solve for x. MP 6

$3 + \sqrt{3x+2} = 9$

- A $\frac{68}{5}$
- B $\frac{34}{3}$
- C $\frac{22}{9}$
- D 12

81. Solve for x. MP 6

$\sqrt{9x+18} \geq x - 2$

82. **MULTI-PART** Consider the following equation.
$\sqrt{x-4} + 1 = 5$ MP 1 3, 6,

a. Which lines of the work shown correctly belong in the solution of the equation above? Choose all that apply.

- A $\sqrt{x-4} = 4$
- B $\left(\sqrt{x-4}\right)^2 + 1^2 = 4$
- C $\left(\sqrt{x-4}\right)^2 = 4$
- D $x - 4 = 16$
- E $\left(\sqrt{x-4}\right)^2 = 24$
- F $x^2 = 20$

b. What is the solution to $\sqrt{x-4} + 1 = 5$?

- A $\sqrt{20}$
- B $\sqrt{20}$
- C 4
- D 20

c. How can you check the answer?

d. Without doing the arithmetic, rearrange the equation to show the solution. (Write the equation as $x = \ldots$ with an arrangement of 4, 1, and 5 on the other side.)

83. a. Why does the solution to $11 + \sqrt{11x - 123} \leq 14$ have two parts? MP 1

b. Solve for both parts of the solution of $11 + \sqrt{11x - 123} \leq 14$.

c. Does the inequality have a real solution?

84. Find the height of a right triangle whose area is 27 square centimeters with a base of 3 cm and a height of $\sqrt{3x-5}$ centimeters. MP 1, 6

- A $\frac{329}{3}$
- B $\frac{34}{3}$
- C 18
- D 21

EXTEND 6

Graphing Technology Lab

Solving Radical Equations

You can use a TI-83/84 Plus graphing calculator to solve radical equations and inequalities. One way to do this is to rewrite the equation or inequality so that one side is 0. Then use the zero feature on the calculator.

Mathematical Practices

MP **4** Model with Mathematics

Example 1 Radical Equation

Solve $\sqrt{x} + \sqrt{x + 2} = 3$.

Step 1 Rewrite the equation.

- Subtract 3 from each side of the equation to get $\sqrt{x} + \sqrt{x + 2} - 3 = 0$.
- Enter the function $y = \sqrt{x} + \sqrt{x + 2} - 3$ in the **Y=** list.

KEYSTROKES: [Y=] [2nd] [√‾] [X,T,θ,n] [ )] [+] [2nd] [√‾] [X,T,θ,n] [+] 2 [ )] [−] 3 [ENTER]

Step 2 Use a table.

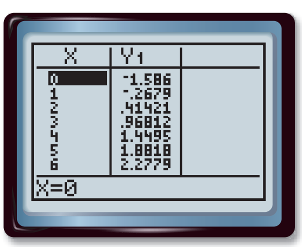
- You can use the **TABLE** function to locate intervals where the solution(s) lie. First, enter the starting value and the interval for the table.

KEYSTROKES: [2nd] [TBLSET] 0 [ENTER] 1 [ENTER]

Step 3 Estimate the solution.

- Complete the table and estimate the solution(s).

KEYSTROKES: [2nd] [TABLE]

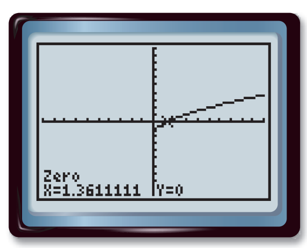


Because the function changes sign from negative to positive between $x = 1$ and $x = 2$, there is a solution between 1 and 2.

Step 4 Use the **ZERO** feature.

- Graph the function in the standard viewing window; then select **ZERO** from the **CALC** menu.

KEYSTROKES: [ZOOM] 6 [2nd] [CALC] 2



[−10, 10] scl: 1 by [−10, 10] scl: 1

Place the cursor on a point at which $y < 0$ and press [ENTER] for the **LEFT BOUND**. Then place the cursor on a point at which $y > 0$ and press [ENTER] for the **RIGHT BOUND**. You can use the same point for the **GUESS**. The solution is about 1.36. This is consistent with the estimate made by using the **TABLE**.

(continued on the next page)

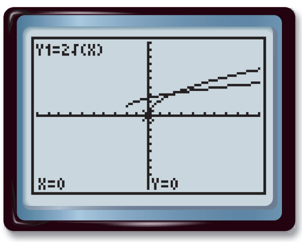
Graphing Technology Lab
Solving Radical Equations *Continued*

Example 2 Radical Inequality

Solve $2\sqrt{x} > \sqrt{x+2} + 1$.

Step 1 Graph each side of the inequality and use the **TRACE** feature.

- In the **Y=** list, enter $y_1 = 2\sqrt{x}$ and $y_2 = \sqrt{x+2} + 1$. Then press [GRAPH].



[−10, 10] scl: 1 by [−10, 10] scl: 1

- Press [TRACE]. You can use [▲] or [▼] to switch the cursor between the two curves.

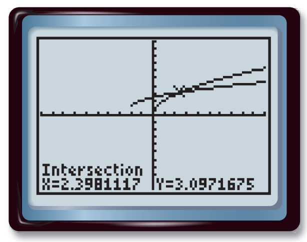
The calculator screen above shows that, for points to the left of where the curves cross, **Y1** < **Y2** or $2\sqrt{x} < \sqrt{x+2} + 1$. To solve the original inequality, you must find points for which **Y1** > **Y2**. These are the points to the right of where the curves cross.

Step 2 Use the **intersect** feature.

- You can use the **intersect** feature on the **CALC** menu to approximate the x-coordinate of the point at which the curves cross.

KEYSTROKES: [2nd] [CALC] 5

- Press [ENTER] for each of **FIRST CURVE?**, **SECOND CURVE?**, and **GUESS?**.



[−10, 10] scl: 1 by [−10, 10] scl: 1

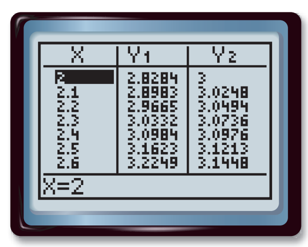
The calculator screen shows that the x-coordinate of the point at which the curves cross is about 2.40. Therefore, the solution of the inequality is about $x > 2.40$. *Use the symbol > in the solution because the symbol in the original inequality is >.*

Step 3 **Use the TABLE feature to check your solution.**

- Start the table at 2 and show x-values in increments of 0.1. Scroll through the table.

KEYSTROKES: [2nd] [TBLSET] 2 [ENTER] .1 [ENTER] [2nd] [TABLE]

Notice that when x is less than or equal to 2.4, **Y1** < **Y2**. This verifies the solution $\{x \mid x > 2.40\}$.



Exercises

Use a graphical method to solve each equation or inequality.

1. $\sqrt{x+4} = 3$

2. $\sqrt{3x-5} = 1$

3. $\sqrt{x+5} = \sqrt{3x+4}$

4. $\sqrt{x+3} + \sqrt{x-2} = 4$

5. $\sqrt{3x-7} = \sqrt{2x-2} - 1$

6. $\sqrt{x+8} - 1 = \sqrt{x+2}$

7. $\sqrt{x-3} \geq 2$

8. $\sqrt{x+3} > 2\sqrt{x}$

9. $\sqrt{x} + \sqrt{x-1} < 4$

10. **WRITING IN MATH** Explain how you could apply the technique in the first example to solving an inequality.

Study Guide and Review

Go Online! for Vocabulary Review Games and key vocabulary in 13 languages

Study Guide

Key Concepts

Operations on Functions (Lessons 5-1 and 5-2)

Operation	Definition
Sum	$(f + g)(x) = f(x) + g(x)$
Difference	$(f - g)(x) = f(x) - g(x)$
Product	$(f \cdot g)(x) = f(x) \cdot g(x)$
Quotient	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0$
Composition	$[f \circ g](x) = f[g(x)]$

Inverse Functions and Relations (Lesson 5-3)

- Two functions are inverses if and only if both their compositions are the identity function.
- The inverse relation is the set of ordered pairs obtained by exchanging the coordinates of each ordered par.
- The inverse of a function can be found by exchanging the independent and dependent variables (x and y, respectively) and then solving for the dependent variable (y).

Square Root and Cube Root Functions (Lessons 5-4 and 5-5)

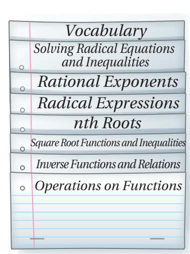
- The domain of a square root function is limited to values for which the function is defined.
- The same techniques used to transform the graph of other functions can be applied to the graphs of square root and cube root functions.

Solving Radical Equations (Lesson 5-6)

- To solve a radical equation: **Step 1** Isolate the radical on one side of the equation. **Step 2** Raise each side of the equation to a power equal to the index. **Step 3** Solve the resulting polynomial equation.

FOLDABLES® Study Organizer

Use your Foldable to review the chapter. Working with a partner can be helpful. Ask for clarification of concepts as needed.



Key Vocabulary

composition of functions (p. 322)
cube root function (p. 345)
extraneous solution (p. 352)
inverse function (p. 329)
inverse relation (p. 329)
inflection point (p. 345)
radical equation (p. 352)
radical function (p. 338)
radical inequality (p. 354)
square root function (p. 338)

Vocabulary Check

Choose a word or term that best completes each statement.

1. If both compositions result in the ____________, then the functions are inverse functions.
2. In a(n) ____________, the results of one function are used to evaluate a second function.
3. Equations with radicals that have variables in the radicands are called ____________.
4. Two relations are ____________ if and only if one relation contains the element (b, a) when the other relation contains the element (a, b).
5. When solving a radical equation, sometimes you will obtain a number that does not satisfy the original equation. Such a number is called a(n) ____________.
6. The square root function is a type of ____________.

Concept Check

7. Explain how to algebraically determine the inverse of a function.
8. Explain how to determine the domain of a square root function.

Lesson-by-Lesson Review

5-1 Operations with Functions

Given $f(x) = 2x + 9$ and $g(x) = x^2 + 2x + 1$, find each function.

9. $(f + g)(x)$
10. $(f - g)(x)$
11. $(f \cdot g)(x)$
12. $\left(\frac{f}{g}\right)(x)$

Given $f(x) = 10x$ and $g(x) = x^3 - 8$, find each function.

13. $(f + g)(x)$
14. $(f - g)(x)$
15. $(f \cdot g)(x)$
16. $\left(\frac{f}{g}\right)(x)$

Example 2

Given $f(x) = 3x + 1$ and $g(x) = x^3 + 1$, find each function.

a. $(f - g)(x)$

$(x - g)(x) = f(x) - g(x)$	Subtraction of functions
$= (3x + 1) - (x^3 + 1)$	Substitution
$= 3x - x^3$	Simplify.

b. $\left(\frac{f}{g}\right)(x)$

$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	Division of functions
$= \frac{(3x + 1)}{(x^3 + 1)}$	Substitution

5-2 Composition of Functions

Find $[f \circ g](x)$ and $[g \circ f](x)$.

17. $f(x) = 2x + 1$, $g(x) = 4x - 5$
18. $f(x) = x^2 + 1$, $g(x) = x - 7$
19. $f(x) = x^2 + 4$, $g(x) = -2x + 1$
20. $f(x) = 4x$, $g(x) = 5x - 1$
21. $f(x) = x^3$, $g(x) = x - 1$
22. $f(x) = x^2 + 2x - 3$, $g(x) = x + 1$

23. **MEASUREMENT** The formula $f = 3y$ converts yards y to feet f and $f = \frac{n}{12}$ converts inches n to feet f. Write a composition of functions that converts yards to inches.

Example 2

If $f(x) = x^2 + 3$ and $g(x) = 3x - 2$, find $g[f(x)]$ and $f[g(x)]$.

$g[f(x)] = 3(x^2 + 3) - 2$	Replace $f(x)$ with $x^2 + 3$.
$= 3x^2 + 9 - 2$	Distributive Property
$= 3x^2 + 7$	Simplify.
$f[g(x)] = (3x - 2)^2 + 3$	Replace $g(x)$ with $3x - 2$.
$= 9x^2 - 12x + 4 + 3$	Multiply.
$= 9x^2 - 12x + 7$	Simplify.

5-3 Inverse Functions and Relations

Find the inverse of each function. Then graph the function and its inverse.

24. $f(x) = 5x - 6$
25. $f(x) = -3x - 5$
26. $f(x) = \frac{1}{2}x + 3$
27. $f(x) = \frac{4x + 1}{5}$
28. $f(x) = x^2$
29. $f(x) = (2x + 1)^2$

30. **SHOPPING** Samuel bought a computer. The sales tax rate was 6% of the sale price, and he paid $50 for shipping. Find the sale price if Samuel paid a total of $1322.

Example 3

Find the inverse of $f(x) = -2x + 7$.

Rewrite $f(x)$ as $y = -2x + 7$. Then interchange the variables and solve for y.

$x = -2y + 7$	Interchange the variables.
$2y = -x + 7$	Solve for y.
$y = \frac{-x + 7}{2}$	Divide each side by 2.
$f^{-1}(x) = \frac{-x + 7}{2}$	Rewrite using function notation.

5-4 Graphing Square Root Functions

Graph each function. State the domain and range.

31. $f(x) = \sqrt{3x}$

32. $f(x) = -\sqrt{6x}$

33. $f(x) = \sqrt{x - 7}$

34. $f(x) = \sqrt{x + 5} - 3$

35. $f(x) = \frac{3}{4}\sqrt{x - 1} + 5$

36. $f(x) = -\frac{1}{3}\sqrt{x + 4} - 1$

37. **GEOMETRY** The area of a circle is given by the formula $A = \pi r^2$. What is the radius of a circle with an area of 300 square inches?

Example 4

Graph $f(x) = \sqrt{x + 1} - 2$. State the domain and range.

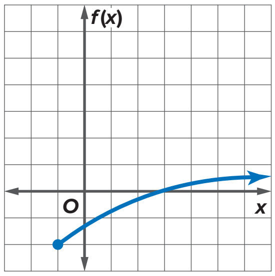
Identify the domain.

$x + 1 \geq 0$ Write the radicand as greater than or equal to 0.

$x \geq -1$ Subtract 1 from each side.

Make a table of values for $x \geq -1$ and graph the function.

x	$f(x)$
−1	−2
0	−1
1	−0.59
2	−0.27
3	0
4	0.24
5	0.45



The domain is $\{x \mid x \geq -1\}$, and the range is $\{f(x) \mid f(x) \geq -2\}$.

5-5 Graphing Cube Root Equations

Graph each function.

38. $f(x) = \sqrt[3]{2x}$

39. $f(x) = \sqrt[3]{-4x}$

40. $f(x) = \sqrt[3]{1 - x}$

41. $f(x) = 2 - \sqrt[3]{x}$

42. $f(x) = \sqrt[3]{x + 4} + 1$

43. $f(x) = 2\sqrt[3]{x - 1} + 2$

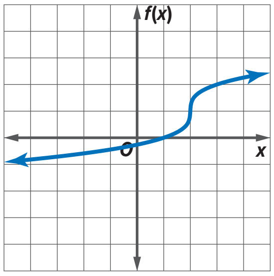
44. **GEOMETRY** The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$. What is the radius of a sphere with a volume of 400 cubic inches?

Example 5

Graph the function $f(x) = \sqrt[3]{x - 2} + 1$.

Make a table of values and graph the function.

x	$f(x)$
−5	−0.91
−4	−0.82
−3	−0.71
−2	−0.59
−1	−0.44
0	−0.26
1	0
2	1
3	2
4	2.26
5	2.44



Study Guide and Review *Continued*

5-6 Solving Radical Equations

Solve each equation.

45. $\sqrt{x-3}+5=15$

46. $-\sqrt{x-11}=3-\sqrt{x}$

47. $4+\sqrt{3x-1}=8$

48. $\sqrt{m+3}=\sqrt{2m+1}$

49. $\sqrt{2x+3}=3$

50. $(x+1)^{\frac{1}{4}}=-3$

51. $a^{\frac{1}{3}}-4=0$

52. $3(3x-1)^{\frac{1}{3}}-6=0$

53. **PHYSICS** The formula $t=2\pi\sqrt{\frac{\ell}{32}}$ represents the swing of a pendulum, where t is the time in seconds for the pendulum to swing back and forth and ℓ is the length of the pendulum in feet. Find the length of a pendulum that makes one swing in 2.75 seconds.

Solve each inequality.

54. $2+\sqrt{3x-1}<5$

55. $\sqrt{3x+13}-5\geq 5$

56. $6-\sqrt{3x+5}\leq 3$

57. $\sqrt{-3x+4}-5\geq 3$

58. $5+\sqrt{2y-7}<5$

59. $3+\sqrt{2x-3}\geq 3$

60. $\sqrt{3x+1}-\sqrt{6+x}>0$

Example 6

Solve $\sqrt{2x+9}-2=5$.

$\sqrt{2x+9}-2=5$	Original equation
$\sqrt{2x+9}=7$	Add 2 to each side.
$(\sqrt{2x+9})^2=7^2$	Square each side.
$2x+9=49$	Evaluate the squares.
$2x=40$	Subtract 9 from each side.
$x=20$	Divide each side by 2.

Example 7

Solve $\sqrt{2x-5}+2>5$.

$\sqrt{2x-5}\geq 0$	Radicand must be ≥ 0.
$2x-5\geq 0$	Square each side.
$2x\geq 5$	Add 5 to each side.
$x\geq 2.5$	Divide each side by 2.

The solution must be greater than or equal to 2.5 to satisfy the domain restriction.

$\sqrt{2x-5}+2>5$	Original inequality
$\sqrt{2x-5}>3$	Subtract 2 from each side.
$(\sqrt{2x-5})^2>3^2$	Square each side.
$2x-5>9$	Evaluate the squares.
$2x>14$	Add 5 to each side.
$x>7$	Divide each side by 2.

Because $x\geq 2.5$ contains $x>7$, the solution of the inequality is $x>7$.

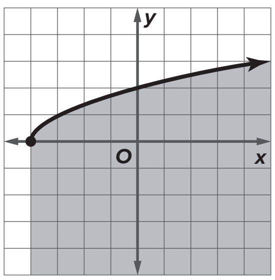
CHAPTER 5
Practice Test

Go Online! for another Chapter Test

Determine whether each pair of functions are inverse functions. Write *yes* or *no*. Explain your reasoning.

1. $f(x) = 3x + 8, g(x) = \frac{x-8}{3}$
2. $f(x) = \frac{1}{3}x + 5, g(x) = 3x - 15$
3. $f(x) = x + 7, g(x) = x - 7$
4. $g(x) = 3x - 2, f(x) = \frac{x-2}{3}$

5. **MULTIPLE CHOICE** Which equation represents the graph below?



A $y = \sqrt{x-4}$

B $y = \sqrt{x+4}$

C $y = \sqrt[3]{x-4}$

D $y = \sqrt[3]{x+4}$

If $f(x) = 3x + 2$ and $g(x) = x^2 - 2x + 1$, find each function.

6. $(f + g)(x)$
7. $(f \cdot g)(x)$
8. $(f - g)(x)$
9. $\left(\frac{f}{g}\right)(x)$

Solve each equation.

10. $\sqrt{a+12} = \sqrt{5a-4}$
11. $\sqrt{3x} = \sqrt{x-2}$
12. $4\left(\sqrt[4]{3x+1}\right) - 8 = 0$
13. $\sqrt[3]{5m+6} + 15 = 21$
14. $\sqrt{3x+21} = \sqrt{5x+27}$
15. $1 + \sqrt{x+11} = \sqrt{2x+15}$
16. $\sqrt{x-5} = \sqrt{2x-4}$
17. $\sqrt{x-6} - \sqrt{x} = 3$

If $f(x) = 3x + 2$ and $g(x) = x^2 - 2x + 1$, find each function.

18. $[f \circ g](x)$
19. $[g \circ f](x)$

Graph each function. State the domain and range of each function.

20. $y = 2 + \sqrt{x}$
21. $y = \sqrt{x+4} - 1$

22. **MULTIPLE CHOICE** What is the domain of $f(x) = \sqrt{2x+5}$?

A $\left\{x \middle| x > \frac{5}{2}\right\}$

B $\left\{x \middle| x > -\frac{5}{2}\right\}$

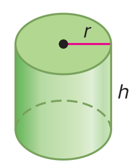
C $\left\{x \middle| x \geq \frac{5}{2}\right\}$

D $\left\{x \middle| x \geq -\frac{5}{2}\right\}$

Graph each function. State the domain and range of each function.

23. $f(x) = 4 - \sqrt[3]{2x}$
24. $f(x) = 2\sqrt[3]{x+2} - 4$

25. **MULTIPLE CHOICE** The radius of the cylinder below is equal to the height of the cylinder. The radius r can be found using the formula $r = \sqrt[3]{\frac{V}{\pi}}$. Find the radius of the cylinder if the volume is 500 cubic inches.



A 2.53 inches

B 5.42 inches

C 7.94 inches

D 24.92 inches

Solve each inequality.

26. $\sqrt{4x-3} < 5$
27. $-2 + \sqrt{3m-1} < 4$
28. $2 + \sqrt{4x-4} \leq 6$
29. $\sqrt{2x+3} - 4 \leq 5$
30. $\sqrt{b+12} - \sqrt{b} > 2$
31. $\sqrt{y-7} + 5 \geq 10$
32. $\sqrt{a-5} - \sqrt{a+7} \leq 4$
33. $\sqrt{c+5} + \sqrt{c+10} > 2$

Find the inverse of each function, if it exists.

34. $f(x) = -2x + 8$
35. $f(x) = \frac{x+1}{3}$

Use the horizontal line test to determine whether the inverse of each function is also a function.

36. $f(x) = -4x^2$
37. $f(x) = x^3 - 1$

Preparing for Assessment

Performance Task

Provide a clear solution to each part of the task. Be sure to show all of your work, include all relevant drawings, and justify your answers.

APPLY MATH Two students at a university are each research assistants. Johanna is a physics research assistant and Ravi is a biology research assistant. They are both performing various calculations on data collected in their labs.

Part A After studying production of two different varieties of tomatoes grown in two identical gardens, Ravi comes up with the following equations to represent the tomatoes' production rates:

Variety A: $t = 2d^2 + 4d - 2$ Variety B: $t = d^2 + d + 1$

In Ravi's models, t represents the total number of tomatoes produced in each garden and d represents the number of whole days since the plants first started producing tomatoes.

- Write an expression that models the total production rates of both varieties.
- Determine which variety is more productive. Explain your reasoning.
- Write an expression that models how much more productive one variety is than the other.

Part B While studying the decline of a certain kind of cell in children with certain illnesses, Ravi calculates the following functions to represent how the decrease in the numbers of cells c as a function of the number of days d the child has the illness:

Illness A: $c_1(d) = 1218e^{-0.05d}$ Illness B: $c_2(d) = 420e^{-0.125d}$

Ravi also finds that if a child has *both* illnesses, because of the increased trauma to the immune system, the total decrease in cells is the product of the two functions, rather than the sum.

- Write a function that models the decrease in the number of cells if a child had *both* illnesses.

Part C Ravi is working with data collected by researchers all over the world concerning bacteria growth in different bodies of water with different temperatures. The data collected is in Kelvin, Celsius, and Fahrenheit. The formula to convert Kelvin to Celsius is $T_{(C)} = T_{(K)} - 273.15$, and the formula to convert Celsius to Fahrenheit is $T_{(F)} = \frac{9}{5}T_{(C)} + 32$.

- Write a composition function that can be used to directly convert Kelvin to Fahrenheit.

Part D Johanna is working on a project involving asteroids stuck in the orbit of a planet. The formula for escape velocity is $v \geq \sqrt{\frac{2\,GM}{r}}$, where v is the necessary velocity needed to escape orbit, G is the gravitational constant, M is the mass of the planet, and r is the radius of the planet. The gravitational constant is 6.673×10^{-11}.

- If the mass of the Earth is 5.98×10^{24}, in kilograms, and the escape velocity needed for an asteroid to leave Earth's orbit is 11,184 meters per second, determine the radius of the Earth. (Note: do not worry about units.)

Part E

Johanna next turns her focus to a project involving springs. The period of oscillation of a spring can be found using the formula $T = 2\pi\sqrt{\frac{m}{k}}$, where T is the period, m is the mass of the oscillating body, and k is a constant.

- Write an equation for the mass of the oscillating body, in terms of the period.

Test-Taking Strategy

Example

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

Given the equation $\sqrt{\frac{2x}{3y}} = 8$, which of the following represents y in terms of x?

A $y = \frac{x}{32}$

B $y = \frac{\sqrt{8x}}{3}$

C $y = \frac{x}{96}$

D $y = \frac{\sqrt{2x}}{3}$

Step 1 **What information is given? Are any of the following involved: earlier quantities, answer choices that are easy to test, or inverse operations?**
An equation is given. Because I need to solve for a variable, I will use inverse operations.

Step 2 **How will working backward help solve this problem?**
I'll use inverse operations to "undo" operations and isolate the variable y.

Step 3 **How can you check your answer? What is the correct answer?**
I can "work forward" by beginning with my answer and seeing if I arrive at the same result given in the problem. Choice C is correct.

Test-Taking Tip

Strategies for Working Backward
In certain math problems, you are given information about an end result, but you are asked to find out something that happened earlier. In other problems, it can be faster to work from the answer choices, instead of from the problem. Finally, you'll sometimes need to use inverse operations, which is a specific way of working backward. You can work backward to solve all of these kinds of problems.

Apply the Strategy

Read the problem. Identify what you need to know. Then use the information in the problem to solve.

An object is shot straight upward into the air with an initial speed of 800 feet per second. The height h that the object will be after t seconds is given by the equation $h = -16t^2 + 800t$. When will the object reach a height of 10,000 feet?

A 10 seconds

B 25 seconds

C 100 seconds

D 625 seconds

Answer the questions below.

a. What information is given? Are any of the following involved: earlier quantities, answer choices that are easy to test, or inverse operations?

b. How will working backward help solve this problem?

c. How can you check your answer?

d. What is the correct answer?

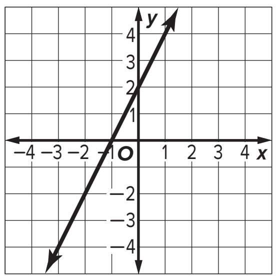
Preparing for Assessment

Cumulative Review

Go Online! for Standardized Test Practice

Read each question. Then fill in the correct answer on the answer document provided by your teacher or on a sheet of paper.

1. What is the inverse of function $f(x)$ graphed below?



○ **A** $f^{-1}(x) = \frac{2}{x-2}$

○ **B** $f^{-1}(x) = \frac{x-2}{2}$

○ **C** $f^{-1}(x) = \frac{x}{2} - 2$

○ **D** $f^{-1}(x) = -2x - 2$

2. For what value of x is $\sqrt{x-3} - 9 = 6$?

3. For what value of x is $9 + \sqrt{5x+5} = 24$

○ **A** −24 ○ **C** 30

○ **B** 18 ○ **D** 44

4. The formula $d = \sqrt[3]{6t^2}$ represents d, the distance in millions of miles a planet is from the Sun, in terms of t, the number of Earth days it takes for the planet to orbit the Sun. It takes Venus 224.7 Earth days to complete one orbit. To the nearest tenth, how many millions of miles is Venus from the Sun?

☐ **A** 37.0

☐ **B** 67.2

☐ **C** 550.4

☐ **D** 100,980.2

☐ **E** 302,940.5

5. Given the functions $f(x) = x^2 - 9$ and $g(x) = x + 1$, find each of the following functions:

$(f + g)(x)$ ________

$(f - g)(x)$ ________

6. What is $(f \cdot g)(2)$ and $\left(\frac{f}{g}\right)(-2)$ for the functions $f(x) = x^2 - 36$ and $g(x) = x + 4$?

○ **A** −24; −14

○ **B** −128; −24

○ **C** −192; −16

○ **D** 212; 32

7. Determine whether each pair of functions are inverse functions.

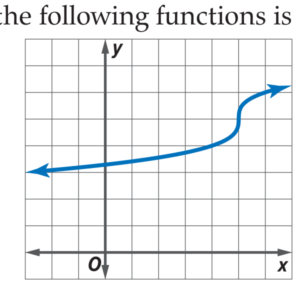
$f(x) = x + 1$ $g(x) = -x - 1$

$f(x) = (x + 4)^2$ $g(x) = \sqrt{x} - 4$

$f(x) = -\frac{1}{2}x - 8$ $g(x) = -2x - 16$

$f(x) = -\frac{1}{16}x^2 - 1$ $g(x) = 4\sqrt{x} + 1$

8. Which of the following functions is graphed below?



○ **A** $f(x) = \sqrt[3]{x-5} - 5$

○ **B** $f(x) = \sqrt[3]{x-5} + 5$

○ **C** $f(x) = \sqrt[3]{x+5} - 5$

○ **D** $f(x) = \sqrt[3]{x+5} + 5$

Test-Taking Tip

Question 7 Remember, two functions are inverse functions if both of their compositions are the identity function.

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9. Solve the equation.

$\sqrt[3]{2x + 5} + 8 = 11$

[ ]

10. What is the value of x for the equation $4 + \sqrt[3]{9x + 81} = 10$?

- **A** -37
- **B** 2
- **C** 15
- **D** 244

11. Andrew works at a suit store where he earns $360 per forty-hour work week, plus a 10% commission on sales over $500 each week, which can be represented by the functions $f(x) = 0.1x$ and $g(x) = x - 500$, where x is the amount Andrew makes in sales in a week. Assuming Andrew made enough to earn a commission, which composition would give his commission: $(f \circ g)(x)$ or $(g \circ f)(x)$? Explain your reasoning.

12. Which of the following belong to the solution set of $10 - \sqrt{4x + 20} > 2$?

- **A** -3
- **B** 6
- **C** 11
- **D** 16
- **E** 21

13. If $a = 10$ and $b = 13$, what is the value of $(6\sqrt{a} + 2\sqrt{b})(6\sqrt{a} - 2\sqrt{b})$?

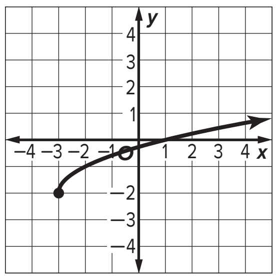
14. What is $(f \circ g)(-2)$ and $(g \circ f)(-2)$ for the functions $f(x) = x^2 - 100$ and $g(x) = x + 10$?

- **A** $-134; -28$
- **B** $-36; -86$
- **C** 192; 28
- **D** 242; 64

15. Let $f(x) = x^2 - 1$. What is the value of $f[f(2)]$?

[ ]

16. Which of the following functions is graphed below?



- **A** $y = \sqrt{x - 2} - 3$
- **B** $y = \sqrt{x + 2} - 3$
- **C** $y = \sqrt{x - 3} - 2$
- **D** $y = \sqrt{x + 3} - 2$

Need Extra Help?

If you missed Question...	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Go to Lesson...	5-3	5-6	5-6	5-5	5-1	5-1	5-3	5-5	5-6	5-6	5-2	5-6	5-4	5-4	5-1	5-4

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