

Levels A-I

Guided Support

Teacher Guide Sampler



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Compose and Decompose Tens and Ones

Mathematical Context

Prior to this module, students have had experience counting groups of objects forward and backward within 20, creating sets of 1 to 10 objects, recognizing numerals to 20, comparing sets of up to 10 objects, and composing and decomposing numbers to 10. In this module, students extend these number sense strategies to compose and decompose tens and ones to 19.

Lessons will focus on the use of models to show numbers more than one way. The use of pictures, objects, and tens frames builds understanding as students learn basic number sense that will apply to the study of operations in later lessons.

Vocabulary

ones

Standards

See online module information or Program Overview for standards alignment.

Lesson Title	Goal	Prerequisite Skills
Compose Tens and Ones from 11 to 15	Students will combine a set of 10 ones and some more ones into a set of 11 to 15.	To be successful with combining a group of 10 ones and some more ones to make numbers 11 through 15, students need to be able to identify and count a group of 10 ones and count forward to include the additional ones. Students need to be able to recognize numbers and number names for 1 through 15.
Decompose Tens and Ones from 11 to 15	Students will decompose a set of 11 to 15 objects into a set of 10 ones and some more ones.	To be successful with decomposing numbers 11 to 15 into a set of 10 ones and some more ones, students need to be able to count to 15 and be able to identify a set of up to 10 objects in a ten frame. Students also need to be able to recognize numbers and number names for 1 through 15.
Compose Tens and Ones from 16 to 19	Students will combine a group of 10 ones and some more ones into a set of 16 to 19.	To be successful with combining a group of 10 ones and some more ones to make numbers 16 through 19, students need to be able to identify and count a group of 10 ones and count forward to include the additional ones. Students need to be able to recognize numbers and number names for 1 through 19.
Decompose Tens and Ones from 16 to 19	Students will decompose a set of 16 to 19 objects into a group of 10 ones and some more ones.	To be successful with decomposing numbers 16 to 19 into a group of 10 ones and some more ones, students need to be able to count to 19 and be able to identify a group of up to 10 in a ten frame. Students also need to be able to recognize numbers and number names for 1 through 19.

Compose Tens and Ones from 11 to 15

Goal Students will combine a set of 10 ones and some more ones into a set of 11 to 15.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Connecting cubes (10 yellow and 5 red per student)
- Number cards (11–15, 1 set)
- Construction paper (2 sheets per student)
- Blackline Master: *Make Numbers 11 to 15* (1 per student)

Begin the Activity

Explain that students will use connecting cubes to combine a group of 10 ones and some more ones to make a number from 11 to 15.

Give each student a copy of *Make Numbers 11 to 15*. [Let's make numbers using a group of ten ones and some more ones.](#) Place two sheets of paper side by side on a table or on the floor. Put 10 yellow connecting cubes together in a train on the left sheet and one red cube on the right sheet. Point to the train of 10 cubes as you count them aloud. [This is a group of 10 cubes.](#) Point to the one cube left over. [There is one more cube. Count forward with me from 10 as I put all of these cubes together: 10, 11.](#) Put the train of 11 cubes on the table or floor and place number card 11 below the train. [Ten cubes and one more cube make 11 cubes. Ten ones and one more one make 11.](#) Guide students to complete the first row on their worksheets. When they are finished, lead students to say the sentence: [Ten ones and one more one make 11.](#)

Place two sheets of construction paper side by side in front of each student. Place a train of 10 yellow cubes on the left sheet and two separate red cubes on the right sheet. Point to the train of yellow cubes. [How many cubes are here? 10](#) Point to the two red cubes. [How many cubes are here? 2](#) [Let's put the cubes together to see what number they make.](#) Guide students to connect the two red cubes to the 10 yellow cubes as they count forward from 10: [10, 11, 12. A group of 10 cubes and two more cubes make 12 cubes. A group of 10 ones and two more ones make 12.](#) Hold up number card 12. Guide students to complete the second row on their worksheets. When they are finished, lead students to say the sentence: [Ten ones and two more ones make 12.](#)

Repeat the process for numbers 13, 14, and 15. Make sure students can describe making each number in terms of a group of 10 ones and how many more ones.

Conclude the Activity

Place a train of 10 yellow cubes and five separate red cubes on a sheet of construction paper. Show students number card 12 and number card 15. Which number did I make? 15 How do you know? Sample answer: Ten ones and five more ones make 15. Repeat the process for numbers 11–15 as time allows.

Questions

- Describe how you made the number 12 using connecting cubes. Sample answer: I made a group of 10 cubes and connected two more cubes. Ten cubes and two more cubes make 12 cubes. Ten ones and two more ones make 12.
- Show students number card 11 and number card 14. A group of 10 ones and four more ones make which number? 14
- Show students number card 13 and number card 15. A group of 10 ones and three more ones make which number? 13

Decompose Tens and Ones from 11 to 15

Goal Students will decompose a set of 11 to 15 objects into a set of 10 ones and some more ones.

Guided Support 15 MINUTES



Materials

- Connecting cubes (15 per student)
- Number cards (11–15, 1 set)
- Construction paper (2 sheets per student)
- Blackline Master: *Take Apart Numbers 11 to 15* (1 per student)

Begin the Activity

Explain that students will use connecting cubes to take apart, or separate, numbers 11 through 15 into a group of 10 ones and some more ones.

Give each student a copy of *Take Apart Numbers 11 to 15*. Place two sheets of paper side by side on a table or on the floor. Hold up number card 11. **Let's show the number 11.** Connect 11 cubes into a train, counting aloud as you connect. **Here are 11 cubes. Let's take apart this group of 11 cubes into a group of 10 ones and some more ones.** Count out 10 cubes, keeping them connected in a train, and separate them from the train of 11. Place the train of 10 cubes on the left sheet of construction paper. **This is a group of 10 ones.** Point to the one cube left over. **There is one more one.** Place the one cube on the right sheet of construction paper. **Eleven is 10 ones and one more one.** Guide students to complete the first row on their worksheets. When they are finished, lead students to say the sentence: **Eleven is 10 ones and one more one.**

Hold up number card 12. **Let's model the number 12.** Give each student 12 connecting cubes. Guide them to count and connect the 12 cubes into a train. **Here are 12 connecting cubes. Let's take apart the group of 12 cubes into a group of 10 ones and some more ones.** Guide students to separate a train of 10 connecting cubes as you modeled in the previous example. Have students place the train of 10 cubes on the left sheet of paper. **This is a group of 10 ones. How many more ones are left? 2** Have students place those two cubes on the right sheet of paper. **Twelve is 10 ones and two more ones.** Guide students to complete the second row on their worksheets. When they are finished, lead students to say the sentence aloud: **Twelve is 10 ones and two more ones.**

Repeat the process for numbers 13, 14, and 15. Make sure students can describe each number in terms of 10 ones and how many more ones.

Conclude the Activity

Show students number card 14. Place a train of 10 connecting cubes and four separate cubes on one sheet of construction paper with *10 ones and 4 more ones* written below it. On another sheet of construction paper, place a train of ten connecting cubes and one separate cube with *10 ones and 1 more one* written below it. **Which paper shows the number 14? Students should point to the group of 10 ones and four more ones. How do you know? Sample answer: 14 is a group of 10 ones and 4 more ones.**

Questions

- Describe how to take apart 12 connecting cubes into a group of 10 ones and some more ones. **Sample answer: Count out a group of 10 cubes from the 12 cubes and there are two more cubes left. 12 is a group of 10 ones and two more ones.**
- Show students number card 11 and 11 connecting cubes. **Eleven is a group of 10 ones and how many more ones? 1** Have students explain their answer. **Sample answer: I broke apart 10 cubes and counted 1 more.**
- Show students number card 13 and 13 connecting cubes. **Thirteen is a group of 10 ones and how many more ones? 3** Have students explain their answer. **Sample answer: I broke apart 10 cubes and counted 3 more.**

Compose Tens and Ones from 16 to 19

Goal Students will combine a group of 10 ones and some more ones into a set of 16 to 19.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Connecting cubes (10 yellow and 9 red per student)
- Number cards (15–19, 1 set)
- Construction paper (2 sheets per student)
- Blackline Master: *Make Numbers 15 to 19* (1 per student)

Begin the Activity

Explain that students will use connecting cubes to combine a group of 10 ones and some more ones to make a number from 15 to 19.

Give each student a copy of *Make Numbers 15 to 19*. **Let's make numbers using a group of 10 ones and some more ones.** Place two sheets of paper side-by-side on a table or on the floor. Put a train of 10 yellow cubes on the left sheet and five separate red cubes on the right sheet. Point to the train of 10 cubes as you count them aloud. **This is a group of 10 cubes.** Point to the five red cubes as you count them aloud. **There are five more cubes. Count with me as I put all of these cubes together: 10, 11, 12, 13, 14, 15.** Put the train of 15 cubes on the table or floor and place number card 15 below the train. **10 cubes and 5 more cubes make 15 cubes. 10 ones and 5 more ones make 15.** Guide students to complete the first row on their worksheets. When they are finished, lead students to say the sentence: **10 ones and 5 more ones make 15.**

Place two sheets of construction paper side by side in front of each student. Place a train of 10 yellow cubes on the left sheet and six separate red cubes on the right sheet. Point to the train of 10 yellow cubes. **How many cubes are here? 10** Point to the six red cubes. **How many cubes are here? 6** **Let's put the cubes together to see what number they make.** Guide students to connect the six red cubes to the train of 10 yellow cubes as they count on: **10, 11, 12, 13, 14, 15, 16.** **A group of 10 cubes and 6 more cubes make 16 cubes. A group of 10 ones and 6 more ones make 16.** Hold up number card 16. Guide students to complete the second row on their worksheets. When they are finished, lead students to say the sentence: **10 ones and 6 more ones make 16.**

Repeat the process for numbers 17, 18, and 19. Make sure students can describe making each number in terms of a group of 10 ones and how many more ones.

Conclude the Activity

Place a train of 10 yellow cubes and nine separate red cubes on a sheet of construction paper. Show students number card 18 and number card 19. Which number did I make? 19 How do you know? Sample answer: 10 ones and 9 more ones make 19. Repeat the process for numbers 16–19.

Questions

- Describe how you made the number 18 using connecting cubes. Sample answer: I made a group of 10 cubes and connected 8 more cubes. 10 cubes and 8 more cubes make 18 cubes. 10 ones and eight more ones make 18.
- Show students number card 16 and number card 17. A group of 10 ones and six more ones make which number? 16 A group of 10 ones and seven more ones make which number? 17

Decompose Tens and Ones from 16 to 19

Goal Students will decompose a set of 16 to 19 objects into a group of 10 ones and some more ones.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Connecting cubes (19 per student)
- Number cards (15–19, 1 set)
- Construction paper (2 sheets per student)
- Blackline Master: *Take Apart Numbers 15 to 19* (1 per student)

Begin the Activity

Tell students they will use connecting cubes to take apart the numbers 15 through 19 into a set of 10 ones and some more ones.

Give each student a copy of *Take Apart Numbers 15 to 19*. Place two sheets of paper side by side on a table or on the floor. Hold up number card 15. **Let's show the number 15 with cubes.** Connect 15 cubes into a train, counting aloud as you connect. **Here are 15 cubes. Let's take apart the 15 cubes into a group of 10 ones and some more ones.** Count out 10 cubes, keeping them connected in a train, and separate them from the train of 15. Place the train of 10 cubes on the left sheet of construction paper. **This is a group of 10 ones.** Point to the five cubes left over. **There are five more ones.** Place the five cubes on the right sheet of construction paper. **Fifteen is a group of 10 ones and five more ones.** Guide students to complete the first row on their worksheets. When they are finished, lead students to say the sentence aloud: **Fifteen is 10 ones and five more ones.**

Repeat the process for numbers 16, 17, 18, and 19. Make sure students can describe each number in terms of 10 ones and how many more ones.

Conclude the Activity

Show students number card 19. Place a train of 10 connecting cubes and nine separate cubes on one sheet of construction paper with *10 ones and 9 more ones* written below it. On another sheet of construction paper, place a train of 10 connecting cubes and eight separate cubes with *10 ones and 8 more ones* written below it. **Which paper shows the number 19? Students should point to the group of 10 ones and nine more ones. How do you know? Sample answer: 19 is a group of 10 ones and nine more ones.**

Questions

- Tell how to take apart 18 connecting cubes into a group of 10 ones and some more ones. Sample answer: I can count out a group of 10 cubes from the 18 cubes and have eight cubes left. Eighteen is a group of 10 ones and eight more ones.
- Show students number card 16 and 16 connecting cubes. Have students use the cubes to demonstrate their understanding. Sixteen is a group of 10 ones and how many more ones? six
- Show students number card 18 and 18 connecting cubes. Eighteen is a group of 10 ones and how many more ones? eight Have students use the cubes to demonstrate their understanding.

Join/Take-Apart Word Problems to 10

Mathematical Context

Previously, students have had lessons in counting and comparing groups of objects, using add-to concepts to add, and using take-from concepts to subtract. In this module, students will represent and solve put-together and take-apart word problems, within 10. Students need to be able to add and subtract within 10 and to decompose numbers within 10. They also must have a strong understanding of *equals* since students will now be seeing addends to the right of the equals sign. Students will use manipulatives, drawings, and equations to represent and solve put-together/take-apart problems with total unknown, one addend unknown, or both addends unknown, within 10. These put-together/take-apart problem-solving skills will be useful as students encounter future lessons dealing with problems within 20 and beyond.

Vocabulary

add
addend
difference
subtract
sum

Standards

See *online module information or Program Overview for standards alignment.*

Lesson Title	Goal	Prerequisite Skills
Total Unknown to 10 (Join/Take Apart)	Students will use models and equations to solve for total unknown in put-together/take-apart word problems within 10.	To be successful with solving put-together/take-apart word problems, students need to be able to add sums within 10.
Addend Unknown to 10 (Join/Take Apart)	Students will use models and equations to solve for addend unknown in put-together/take-apart word problems with sums and differences within 10.	To be successful with solving put-together/take-apart word problems, students need to be able to add and subtract within 10.
Addends Unknown to 10 (Join/Take Apart)	Students will use models and equations to solve for addends unknown in put-together/take-apart word problems within 10.	To be successful with solving put-together/take-apart word problems with both addends unknown, students need to be able to solve put-together/take-apart problems with one addend unknown. Students need to be able to decompose numbers to 10. Students also need a very strong understanding of <i>equals</i> in order to recognize that the addends may be shown on the left side or right side of the equals sign.

Total Unknown to 10 (Join/Take Apart)

Goal Students will use models and equations to solve for total unknown in put-together/take-apart word problems within 10.

Guided Support 15 MINUTES



Materials

- Dry-erase markers (1 per group)
- Two-color counters (10 per group)
- Blackline Master: *Part-Part-Whole (+ and =) Mat*

Begin the Activity

Divide students into partners or have students work individually if numbers are insufficient for group work. Give each group a *Part-Part-Whole (+ and =) Mat*, a dry-erase marker, and a set of two-colored counters. Tell students they will be using counters to solve word problems.

Explain to students that they will be modeling word problems using counters. Tell students that red counters will represent red birds and yellow counters will represent yellow birds in the first word problem. **There are three red birds and six yellow birds on the tree. How many birds are on the tree?** Repeat the beginning of the problem (**There are three red birds . . .**) and instruct students to place three red counters on the first part of the mat and label it 3. **We also know that there are six yellow birds on the tree.** Instruct students to place six yellow counters on the second part of the mat and label it 6. **Now, let's find out how many birds there are in the whole tree.** Tell students to place all of the counters on the mat together in the whole section. Count the number of counters together out loud as a class. **There are nine birds in all. Nine is the sum. Label the whole 9.** Instruct students to read the equation formed by the mat. **What does the 3 in the number sentence show? 3 red birds What does the 6 show? 6 yellow birds What does the 9 show? 9 birds in the tree**

Repeat this activity for the following word problem: *I have 8 red tomatoes. I have 2 yellow tomatoes. How many tomatoes do I have in all?*

Conclude the Activity

Ask students to make up word problems. Then have students share their word problems with the class. Have students model and solve each word problem using counters and the workmat.

Questions

- How do you use a picture to solve a problem? Sample answer: The first part represents the first amount in the picture. The second part represents the second amount in the picture. You count the two parts together to find the whole.
- What symbol tells you that you are adding? the plus sign

Addend Unknown to 10 (Join/Take Apart)

Goal Students will use models and equations to solve for addend unknown in put-together/take-apart word problems with sums and differences within 10.

Guided Support 15 MINUTES



Materials

- Blackline Master: *Part-Part-Whole Mat 2* (2 per student)
- Blackline Master: *Whole-Part-Part Mat 2* (2 per student)
- Two-color counters (10 per group)

Begin the Activity

Give each student *Part-Part-Whole Mat 2* and two-colored counters. Tell students they will use counters to solve word problems.

Explain to students that they will hear a word problem about red and yellow birds and then break the problem down into parts to solve it. Tell students that red counters will represent red birds and yellow counters will represent yellow birds. Show the word problem as you read it aloud with the students: **There are 10 birds. Three birds are red. The rest are yellow. How many birds are yellow?**

There are 10 birds. Instruct students to trace or draw 10 counters in a single row in the bottom box (whole) of *Part-Part-Whole Mat 2*. Students will be placing counters over these circle drawings so guide students to approximate the size and spacing of each circle.

Three are red. Instruct students to place three red counters in the top left hand box (part) and label this section 3. Continue with the word problem. **The rest are yellow. We can use the red counters and the counter drawings to find out how many birds are yellow?** Instruct students to move the three red counters to cover the first three counter drawings. **Since the rest of the birds are yellow, cover the rest of the counter drawings with yellow counters. How many yellow counters are there? 7** Instruct students to move the red counters back to the top left box (part) and to move the yellow counters to the top right box (part). Have them label the top right section 7. **How many yellow birds are there? 7** **How do you know? Sample answer: The yellow counters show yellow birds and there are seven yellow counters.** Explain that 3 and 7 are the addends and 10 is the sum. To show the sum, have students write 10 after the word *is* in the Whole section of the mat. Read the statement with the students that is formed by the Part-Part-Whole Mat. **Three and seven is 10.**

Tell students that they can solve the problem another way. Have them use *Whole-Part-Part Mat 2*. Repeat the original problem and break it down to solve using subtraction. Have students trace or draw 10 counters in the Whole section of the mat and label it 10. Have them use 3 red counters and cover three counter drawings at the right end of the row, count to see how many more counters are needed, and then place the yellow counters over the remaining seven counter drawings. **How many counters are there in all? 10 How many are red? 3** Have students slide the red counters to the bottom left part of the mat and write the label 3. **How many yellow counters are left? 7 How do you know many yellow birds there are? Sample answer: There are seven yellow counters left. The counters show yellow birds. So, there are seven yellow birds.** Guide students to place these counters in the bottom right part of the mat and write the label 7. Read the statement with the students that is formed by the *Whole-Part-Part Mat*. **Ten take away three is seven.** Review how the difference of 7 is the number of yellow birds.

Conclude the Activity

Have students model and solve the following word problem on their own and explain their work: **There are eight fish. Three of them are red. The rest are yellow. How many fish are yellow? 5 How do you know? Sample answer: The word problem says that there were eight fish in all. I subtracted the number of red fish from eight to find the rest of the fish that are yellow. Eight minus three is five.**

Questions

Provide 10 counters, *Part-Part-Whole Mat 2*, and *Whole-Part-Part Mat 2* for each student.

- Tell students to use counters and mat to solve this problem: **I have nine counters. Two are red. The rest are yellow. How many yellow counters do I have? 7**
- Tell a word problem of your own. Use counters and drawings to find the answer. **Sample answer: I have 10 counters. Five are red. The rest are yellow. How many counters are yellow? The answer is: 5 are yellow.**

Addends Unknown to 10 (Join/Take Apart)

Goal Students will use models and equations to solve for addends unknown in put-together/take-apart word problems within 10.

Guided Support 15 MINUTES



Materials

- Counters (10 per group)
- Dry-erase markers (1 per group)
- Blackline Master: *Take Apart More Than One Way* (1 per group)
- Blackline Master: *Take Apart More Than One Way (= and +)* (optional, 1 per group)

Begin the Activity

Before the lesson, laminate one *Take Apart More Than One Way* mat for each group. Divide students into partners or small groups. Explain to students that they are going to use counters to model word problems. They will pretend that the counters are round stickers.

Write the following word problem on the board and then read it with the students.

There are 10 stickers.
How many stickers can be on my binder?
How many stickers can be in my notebook?

Tell students that they are going to draw a picture on their *Take Apart More Than One Way* mat with counters to find a solution to the problem. **There are 10 stickers.** Make sure that each group places ten counters in the whole section of their *Take Apart More Than One Way* mat. **Some stickers are in my notebook.** Tell students to move some counters to the left part to represent the stickers in your notebook. **The rest of the stickers are on my binder.** Tell students to move the rest of the counters to the right part to represent the stickers in your binder. Explain that since it says “the rest,” they have to put all of the leftover counters on the right side. **How many stickers could be in each place? Sample answer: Four stickers could be in your notebook and six stickers could be on your binder.**

Have students count and record the numbers in each section on the *Take Apart More Than One Way* mat. Then allow students to share their solutions with the class. When students share their answers, have the class make models to check each answer.

Conclude the Activity

Record the solutions on the board. *Are there any other solutions?* You should get a list like this once the class thinks they have found all of the solutions:

$$1 + 9 = 10$$

$$2 + 8 = 10$$

$$3 + 7 = 10$$

$$4 + 6 = 10$$

$$5 + 5 = 10$$

$$6 + 4 = 10$$

$$7 + 3 = 10$$

$$8 + 2 = 10$$

$$9 + 1 = 10$$

Questions

- How do you know how many items go in the *Whole* section of the *Take Apart More Than One Way* mat? *Sample answer:* The total number of items goes in the *Whole* section.
- How do you know how many to put in the second place? *Sample answer:* You put any leftover items in the second place.
- Why are there many different answers for these word problems? *Sample answer:* The problems say “some.” They do not tell you how many items go in one place.

Whole Numbers on a Number Line

Mathematical Context

In previous lessons, students have experienced various opportunities to use counting skills, skip counting, basic addition facts, and basic subtraction facts. They have used images and models to build understanding of these concepts. In this lesson, students will apply and extend these skills by representing points, skip counting by fives and tens, and exercising addition and subtraction of whole numbers on number lines. The use of number lines provides students opportunities to visualize mathematical concepts that will apply to future lessons in skip counting greater numbers, understanding magnitude of numbers, and finding sums and differences within greater numbers.

Vocabulary

add
addend
difference
even number
number line
number sentence
odd number
pair (verb)
pairs (noun)
set of whole numbers
skip count
subtract
sum
whole number

Standards

See *online module information or Program Overview for standards alignment.*

Lesson Title	Goal	Prerequisite Skills
Whole Numbers on a Line	Students will place points on a number line to represent whole numbers.	To be successful with placing points on a number line, students need to be able to identify and sequence whole numbers, identify patterns such as fives and tens, and identify whole numbers on a number line.
Graphs of Whole Numbers	Students will identify whole numbers represented by points on a number line.	To be successful with identifying the whole numbers represented by points on a number line, students need to be able to count by ones to 100 starting and ending with any number in the sequence. They should also be able to identify counting patterns.
Graph Whole Numbers	Students will place points on a number line to represent whole number sums and differences.	To be successful with placing points on a number line to represent sums and differences, students need to be able to identify whole numbers represented by points on a number line as well as find points on a number line given whole numbers. They should be able to count by ones to 100, starting and ending with any number in the sequence. Additionally, students should be able to add and subtract within 100.

Whole Numbers on a Line

Goal Students will place points on a number line to represent whole numbers.

Guided Support 15 MINUTES



Materials

- Number line assembled from *Make a Number Line 2* and *Make a Number Line 3* (1 per student)
- Scissors
- Glue
- Counters (3 colors if possible, 100 per student)
- Number cards (1–100, 1 set)
- Construction paper (2 pieces per student)

Begin the Activity

Distribute counters and a number line to each student. Explain that students will place the counters on tick marks to represent points on the number line. Point out the number labels and tell students that a point placed on a tick mark represents the number directly below the tick mark. Display the 10 number card. Count the tick marks from 1 to 10 with students. Then have them place a counter on the tick mark above 10 to represent 10. Count with students by tens from 10 to 100 as they point to each multiple of 10 on the number line.

Display the 20 number card. Have students place a counter on their number line to represent 20. If possible, use different colors for tens, fives, and ones. Continue this process until all the tens are plotted.

Display the 5 number card. Have students place a counter on the number line to represent 5. Count the tick marks from 1 through 5, if necessary. Count with students by fives from 5 to 100 as they point to each multiple of 5 on the number line.

Display the 15 number card. Have students place a counter on their number line to represent 15. Continue the process until all the fives are plotted. If necessary, cover parts of the number line with construction paper to focus attention on one section. For example, place one piece of construction paper over the number line just before 25 and the second piece on the number line just after 50 to focus on the 25 through 50 section.

Next, display the number cards for the numbers 1 through 9 and have students place a counter on the number line to represent each number as it is presented. Number cards can be presented in numerical order or in random order, depending on student needs. Continue displaying number cards for the remaining numbers on the number line for students to plot with counters. Present numbers within decades in numerical order, or in random order, depending upon student needs. When students are ready, present numbers in adjacent decades, then skip around on the number line. Continue to use the construction paper to focus attention on segments of the number line as needed.

Conclude the Activity

Discuss with students different strategies for placing points on a number line. Guide them to mention that tick marks are labeled with numbers and that a point on a tick mark represents the number below that tick mark. Possible strategies might include looking for patterns such as tens, fives, or ones.

Questions

- **How did you use counters in this activity?** Sample answer: I used counters to represent points on the number line. I placed a counter on the tick mark above a number to represent that number on the number line.
- **Explain how to place a point that represents 15 on the number line.** Sample answer: I would start at 5 and count by fives to find 15 on the number line. Then I would place a point on the tick mark directly above 15.
- **Explain how to place a point that represents 82 on the number line.** Sample answer: I would count by tens to find 80 on the number line. Then I would count by ones from 80 to 82. I would place a point on the tick mark directly above 82.

Graphs of Whole Numbers

Goal Students will identify whole numbers represented by points on a number line.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Scissors (1 per student)
- Glue sticks (1 per student)
- Blackline Master: *Blank Number Line* (2 per student)

Begin the Activity

Distribute one copy of *Blank Number Line*, scissors, and glue sticks to students. Point to the first blank number line and tell students that they will use it to make a number line that begins with 0 and shows whole numbers through 10. Point to the tick marks and explain that students will draw points on the tick marks and identify the whole number represented by each point. Have students cut out the first blank number line. **Point to the first tick mark after 0. Draw a point on that tick mark. What is the first whole number after 0? 1 What number does the point stand for? 1** Guide students to label the first point, 1. Continue this process to label each point from 2 to 10.

Have students cut out the second blank number line and glue it to the end of the 1 to 10 strip to form a 0 to 20 number line. Direct students to place a point on the last tick mark. **What number does this point stand for? 20 How do you know? Sample answer: I counted tick marks from 10.** Have students cut out the remaining blank number lines from *Blank Number Line*. As they glue each strip to the number line, have them place a point at the last tick mark and label the tick marks 30, 40, and 50.

Next, guide students to place a point at the tick mark halfway between 10 and 20. **What number does this point stand for? 15 How do you know? Sample answer: 15 is halfway between 10 and 20.** Repeat the process to have students place points and labels at 25, 35, and 45.

Have students place points on the tick marks for 21 and 28. **What number does the first point stand for? 21 How do you know? Sample answer: The first whole number after 20 is 21.** Have them label the point at 21. **What number does the second point stand for? 28 How do you know? Sample answer: I counted tick marks beginning at 25: 25, 26, 27, 28.** Have them label the point at 28. Next have students label points on the tick marks for 33, 36, and 37. **What number does the first point stand for? 33 How do you know? Sample answer: I counted tick marks beginning at 30: 31, 32, 33. What numbers do the next two points stand for? 36 and 37 How do you know? Sample answer: I counted along the number line and these are the first two whole numbers after 35.** Repeat this process to have students label more points on the number line.

Conclude the Activity

As time permits, have students cut out strips from the second copy of *Blank Number Line* to complete the number line through 100. Depending on the needs of your students, you might have them place points and label each consecutive decade either in numerical order or random order. Or if your students are ready, you might have them place points and label decades in different sections of the number line or even label points within different decades.

Questions

Display a section of number line 61 to 80 with 61, 65, 70, and 80 labeled.

- Draw a point at 63. **What number does this point stand for? 63** **How do you know? Sample answer: I counted whole numbers beginning at 61: 61, 62, 63.**
- Draw a point at 75. **What number does this point stand for? 75** **How do you know? Sample answer: It is halfway between 70 and 80.**
- Draw points at 66 and 67. **What numbers do these points stand for? 66, 67** **How do you know? Sample answer: They are the first two whole numbers after 65.**

Graph Whole Numbers

Goal Students will place points on a number line to represent whole number sums and differences.

Guided Support 15 MINUTES



Materials

- Counters (2 per student)
- Blackline Master: *Make a Number Line 2* (1 per student)
- Blackline Master: *Make a Number Line 3* (1 per student)

Begin the Activity

Assemble a number line for each student prior to the activity. Distribute one 0–50 number line and counters to each student. Tell students that they will practice adding and subtracting on a number line. Write $15 + 5$ on the board. Tell students to find the sum of $15 + 5$ on their number lines. **At what number will we start? 15** **How can we use a counter to show that point? Sample answer: Place a counter on the tick mark that represents 15 on the number line.** Monitor students to see that the counters are placed correctly.

Remind students that the second number in the sentence is the number they will add. **How many will we add? 5** Remind students to add on a number line they move or “hop” to the right. Guide students to use their pointer finger to make 5 hops on the number line. Have them start at 15 and touch each tick mark in turn, moving left to right, as they count the 5 hops together. **What number are we pointing to? 20** **What is the sum of $15 + 5$? 20** **How can we use a counter to show the sum? Sample answer: Place a counter on the tick mark above 20 on the number line.** Have students place the counter on 20.

Write $34 + 12$ on the board. Have students follow the same process to find the sum. Monitor students to ensure that they start at the correct number, make the correct number of hops in the correct direction on the number line, and place the point on the number that shows the sum. Repeat with other sums within 50 until students are proficient at finding sums on the 0–50 number line.

Write $18 - 6$ on the board. Tell students that they will find the difference of $18 - 6$ on their number lines. Remind students to hop back on a number line to subtract. **At what number will we start? 18** How can we use a counter to show that point? **Sample answer: Place a counter on the tick mark that represents 18 on the number line.** Monitor students to see that the counters are placed correctly. **How many will we subtract? 6** How can we subtract 6 from 18 on the number line? **Sample answer: Start at 18 and make 6 hops back on the number line.** Guide students to use their pointer finger to make the hops on the number line. Have them start at 18 and touch each tick mark in turn, moving right to left, as they count the 6 hops together. **What number are we pointing to? 12** **What is $18 - 6$? 12** How can we use a counter to show the difference? **Sample answer: Place a counter on the tick mark that represents 12 on the number line.** Have students place the counter on 12.

Write $25 - 8$ on the board. Have students follow the same process to find the difference. Monitor students to ensure that they correctly place the point at the starting number, make the correct number of hops in the correct direction on the number line, and place the point on the number that shows the difference. Repeat with other differences within 50 until students are proficient at finding differences on the 0–50 number line.

Conclude the Activity

Assemble a 51–100 number line for each student before the activity. Distribute a 51–100 number line to each student. Tell students they will use counters to add and subtract on the number line. Present addition and subtraction problems such as the following: $52 + 8$ **60**; $65 - 5$ **60**; $83 - 7$ **76**; $61 + 12$ [**73**]; $83 + 14$ **97**; $95 - 8$ **87**; $78 - 6$ **72**; $74 + 11$ **85**.

Questions

Have students use number lines and counters to answer the questions.

- Which point on the number line shows the sum of $52 + 8$? **60**
- Explain how to show $61 + 12$ on the number line. **Sample answer: Place a point at 61. Count or hop forward on the number line twelve times. The last hop will end at 73 so place a point at 73.**
- Explain how to show $95 - 8$ on the number line. **Sample answer: Place a point at 95. Count or hop back on the number line eight times. The last hop will end at 87 so place a point at 87.**

Introduction to Fractional Numbers

Mathematical Context

In this module, students relate the numeric representation of fractions to visual and word representations. Students have been discovering equal parts of wholes through models and drawings. They have become familiar with the language of fractions when dividing squares, circles, and rectangles, and have been using the words *halves*, *thirds*, and *fourths*. This foundation will prepare students for reading, writing, and comparing with them.

Vocabulary

denominator
equal parts
equivalent fractions
fraction
number line
numerator
unit fraction

Standards

See *online module information or Program Overview for standards alignment.*

Lesson Title	Goal	Prerequisite Skills
Equal Parts of a Whole	Students will demonstrate understanding of equal parts of a whole.	To be successful with understanding parts of a whole, students need to be able to identify basic shapes and recognize the difference between same-size and different-size.
Understand Unit Fractions	Students will identify unit fractions.	To be successful with identifying unit fractions, students need to be able to divide objects into equal parts, and understand that fractions show parts of the whole.
Fractions with Numerators Greater Than 1	Students will demonstrate understanding of fractions with numerators greater than 1.	To be successful with fractions with numerators greater than 1, students must be familiar with unit fractions and interpreting fractions from visual representations.
Fractions Equivalent to 1 Whole	Students will recognize and generate fractions equal to one.	To be successful with recognizing and generating fractions equal to one, students need to be able to identify both the numerator and the denominator of a fraction.
Intervals on a Number Line	Student will demonstrate understanding of number line intervals.	To be successful with partitioning a number line into fractional parts, students should have an understanding of numerator and denominator. Students also build on their ability to partition shapes into equal parts.
Unit Fractions on a Number Line	Students will demonstrate understanding of unit fractions on a number line.	To be successful with unit fractions on a number line, students need to be able to understand unit fractions, and how to place fractions on a number line.

Lesson Title	Goal	Prerequisite Skills
Fractions on a Number Line	Students will identify fractions on a number line.	To be successful with fractions on a number line, students need to be able to understand the terms numerator and denominator. Students must also have experience working with intervals on number lines.
Fractions Greater Than 1 on Number Lines	Students will identify fractions greater than 1 on a number line.	To be successful with fractions greater than one on a number line, students need to have an understanding of the relationship between the numerator and denominator. Students must also have experience working with intervals on number lines and labeling a number line with fractions less than one.
Mixed Numbers on a Number Line	Students will identify mixed numbers on a number line.	To be successful with identifying numbers greater than 1 on the number line as a mixed number, students must be familiar with number lines divided into fractional units, such as halves, fourths, thirds, and eighths. Students should also be able to label the points up to 1 as fractions and to continue labeling the points greater than 1 as (improper) fractions, such as five fourths, six fourths, and so on.

Equal Parts of a Whole

Goal Students will demonstrate understanding of equal parts of a whole.

Guided Support 15 MINUTES



Materials

- Paper rectangles (4 for teacher and 6 per student)
- Scissors (1 pair per student)

Begin the Activity

Show students one of the paper rectangles. Tell them to pretend that it is a tray of lasagna. Tell them that you want to share it with four people. Cut the paper rectangle into four parts that are different shapes and sizes. Explain that this is not a fair way to share the lasagna because the parts are different sizes and each person would not get the same amount.

Cut another sheet of paper into four equal parts and explain that this is a fair way to share the lasagna because the parts are all the same size. Since the parts are the same size, they are equal. The lasagna is divided into four equal parts. The four equal parts are called fourths. Compare these parts to the first model and explain that even though there are four parts in the first model, they are not fourths because they are not the same size.

Cut a paper rectangle into sixths and another into six unequal parts. Work with students to identify the parts of each as equal or unequal. Guide them to explain that equal parts are the same size. Use the terms fair and unfair to help the students differentiate equal and unequal parts of a whole. Tell students that the six equal parts of the whole are called sixths.

Work with the students to model halves and two unequal parts, thirds and three unequal parts, and eighths and eight unequal parts. For each, name the fractions for the equal parts and point out that the unequal parts are not named by the fractions.

Conclude the Activity

Display the models. Point to them one at a time and ask students to identify the parts of each as equal or unequal. For the models with equal parts ask them the fraction name of the equal parts.

Questions

- Which of these models show thirds? Students point out the model with three equal parts.
- Point to the model with three unequal parts. Why does this model not show thirds? The parts are not all the same size.
- Point to the model showing sixths. What are the six equal parts of this model called? sixths
- How can you tell if the parts of a whole are equal? Sample answer: Equal parts are the same size.
- How many equal parts does a model that shows eighths have? 8

Understand Unit Fractions

Goal Students will identify unit fractions.

Guided Support 15 MINUTES



Materials

- Construction paper (1 piece per student and 1 piece for teacher)
- Connecting cubes (8 per student)

Begin the Activity

Review the terms *fractions*, *unit fractions*, *numerator*, and *denominator*. Explain to students, using a piece of construction paper, that they will see how a whole can be divided into equal parts, and how those parts relate to the whole. Explain that connecting cubes can also show how one part of a whole is a unit fraction.

Give each student a sheet of construction paper. Explain that this is our whole, since it is the whole sheet of paper. Fold your piece of paper in half, and ask, **How many parts now make up the whole?** **2** Have students fold their paper as well so they can see how the whole is divided into smaller parts, yet is still the whole.

Model how to fold the paper in half again, and discuss how many parts the whole sheet is divided into. **4** Discuss how the number of parts, when together, still makes up the whole. Have students count the number of parts, and write the denominator on the board. Cover one of the parts, ask, **How many parts am I covering?** **1**, and write that as the numerator. Discuss how the numerator tells us the number of parts covered and the denominator tells how many total parts. Repeat with folding to make eighths as well.

Distribute eight connecting cubes to each student. Have them link all eight together. Explain this is similar to the sheet of paper, and represents one whole. Ask, **How many parts make up the whole?** **8** Write the denominator on the board, and have the students take one cube and pull it away slightly from the whole. Ask, **How many pieces did you take away?** **1** Write this as the numerator and discuss how the numerator tells or represents the number of pieces that were taken. Explain a fraction that has 1 in the numerator is called a unit fraction, because it tells what one unit of the whole looks like. Repeat using the number 6 as the whole, then 4, 3, 2, etc...

Conclude the Activity

After finishing the activity, ask the students to explain what a unit fraction is and what it can tell about the fraction and the whole.

Questions

- Where do we write the total number of equal parts in the whole? Sample answers: on the bottom; in the denominator.
- What does the top number, the numerator, tell us? Sample answer: It tells the number of equal parts taken or used.
- What does a unit fraction tell us? Sample answer: It tells what one piece of the whole will look like; it always has a 1 in the numerator.

Fractions with Numerators Greater Than 1

Goal Students will demonstrate understanding of fractions with numerators greater than 1.

Guided Support 15 MINUTES



Materials

- Blackline Master: *Matching Fractions to Pictures* (1 per student)
- Scissors (1 pair per student)
- Colored markers (various colors per student)

Begin the Activity

Explain that we can use fractions to name parts of a whole. Distribute *Matching Fractions to Pictures*. Direct students to the first shape, which is a rectangle divided in half. Ask students to choose a color to shade one part of the whole. Tell students that, in the next box, they are going to name the shaded part with a fraction. Direct students to the box to the right of the rectangle. Ask students to point to the small box above the fraction bar. Tell students that this is where we will write the numerator. Write the word “numerator” on the board. The numerator tells us how many parts are used or shaded. **How many parts are shaded?** 1 Have students write “1” in the box above the fraction bar. Write a 1 with a fraction bar below it next to the word “numerator” on the board.

Have students point to the small box below the fraction bar. Tell students that this is where they will write the denominator. Write the word “denominator” on the board below the word “numerator”. The denominator tells us how many total parts are in the whole. **How many total parts are in the whole rectangle?** 2 Have students write “2” in the box below the fraction bar. Write a 2 below the fraction bar on the board. Explain that the name for the shaded part of this shape is *one-half*.

Repeat this questioning with the rectangles divided into thirds. First, students will shade $\frac{1}{3}$ of the rectangle and write the fraction $\frac{1}{3}$ in the box to the right. Then students will shade $\frac{2}{3}$ of the rectangle and write the fraction $\frac{2}{3}$ in the box to the right.

Repeat this questioning with the rectangles divided into fourths. Students shade parts of the rectangles and name fractions for $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$.

Repeat this questioning with the rectangles divided into eighths. Students can choose the amount of eighths to shade and then name the fraction.

When students are finished shading and naming, they can out the cards, separating each of the rectangles away from the fractions. Then students can mix up the cards and match the fraction to the picture it describes.

Check each match to informally assess understanding and correct errors.

Conclude the Activity

After students have had a chance to match their fractions to the pictures they describe, have students stack their cards and put them aside.

Questions

- Choose a fraction and its matching picture from one of the student's stacks. Display the pair of cards. Point to the numerator. **What does the numerator tell us about this shape?** **Sample answer: The numerator tells me how many parts of the shape are shaded.**
- Use the same fraction and matching picture. Point to the denominator. **What does the denominator tell us about this shape?** **Sample answer: The denominator tells me into how many total parts the shape has.**
- Choose one of the student's picture cards to display. **What fraction names the shaded parts of this shape?** **Sample answer: (This will vary depending on the picture card chosen.) For the picture of a rectangle with two-thirds shaded, the fraction $\frac{2}{3}$ names the shaded parts.**

Fractions Equivalent to 1 Whole

Goal Students will recognize and generate fractions equal to one.

Guided Support 15 MINUTES



Materials

- Blackline Master: *Number Lines for Equivalent Fractions* (1 per student)

Begin the Activity

Display *Number Lines for Equivalent Fractions*. Explain that you will divide the first number line into fourths. Assist students in dividing the first number line into fourths. Demonstrate labeling the intervals on the first number line ($\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$). Assist students in labeling their number lines.

Next, explain that you will divide the second number line in half. Assist students in dividing the second number line in half. Demonstrate labeling the intervals on the second number line ($\frac{1}{2}, \frac{2}{2}$). Assist students in labeling their number lines.

Which fraction on the first number line is equal to one whole? $\frac{4}{4}$ Encourage students to identify that $\frac{4}{4}$ equals 1 whole on the first number line. You may need to remind students that the denominator represents how many parts the number line is divided into and the numerator represents the number of parts accounted for. All the parts are needed to make 1 whole.

Which fraction on the second number line is equal to one whole? $\frac{2}{2}$ Encourage students to identify that $\frac{2}{2}$ equals 1 whole on the second number line.

What are two names for one whole? $\frac{4}{4}$ and $\frac{2}{2}$ Help students recognize that $\frac{4}{4}$ and $\frac{2}{2}$ are two names for one whole.

Explain that you will divide the third number line into thirds. Assist the students in dividing their third number line into thirds. Have students label the first two fractions on this number line.

What is another name for one whole on the number line? $\frac{3}{3}$ Write $\frac{3}{3}$ underneath 1 whole on the third number line.

Conclude the Activity

Have students divide the fourth number line into fifths and label it. Encourage students to share any patterns that they noticed. Help students notice that both the numerator and denominator are the same in fractions that equal one whole.

Questions

- What are some fractions that equal one whole? Sample answer: $\frac{3}{3}$, $\frac{6}{6}$
- How can you tell if a fraction is equivalent to one whole without using a number line? Sample answer: Fractions equal to 1 whole have the same number in both the numerator and the denominator.

Intervals on a Number Line

Goal Students can demonstrate understanding of number line intervals.

Guided Support 15 MINUTES



Materials

- Long rectangular paper strips (4 per student and 1 for teacher use)

Begin the Activity

Begin by reviewing the terms fraction, numerator and denominator. Tell students you will guide them through an activity that will show them fractions on strips of paper that will serve as number lines. Tell students they will practice folding the rectangles into different fractions.

Hand out four rectangular strips of construction paper to each student. Tell students to put three of the strips aside. Have students hold up one rectangular strip. Tell them that this strip will represent a number line from 0 to 1. Have students write 0 on one end of the strip, and 1 on the opposite end, close to the edges of the paper.

Ask students to fold the strip of paper in half, lining up the ends so that the halves are of equal size. Have students draw a line down the center of the fold. **Into how many equal parts is the number line divided? 2** Tell students that they have divided the whole number line in half. Each side of the fold shows one half of the number line. Now ask students to write the fraction $\frac{1}{2}$ in the each section of the number line/rectangle.

Next, using another strip of paper, students will show fourths. Remind students that the paper strip represents a number line from 0 to 1. Have students write 0 on one end of the strip, and 1 on the opposite end, close to the edges of the paper. Ask students to fold their number line into 2 equal parts by folding it down the middle and aligning the ends. Then fold it again. **Into how many equal parts is the number line divided? 4** Explain that this tells us the denominator. **What is the name of each equal part? $\frac{1}{4}$** Have students write the fraction $\frac{1}{4}$ in each section of the number line/rectangle. Have students fold and label each of the remaining paper strips to show the fractions $\frac{1}{8}$ and then $\frac{1}{3}$. Repeat the questions as needed.

Have students place their number lines in front of them. Review the fraction names of each one. For example, the number line divided into two equal parts is divided into halves. Ask students to point to each paper strip that represents different fraction names.

Conclude the Activity

Draw a number line from 0 to 1. Divide the number line into thirds. Determine how many equal parts there are in the number line by counting aloud along with students. Discuss with students what each interval or part should be labeled and label it correctly. Discuss with students what fraction name can be given to this number line.

Questions

- Draw a number line from 0 to 1 and divide it into fourths. **What fraction name can we give this number line? fourths**
- **What can we label each part or interval on this number line? $\frac{1}{4}$**
- **How do you determine what the denominator of a fraction is? Sample answer: By counting how many equal parts the line has been divided into.**

Unit Fractions on a Number Line

Goal Students will demonstrate understanding of unit fractions on a number line.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Unlined $8\frac{1}{2}$ " \times 11" paper, cut into 3" strips (5 per student)

Begin the Activity

Review the terms fraction, numerator, denominator, and unit fraction with students. Hold up a strip of the paper. Tell students the strip represents the number line between 0 and 1, with the left end of the strip being 0 and the right end being 1. Have students label one of their strips with 0 and 1. Demonstrate showing $\frac{1}{2}$ on a strip by folding the paper in half to make 2 equal parts. Point out how this fold line indicates $\frac{1}{2}$. Demonstrate how to label this fraction. Connect student understanding of the parts of the strip to the terms numerator, denominator, and unit fraction.

Use the remaining four strips to model the fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$. Hold up another strip of paper. Tell students that the strip represents a number line from 0 to 1. Have students label 0 and 1 on either end of the strip. Make sure students write the numbers 0 and 1 close to the end of the strip. Ask students to fold the strip into thirds. Identify the middle third of the strip of paper; each end of the middle $\frac{1}{3}$ will be a fold line. Fold on the fold line from each end of the strip toward the center. **Into how many equal parts is the strip divided? 3** Have students locate the first fold line of the strip. Ask them to think about how they would label this fold line with the fractional part it represents. **What is the denominator of the fraction? 3 Why? There are three equal parts. What is the numerator of the fraction? 1 Why? This is the first section of the number line. It represents one of the three parts. What fraction should we write to label the first fold line on the strip? $\frac{1}{3}$**

Repeat by modeling the fractions $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$ using the remaining strips of paper.

Conclude the Activity

After students build their number lines, have them place one on top of the other. Discuss and identify the numerator and denominator in each fraction. Ask students to compare the fractions on each strip. Encourage them to connect the number of parts on the strip to the denominator for each of the fractions.

Questions

- Name the unit fraction that we labeled on the strips of paper. Sample answer: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{8}$.
- What does the denominator of a unit fraction tell us? How many equal parts the number line or strip is divided into.
- What is the numerator of a unit fraction? Sample answer: It will always be 1.

Repeat questions as necessary for $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{8}$.

Fractions on a Number Line

Goal Students will identify fractions on a number line.

Guided Support 15 MINUTES



Materials

- Construction paper, each piece cut into 5 long, thin rectangles (1 sheet per student)
- Ruler (1 per student)

Begin the Activity

Review the terms fraction, numerator, denominator, and number line. Explain to students that they are going to label fractions on a number line.

Present students with the long, thin rectangles cut from construction paper. Have each student pick up one rectangle and explain that this paper represents a number line. Start by folding the paper in half. Discuss with students how they should label the number line to represent the fold they just made. Have students use a ruler to draw a straight line lengthwise through the center of their construction paper. Guide students through drawing tick marks to represent each fraction. Lead students in labeling $\frac{0}{2}$, $\frac{1}{2}$, and $\frac{2}{2}$. Explain to students how $\frac{0}{2}$ can simply be written as 0, and how $\frac{2}{2}$ can be written as 1.

Have each student pick up another rectangle of construction paper. Students should fold in half and then in half again. Have students open up their rectangles and count how many equal parts they created. Lead students to understand that they created 4 equal sections, so the denominator in all of their fractions will be 4.

Have students again use a ruler to draw a straight line lengthwise through the center of their construction paper.

Guide students through drawing tick marks to represent each fraction. Point to the tick mark for $\frac{3}{4}$ and have students draw a point. Ask students what fraction is located at that point. Explain that labeling the number line will help to identify the fraction. Walk students through the steps to label the number line and identify the fraction.

Repeat the process with thirds, sixths, and eighths using different strips of construction paper for each.

Conclude the Activity

Discuss with students the process they used to divide the pieces of construction paper into equal parts so that they could show fractions on their number lines.

Questions

- For the fraction $\frac{1}{4}$, what is the numerator? 1
- For the fraction $\frac{1}{8}$, what is the denominator? 8
- If you are labeling a number line in eighths, what fraction can you use for 1? $\frac{8}{8}$
- How many equal parts did you make when you folded your paper in half? 2

Fractions Greater Than 1 on Number Lines

Goal Students can identify fractions greater than 1 on a number line.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Construction paper, cut into four long, thin rectangles (1 sheet per student)

Begin the Activity

Tell students they will be making number lines with construction paper to represent fractions. Give each student two pieces of construction paper. Tell the students that they will fold the paper to create fractions on the number line. Model folding the first piece of paper in half and then in half again to create four equal parts. Assist students in folding their paper into fourths.

Discuss how to label the parts of the whole rectangle (1 whole) with the appropriate fractions. Label each fourth of the paper $\frac{0}{4}, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$.

Assist students in folding the second piece of paper into fourths. Have students place the second number line to the right of the first number line. Explain that the first number line represented fractions up to one whole. The second number line continues the first number line.

Discuss how to label the parts of the second rectangle with appropriate fractions. Label each fourth of the paper $\frac{5}{4}, \frac{6}{4}, \frac{7}{4}, \frac{8}{4}$.

Repeat these steps with additional fractions, such as halves or eighths. Progress towards leaving the number lines unlabeled. Place a point on one of the number lines and have students count aloud as they move along the number line to identify the point. Then give students a fraction and have them count aloud as they move along the number line to find the location of the given fraction.

Conclude the Activity

Count through the labeled and unlabeled number lines aloud with students as you point to the location of each fraction on the number line.

Questions

- How does counting help you label the fractions on the number line? **Sample answer:** The numerator always increases by one, just like counting up.
- What do you notice about fractions greater than one? **Sample answer:** The numerator is greater than the denominator.

Mixed Numbers on a Number Line

Goal Students will identify mixed numbers on a number line.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Construction paper, cut into long, thin strips (6 strips per student)

Begin the Activity

Distribute three strips of construction paper of the same color to each student. Explain that number lines will be created with the strips and will be used to identify numbers between whole numbers. Place two strips together, end to end, on a table to show a number line made up of two wholes. Next, demonstrate how to fold one strip into fourths by folding a strip in half and, without unfolding, fold once again. When opened up, the strip will show fourths. Guide students to count the fourths, “one-fourth, two-fourths, three-fourths, four-fourths.” Explain that four fourths make one whole. Have students fold a second strip into fourths and place it end-to-end with the first strip. Count the fourths together saying, “one-fourth, two-fourths, three-fourths, one, one and one-fourth, one and two-fourths, one and three-fourths.” What comes next? **two**

Write the fractions on the board: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, 1, $1\frac{1}{4}$, $1\frac{2}{4}$, $1\frac{3}{4}$, 2. Finally, ask students to fold the third strip, place it at the end of the first two strips, and count again. Continue writing the numbers counted on the board: $2\frac{1}{4}$, $2\frac{2}{4}$, $2\frac{3}{4}$, 3.

Repeat with other fractions. You can fold the same strips once again to show eighths. Then distribute three strips, already folded into thirds, to each student. Repeat the procedure to demonstrate counting numbers greater than one by thirds: $\frac{1}{3}$, $\frac{2}{3}$, 1, $1\frac{1}{3}$, $1\frac{2}{3}$, 2, $2\frac{1}{3}$, $2\frac{2}{3}$, 3.

Conclude the Activity

Using the strips as number lines, ask a student to point out where $1\frac{1}{4}$ is. Repeat for other fractions greater than one. Then point to a fold, for example $2\frac{2}{4}$, and ask what number it represents on the number line.

Questions

- Explain how you count on a fourths number line up to 2. **Sample answer:** I count the number of fourths until I get to one whole. Then I say the one with the next numbers and repeat the fractions $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, until I get to 2.
- How would you find “two and three-fourths” on a fourths strip? **Sample answer:** I count the number of wholes, which is two, and then count number of fourths to get to three fourths.

MODULE

Model Equivalent Fractions

Mathematical Context

In this module, students extend their understanding of fractions to recognizing and generating simple equivalent fractions, including fractions equal to a whole number. Students may have prior knowledge of fractions less than one on a number line and a basic knowledge of fraction meanings and representations. Learning about fractions equivalent to one can build upon an understanding that fractions have equivalents—that is, different fractions can stand for the same amount (quantity, size). This foundation will prepare students to compare, order, add, and subtract fractions.

Vocabulary

equivalent fractions
number line

Standards

See *online module information* or *Program Overview for standards alignment*.

Lesson Title	Goal	Prerequisite Skills
Equivalent Fractions with Models	Students will make equivalent fractions using models.	To be successful with equivalent fractions, students must understand that fractions represent part of a whole and are able to compare fractions to determine if fractions are less than, greater than, or equal to their counterparts.
Equivalent Fractions Using Number Lines	Students will use a number line to find equivalent fractions.	To be successful with understanding equivalent fractions, students must be able to recognize the numerator and denominator, understand equivalence as the same amount, and recognize fractions on a number line.
Recognize Equivalent Fractions	Students will recognize simple equivalent fractions.	To be successful with recognizing simple equivalent fractions, students must know how to identify the numerator and denominator of a fraction. Students should have experience recognizing and generating equivalent fractions with the support of a visual model.
Equivalent Fractions with Whole Numbers	Students will recognize fractions equal to a whole number.	To be successful with finding equivalent whole numbers and fractions, students must be able to understand the meaning of a numerator and denominator, identify fractions on a number line, identify fractions equal to one, and understand fractions as <i>improper fractions</i> . However, the term <i>improper fraction</i> is not yet introduced.

Equivalent Fractions with Models

Goal Students will make equivalent fractions using models.

Guided Support 15 MINUTES



Materials

- Same-sized rectangular strips of paper (3 per student, 3 per instructor)

Begin the Activity

Remind students that **equivalent fractions** are fractions that represent the same amount of the whole. Distribute three same-sized rectangular strips of paper to each student. As the instructor, make sure you have three strips of paper to complete the activity with students. Explain to students that they will use strips of paper to model different fractions. Then, they will compare the models they constructed to test for equivalence.

Have students complete the following:

- Take one strip of paper, and fold it in half lengthwise.
- Label each part of the strip $\frac{1}{2}$.
- Take the second strip of paper, and fold it in half lengthwise. (Emphasize accuracy, so that the folds on both strips of paper align.)
- Fold the second strip in half lengthwise again to create four equal sections. Label each section $\frac{1}{4}$.

Next, have students align the strips of paper on the left, one above the other, and ask them to identify how many fourths there are in one half. Then ask them to complete this number sentence: $\frac{1}{2} = \frac{?}{4}$. $\frac{2}{4}$ Have students identify these fractions as equivalent fractions. Then ask students to use their models to show and explain how they know these fractions are equivalent.

Conclude the Activity

Ask students to take the remaining strip of paper and fold it into halves lengthwise. Then fold each half again lengthwise into two equal parts to make fourths. Finally, fold each $\frac{1}{4}$ into two equal parts to make eighths. Label each part $\frac{1}{8}$. Place the three strips of paper one above the other, making sure to align the left sides.

Questions

- Using the fractional bar models you created, show how the fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are equivalent. Explain your answer. **Sample answer:** You can see by where the fractions line up that $\frac{1}{2}$ is the same amount as $\frac{2}{4}$ and $\frac{4}{8}$. Because these fractions take up the same amount of space, they are equivalent fractions.
- Using the models you created, what fraction is equivalent to $\frac{3}{4}$? **Sample answer:** $\frac{3}{4}$ lines up with $\frac{6}{8}$ so $\frac{3}{4}$ is equivalent to $\frac{6}{8}$.
- Using the models you created, can you find another fraction that is equivalent to $\frac{1}{4}$? **Sample answer:** $\frac{2}{8}$.

Equivalent Fractions Using Number Lines

Goal Students will use a number line to find equivalent fractions.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Fraction tiles (1 set of all tiles to eighths per student)
- Blackline Master: *Number Lines (0–1)*

Begin the Activity

Present the one-whole fraction tile to students. Explain it is one whole.

Discuss with students what it could represent (1 whole cheese stick, 1 whole carrot stick, etc.). Then have students place two $\frac{1}{2}$ -fraction tiles equally aligned on top of the whole fraction tile. Discuss with students what they observe, such as two halves make one whole. Repeat with the $\frac{1}{4}$ -fraction tiles on top of the whole fraction bar. Explain that each fraction tile is one part of a whole. Then present *Number Lines (0–1)*.

First Number Line:

- Refer to the first number line. Have students place the one-whole fraction tile above the number line. Guide students to understand that the number line from 0 to 1 represents one whole, just as the one-whole fraction tile does. Have students make a tick mark on the line at both ends of the one-whole fraction tile to indicate the one whole on the number line. Tell students to mark the first tick mark 0 and the second tick mark 1. Remove the fraction tile.
- Have students place two $\frac{1}{2}$ -fraction tiles (side-by-side) along the line between the 0 and 1 tick marks. Explain that two $\frac{1}{2}$ -fraction tiles make up the whole number line. Tell students that instead of using tiles to mark the halves, we use tick marks.
- Have students remove the second $\frac{1}{2}$ -tile, leaving the first $\frac{1}{2}$ -tile along the line. At the end of the first tile, have students draw a tick mark on the number line and remove the tile. Explain this marks $\frac{1}{2}$ of the number line. Students should label the tick mark $\frac{1}{2}$. Explain that we know this marks $\frac{1}{2}$, because it shows one part out of two total parts.
- Have students point to each section on the number line, not the tick marks, to count the parts of this number line. Counting the tick marks to determine the denominator is a common mistake made by many students. Ensure that students are counting the sections between tick marks.

Second Number Line:

- Refer to the second number line. Have students place the one-whole fraction tile along the number line and make a tick mark on the line at both ends of the 1 whole fraction tile to indicate the one whole on the number line. Tell students to mark the first tick mark 0 and the second tick mark 1. Have students use the $\frac{1}{4}$ -tile to repeat the process of creating tick marks on the second number line.
- As you count the parts of this number line with the students, have them add a fraction tile to each section as you count. As they add a tile, help students label the tick marks by referring to the number of tiles they have placed along the number line. For example, label the third tick mark $\frac{3}{4}$, because the students have placed three $\frac{1}{4}$ -tiles on the number line.
- After labeling each tick mark, remove all the tiles.

Third Number Line:

- Refer to the third number line. Have students place the one-whole fraction tile along the number line. Make a tick mark on the line at both ends of the one-whole fraction tile to indicate the one whole on the number line. Tell students to mark the first tick mark 0 and the second tick mark 1. Have students use the $\frac{1}{8}$ -tile to repeat this process of creating tick marks on the third number line.
- As you count the parts of this number line with students, have them add a fraction tile to each section as you count. As they add a tile, help students label the tick marks by referring to the number of tiles they have placed along the number line. For example, label the third tick mark $\frac{3}{8}$, because the students have placed three $\frac{1}{8}$ -tiles on the number line.
- After labeling each tick mark, remove all the tiles.

Demonstrate forming equivalent fractions. Explain that equivalent fractions are fractions that may look different, but equal the same amount of the whole. [How many fourths are equal to one half?](#) Have students take one $\frac{1}{2}$ -tile and place it along the first number line between the 0- and $\frac{1}{2}$ -tick marks. Explain that this shows $\frac{1}{2}$. Have students experiment with $\frac{1}{4}$ tiles along the second number line to create an equivalent fraction.

Guide students to correctly place two $\frac{1}{4}$ -tiles along the second number line. **How do you know these fractions are equal?** Students may line up the end of the tiles or place them on top of each other to check. Point out that the $\frac{1}{2}$ -tick mark is at the same point on the first number line as the $\frac{2}{4}$ -tick mark is on the second. Mark each tick mark with a dot and remove the tiles. Explain that these dots represent points. Points can be used to mark fractions instead of tiles. **How can we write $\frac{1}{2}$ and $\frac{2}{4}$ to show that they are equivalent fractions?** After discussing, guide students to write $\frac{1}{2} = \frac{2}{4}$ in the box labeled *Equivalent Fractions*.

Now refer to the third number line, separated into eighths. **How many eighths are equal to two-fourths?** Have students experiment with tiles on the second and third number lines to represent these equivalent fractions. Once the students have represented these fractions using tiles, they should add a point to identify two-fourths and four-eighths. **How do you know these fractions are equal?** Guide students to understand that $\frac{4}{8}$ is at the same point on the second number line as $\frac{2}{4}$ is on the first number line. **How can we write $\frac{2}{4}$ and $\frac{4}{8}$ to show that they are equivalent fractions?** Guide students to write $\frac{2}{4} = \frac{4}{8}$ in the box labeled *Equivalent Fractions*.

Repeat with other representative examples, such as finding equivalent fractions for $\frac{1}{4}$, $\frac{3}{4}$, $\frac{2}{8}$, and $\frac{6}{8}$. Note: if further examples are needed, students will have to erase the dots drawn. Gradually reduce students' dependence on the tiles and encourage the use of the points to identify equivalent fractions.

Conclude the Activity

To conclude the lesson, review with students the meaning of equivalent fractions and how to determine equivalent fractions using points on number lines.

Questions

- **What are equivalent fractions? Equivalent fractions are fractions that are equal. They are the same part of the whole.**
- **How do you know fractions are equivalent on number lines? Fractions are equivalent when they are at the same point on the number lines.**

Recognize Equivalent Fractions

Goal Students will recognize simple equivalent fractions.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Fraction tiles (1 set per pair of students)

Begin the Activity

Explain to students that they will be looking for patterns in equivalent fractions. Stack one $\frac{1}{2}$ -tile on top of two $\frac{1}{4}$ -tiles. Explain that they both show the same amount of the whole. Explain that one half is the same size as two fourths, but two fourths is built with two pieces instead of one. Write $\frac{1}{2} = \frac{2}{4}$. Explain that these are equivalent fractions.

The two sets of fraction tiles should remain showing. Begin a new set of fraction tiles to the right of the first group. Place two $\frac{1}{3}$ -tiles down next to each other to build the fraction $\frac{2}{3}$. Then place four $\frac{1}{6}$ -fraction tiles below the tiles, representing $\frac{2}{3}$. **What equation can we write that shows these fractions are equal?** Write $\frac{2}{3} = \frac{4}{6}$ next to the other fraction equation.

Explain that equivalent fractions have patterns. Have students look at the numerators in the first and second set of equivalent fractions. **Do you notice any patterns in the numerators? The first numerator doubles to form the second.** Point out how 1 doubles to make 2 just as the one $\frac{1}{2}$ -tile was split to make two pieces. Also note how 2 doubles to make 4 just like the two $\frac{1}{3}$ -tiles were split to make four pieces. Have students look at the denominators in the first and second set of equivalent fractions. **Do you notice any patterns in the denominators? The first denominator doubles to form the second.** Point out how 2 doubles to make 4. Also note how 3 doubles to make 6. Review the pattern. Explain that when we double the numbers in the first fraction, they form the second fraction. This shows us that the fractions are equivalent. Erase the fractions and write the fractions $\frac{1}{2}$ and $\frac{3}{4}$. **Are these equivalent fractions? Why or Why not? No, because $\frac{1}{2} = \frac{2}{4}$, not $\frac{3}{4}$.** Have students verify their answers using fraction tiles.

Write $\frac{2}{4} = \frac{1}{2}$ and $\frac{4}{6} = \frac{2}{3}$. Explain that these are the same fractions we worked with before, but they are in a different order. Have students check that they are still equivalent by placing two $\frac{1}{4}$ -tiles down first and one $\frac{1}{2}$ below it. Repeat with the $\frac{1}{6}$ - and $\frac{1}{3}$ -tiles. Tell students that sometimes equivalent fractions have a different pattern than the “doubling” pattern you discussed. **Do you notice any patterns in the numerators? Take half of the first numerator to form the second.** Point out that 1 is half of 2 and 2 is half of 4. **Do you notice any patterns in the denominators? Take half of the first denominator to form the second.** Note that 2 is half of 4 and 3 is half of 6. Review the pattern. Explain that when we take half of each numbers in the first fraction they form the second fraction. This shows us that the fractions are equivalent. Erase the fractions and write the fractions $\frac{4}{6}$ and $\frac{1}{3}$. **Are these equivalent fractions? Why or Why not? No, because $\frac{4}{6} = \frac{2}{3}$, not $\frac{1}{3}$.** Have students verify their answer using fraction tiles.

Present students with additional fraction pairs and ask students to determine if they are equivalent or not. Students should offer a reason with their answer. Students should check their reasoning using tiles.

Conclude the Activity

Remind students that when there are no pictures, patterns can help us find equivalent fractions. Review the two patterns with the students.

Questions

- How do we know $\frac{1}{2} = \frac{2}{4}$ without using a picture or tiles? We know they are equal because the numbers in the first fraction double to make the second fraction.
- How do we know $\frac{4}{6} = \frac{2}{3}$ without using a picture or tiles? We know they are equal because we can take half of each number in the first fraction to make the second fraction.

Equivalent Fractions with Whole Numbers

Goal Students will recognize fractions equal to a whole number.

Guided Support 15 MINUTES



Materials

- Blackline Master: *Fractional Number Lines 0–3* (1 per student)
- Fraction tiles (1 set and 9 circular thirds per student)

Begin the Activity

Explain to students that they will be finding fractions that are equivalent to whole numbers.

Example 1

Give each student nine circular thirds fraction pieces. Ask students to assemble the pieces into one complete circle. Together with students, count the number of thirds they used to make one whole circle. Display the equation $1 = \frac{3}{3}$. Then write the equation $2 = \frac{\square}{\square}$. Have students assemble a second circle and count how many thirds equal two wholes. Complete the equation with the students.

$2 = \frac{6}{3}$ Have students assemble a third circle and count how many thirds equal three wholes. Write an equation that describes this model with the students.

$3 = \frac{9}{3}$

Example 2

Give each student twelve $\frac{1}{4}$ -fraction tiles. Display the one-whole bar. Have students assemble the $\frac{1}{4}$ -tiles to make one whole. Display the equation $1 = \frac{4}{4}$. Next, write the equation $2 = \frac{\square}{\square}$. Have students assemble a second whole and count how many fourths equal two wholes. Complete the equation with students. $2 = \frac{8}{4}$ Have students assemble a third whole and count how many fourths equal three wholes. With the students, write an equation that describes this model. $3 = \frac{12}{4}$

Example 3

Provide each student with *Fractional Number Lines 0–3*. Direct students to the first number line. **What whole numbers do you see on this number line? 0, 1, 2, and 3** Explain that the space between each whole number is one whole, just like the one-whole bar you worked with. Demonstrate this by placing the one whole on the number line between 0 and 1. Explain that sometimes space is divided into parts. **How many parts make up the space between 0 and 1? 2 parts** Demonstrate this by placing two $\frac{1}{2}$ -fraction tiles on the number line between 0 and 1. Tell students this means the number line is showing halves. Count in halves as students point to each tick mark and label the top of each tick mark. **What fraction is equal to the whole number 1? $\frac{2}{2}$ What fraction is equal to the whole number 2? $\frac{4}{2}$ What fraction is equal to the whole number 3? $\frac{6}{2}$** Repeat this process with the second and third number lines, labeling and identifying which fractions are equal to whole numbers. If necessary, use the support of fraction tiles on the number lines to help students identify each tick mark.

Second number line: 0, 1, 2, and 3; 3 parts; $\frac{3}{3}$, $\frac{6}{3}$, $\frac{9}{3}$

Third number line: 0, 1, 2, and 3; 4 parts; $\frac{4}{4}$, $\frac{8}{4}$, $\frac{12}{4}$

Conclude the Activity

Explain that there are many fractions that equal each whole number and that they found just a few of these.

Questions

- **What fractions did we find that equal 1? Sample answer: $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$.**
- **What fractions did we find that equal 2? Sample answer: $\frac{4}{2}$, $\frac{6}{3}$, and $\frac{8}{4}$.**
- **How can we find out how many sixths equal 2 wholes? Sample answer: We can use $\frac{1}{6}$ fraction tiles to create 2 wholes. We can use a number line divided into sixths and count how many sixths equal 2.**

Equivalent Fractional Numbers

Mathematical Context

In this module, students will build on their knowledge of decomposing fractions. They will learn that multiplying the numerator and the denominator of a fraction by the same non-zero number will result in a fraction equivalent to the original fraction. Students learn that multiplying the numerator and denominator of a fraction by the same non-zero number (n) corresponds to partitioning a unit fraction piece into (n) smaller pieces and build understanding that the inverse is true for division: dividing the numerator and denominator of a fraction by the same non-zero number (n) corresponds to partitioning unit fraction pieces into (n) larger pieces. Understanding equivalent fractions is a necessary precursor to the ability to adding and subtracting fractions with unlike denominators. This will also be useful as student develop an understanding of the relationship between numbers expressed as decimals and numbers expressed as fractions.

Vocabulary

equivalent fractions

Standards

See online module information or Program Overview for standards alignment.

Lesson Title	Goal	Prerequisite Skills
Equivalent Fractions with Multiplication	Students will use multiplication to recognize equivalent fractions.	To be successful with forming equivalent fractions using multiplication, students need to be able to fluently multiply.
Equivalent Fractions with Division	Students will recognize equivalent fractions with division.	To be successful with forming equivalent fractions using division, students need to be fluent with division facts and understand the relationship between multiplication and division.
Equivalent Fractions	Students will recognize and generate equivalent fractions with multiplication or division.	To be successful with multiplying and dividing to create equivalent fractions, students should be able to recognize equivalent fractions using models. Students should also be able to multiply and divide whole numbers.

Equivalent Fractions with Multiplication

Goal Students will use multiplication to recognize equivalent fractions.

Guided Support 15 MINUTES



Materials

- Paper strips (2 per student)
- Markers

Begin the Activity

Give each student a paper strip. Instruct students to fold the paper in half and color one half. **What fraction of the paper is colored? one half** Instruct students to fold the paper in half again to form fourths. **What fraction of the paper is colored? one half or two fourths** **How did the colored piece change? It changed from one piece to two pieces.** **Is the same amount of the paper colored? Yes, the amount of coloring has not changed.** Explain that this shows that two fourths is equivalent to one half.

Write $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$. Explain that each piece in the one half model was split into two pieces. Tell students this is what happens when each part of the fraction is multiplied by 2.

Distribute another strip of paper to each student. Instruct students to fold the paper in half and color one half. Write $\frac{1 \times 4}{2 \times 4} = ?$ Discuss with students what fraction one half will equal if we split each piece into four pieces. Remind students that when you multiply the numerator and denominator by 4, each piece is splitting into four pieces. Allow students to share their reasoning. Instruct students to fold their paper strip in half two more times. Students should now have a paper strip divided into 8 pieces with 4 pieces colored. **What fraction does one half equal when we split each part into four pieces? $\frac{4}{8}$**

Complete the written number sentence by writing $\frac{4}{8}$.

Conclude the Activity

Discuss with students how the same amount of the paper was colored each time you folded, but each piece was split into more pieces.

Questions

- **What happens when you multiply each part of a fraction by 2? Sample answer: Each piece is split into two pieces.**
- **When you multiply each part of a fraction by the same number, what does it equal? Sample answer: An equivalent fraction.**
- **What makes fractions equivalent? Sample answer: They have the same value or the same amount of the whole.**

Equivalent Fractions with Division

Goal Students will recognize equivalent fractions with division.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Fraction tiles (1 set per student)

Begin the Activity

Show students a $\frac{1}{2}$ fraction tile above two $\frac{1}{4}$ fraction tiles. Explain that we can show $\frac{1}{2}$ is equal to $\frac{2}{4}$ using an equal sign. Write $\frac{1}{2} = \frac{2}{4}$. **How does the numerator change from the first fraction to the second? 1 is multiplied by 2 to equal 2 in the second fraction. How does the denominator change from the first fraction to the second? 2 is multiplied by 2 to equal 4 in the second fraction.** Remind students we can use multiplication to form equivalent fractions.

Move the fraction tiles so the fraction $\frac{2}{4}$ is represented above $\frac{1}{2}$. **How does the numerator change from the first fraction to the second? 2 is divided by 2 to equal 1 in the second fraction. How does the denominator change from the first fraction to the second? 4 is divided by 2 to equal 2 in the second fraction.**

Explain that these two models are related. Explain that understanding multiplication can help you form equivalent fractions using division. Start with the second fraction and think about what you can multiply both parts by to equal the first. This is the same number that the parts of the first fraction can be divided by to form the second. Illustrate this idea by referencing the model. The same number you multiply by in the first model is the same number you divide by when $\frac{2}{4}$ is presented first.

Use other examples of equivalent fractions to explain this concept. Some equivalent pairs to model: $\frac{1}{2}$ and $\frac{3}{6}$, $\frac{2}{6}$ and $\frac{4}{12}$, $\frac{1}{4}$ and $\frac{3}{12}$, $\frac{4}{5}$ and $\frac{8}{10}$, $\frac{2}{4}$ and $\frac{6}{12}$. First display the fraction tiles to show moving from one fraction to the next using multiplication. Then display the tiles to show moving from one fraction to the next using division. Each time write the multiplication and division equations on the board so students can recognize the similarity.

Conclude the Activity

Have students consider $\frac{5}{10} = \frac{1}{2}$. Discuss how students can decide what each part of the first fraction is being divided by to equal the second.

Questions

- How do we know fractions are equivalent? Sample answer: They are the same amount of the whole. The set of tiles used to build each fraction are the same length.
- How does understanding multiplication help you when dividing to form an equivalent fraction? Sample answer: If you begin with the second fraction and work backwards toward the first, you can decide what number is being used to multiply and divide.

Equivalent Fractions

Goal Students will recognize and generate equivalent fractions with multiplication or division.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Fraction tiles (1 set per student)

Begin the Activity

Tell students that they will be using multiplication and division to find equivalent fractions. Explain that when multiplication is used to create an equivalent fraction, the number of pieces it takes to make one whole increases. The opposite is true for division. When division is used to create an equivalent fraction, the number of pieces it takes to make one whole decreases.

Write the equation " $\frac{1}{2} =$ " on the board. Pass out a set of fraction tiles to each student. Have students use the $\frac{1}{2}$ fraction tile piece to find other fractions equivalent to $\frac{1}{2}$. Ask students to name some of the fractions equal to $\frac{1}{2}$ and write them on the board. Discuss how multiplication can be used to create the fractions equal to $\frac{1}{2}$. Write the following example on the board: $\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$. Emphasize that the number used to multiply must be the same for the numerator and denominator.

Next take the fractions the students discovered were equal to $\frac{1}{2}$ and show how division can be used to create the fraction $\frac{1}{2}$. Write the following example on the board: $\frac{3 \div 2}{6 \div 2} = \frac{1}{2}$. Emphasize that the number used to divide must be the same for the numerator and denominator.

Conclude the Activity

Discuss with students ways that they can use multiplication and division to create equivalent fractions. Be sure students understand the fractions are still equal even though the number of pieces needed to create one whole increases or decreases by multiplying or dividing.

Questions

- Which operation is used to create equivalent fractions when the number of pieces used to make one whole increases? **multiplication**
- Which operation is used to create equivalent fractions when the number of pieces used to make one whole decreases? **division**
- How can you use multiplication or division to justify your answers about whether two fractions are equivalent? **Sample answer: If the numerator and denominator of one fraction can be multiplied by the same number or divided by the same number to equal the other fraction, then they are equivalent. If not, then they are not equivalent.**

Fractions as Quotients

Mathematical Context

In this module, understanding fractions as quotients will help students use fractions to solve real-world problems involving comparisons and grouping. Seeing fractional expressions as division helps students later recognize that ratios and percentages can be expressed using fractions. Expressing ratios as fractions is useful because equivalent ratios can be found by writing equivalent fractions.

Vocabulary

denominator divide (division)
fraction
numerator
quotient

Standards

See *online module information or Program Overview for standards alignment.*

Lesson Title	Goal	Prerequisite Skills
Quotient Fractions (Models)	Students will interpret a fraction as the division of the numerator by the denominator by using models.	To be successful with interpreting fractions as quotients using models, students need to be able to apply division to solving word problems. They also need to know the meaning of a fraction and of an improper fraction.
Quotient Fractions (Models & Equations)	Students will interpret a fraction as the division of the numerator by the denominator by using models and equations.	To be successful with interpreting fractions as equations involving division of the numerator by the denominator, students need to be able to use models to represent division and apply division to solving word problems.
Quotient Fractions (Word Problems)	Students will solve word problems involving division of whole numbers resulting in answers in the form of fractions or mixed numbers by using equations.	To be successful with interpreting fractions as quotients, students need to understand what the numerator and the denominator represent in a fraction. Students also need to have a basic understanding of the construction of a division equation.

Quotient Fractions (Models)

Goal Students will interpret a fraction as the division of the numerator by the denominator by using models.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Fraction circles: halves, fourths, sixths (2 of each per student)

Begin the Activity

Give each student 1 fraction circle divided into halves. Tell students to imagine this is an orange. Explain that they need to share this orange between 2 people. Tell students to split the orange into 2 groups. **How much of the orange is in each group?** $\frac{1}{2}$ Point out the numerator. **What does 1 tell us? 1 tells us that there is 1 orange or 1 whole.** Point out the denominator. **What does 2 tell us? 2 tells us that there are 2 groups or 2 people sharing the object.** Explain that students took 1 orange and divided it into 2 groups. Write “1 orange divided into 2 groups. $\frac{1}{2}$ per group.”

Give each student 1 fraction circle divided into fourths. Explain that they need to share this orange among 4 people. Tell students to split the orange into 4 groups. **How much of the orange is in each group?** $\frac{1}{4}$ Point out the numerator. **What does 1 tell us? 1 tells us that there is 1 whole.** Point out the denominator. **What does 4 tell us? 4 tells us that there are 4 groups.** Explain that students took 1 orange and divided it into 4 groups. Write “1 orange divided into 4 groups. $\frac{1}{4}$ per group.”

Give each student 1 fraction circle divided into sixths. Explain that they need to share this orange among 6 people. Tell students to split the orange into 6 groups. **How much of the orange is in each group?** $\frac{1}{6}$ Point out the numerator. **What does 1 tell us? 1 tells us that there is 1 whole.** Point out the denominator. **What does 6 tell us? 6 tells us that there are 6 groups.** Explain that students took 1 orange and divided it into 6 groups. Write “1 orange divided into 6 groups. $\frac{1}{6}$ per group.”

Discuss with students the patterns between the statements on the board and the fraction. Guide students to understand that the numerator always matches the number of wholes and denominator always matches the number of groups the wholes were divided into.

Give each student 2 fraction circles divided into fourths. Explain that they need to share 2 oranges among 4 people. Tell students to split the oranges into 4 groups. **How much of the orange is in each group?** $\frac{2}{4}$ Point out the numerator. **What does 2 tells us? 2 tells us that there are 2 wholes.** Point out the denominator. **What does 4 tell us? 4 tells us that there are 4 groups.** Explain that students took 2 oranges and divided them into 4 groups. Write “2 oranges divided into 4 groups. $\frac{2}{4}$ per group.” Compare this example to the other models.

Conclude the Activity

Review the models students created during the activity and how they related to the fraction each group received.

Questions

- **How is the last model different from the rest?** **Sample answer: In the last model, we divided 2 wholes into groups.**
- **How did this change the fraction?** **Sample answer: The numerator was 2, instead of 1 as it was in the other models.**
- **How would the numerator change if we divided 3 wholes into groups instead of 2?** **Sample answer: The numerator would be 3 instead of 2.**

Quotient Fractions (Models & Equations)

Goal Students will interpret a fraction as the division of the numerator by the denominator by using models and equations.

Guided Support 15 MINUTES



Materials

- 3-inch diameter circles cut from construction paper (3 per student)
- Scissors (1 per student)

Begin the Activity

Give each student 3 construction paper circles. Tell the students that they have 3 muffins that need to be shared equally among 4 people. The circles represent the muffins. Tell the students that they will divide each of the muffins equally among the people by cutting the circles. Each person will get one piece from each muffin. Have students draw a horizontal and vertical line on each circle. Students should then number the 4 parts with 1, 2, 3, and 4. Students should then cut apart the circles and arrange the pieces according to the number labels. Guide students to see that each person gets $\frac{3}{4}$ of a muffin. Write the following equation for the situation on the board.

$$3 \div 4 = \frac{3}{4}$$

Continue to use models or drawings to solve word problems involving fractions as quotients. For example: There are 5 bags of seed to plant in a community garden. Three students volunteer to help plant. Each student will plant an equal amount of seeds. How much of the bags of seed will each student plant? Guide students to draw 5 rectangles to represent the bags of seeds. Point out that there are 5 bags to be shared among 3 students. **How can we divide the 5 bags equally among the 3 students? Sample answer: Divide each bag into 3 parts, 1 part for each student.** Have students divide the first rectangle into 3 equal parts and label the parts with the numbers 1, 2, and 3. Have them continue to divide and label the remaining 4 bags this way. When the students have labeled all of the bags, have them count the parts of each bag that each student will plant. **How much will each volunteer plant? $\frac{5}{3}$** Guide students to see that $\frac{5}{3}$ is the same as $1\frac{2}{3}$.

Conclude the Activity

Review with students how the equation is found for each situation represented in the activity. Emphasize that when a smaller number is divided by a larger number, the answer is a fraction. Also, when a larger number is divided by a smaller number, the answer is a mixed number.

Questions

- If each person gets $\frac{3}{4}$ of a muffin, why is the denominator 4? Sample answer: Each muffin is divided into 4 parts – one for each member of the group.
- How does making a model help you find the amount each person will get? Sample answer: The model shows how many pieces, or fraction of the whole, each person will get.

Quotient Fractions (Word Problems)

Goal Students will solve word problems involving division of whole numbers resulting in answers in the form of fractions or mixed numbers by using equations.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Blackline Master: *Fractions and Division Equations* (1 per student)

Begin the Activity

Explain to students that they will be solving division word problems with fractions as quotients. Go over the lesson vocabulary, division, numerator, and denominator, before beginning the activity.

Tell students that they will be solving four separate word problems. Read the following example problem to students.

There is 1 glass of lemonade that 3 people want to share. How much of the lemonade can each person have?

Work through the problem using *Fractions and Division Equations*. How much lemonade is there? 1 glass One is the numerator of the fraction. How many people are sharing the lemonade? 3 This number is the denominator of the fraction. Continue to explain the problem using the completed example.

Read and solve each of the following word problems with students. Encourage students to use the example on *Fractions and Division Equations* as a guide.

1. There are 5 bags of bird food and 4 bird feeders to fill. How much will each bird feeder get if the food is evenly split? $5 \div 4 = \frac{5}{4}$ or $1\frac{1}{4}$ bags of bird food
2. There are 3 slices of an apple left. 2 friends decide to equally share the slices. How much does each person get? $3 \div 2 = \frac{3}{2}$ or $1\frac{1}{2}$ slices of apple
3. There are 2 bottles of juice left for 5 people to share at a school party. How much juice does each person get? $2 \div 5 = \frac{2}{5}$ of a bottle
4. There are 3 cookies left after a birthday party. The 4 members of the Jones family decide to split them up evenly. How much does each family member get? $3 \div 4 = \frac{3}{4}$ of a cookie

Conclude the Activity

After students have completed the activity, have them explain their answers to two of the word problems.

Questions

- What does the numerator represent in each equation? Sample answer: The numerator of a fraction can be thought of as the total starting amount being split or shared.
- What does the denominator represent in each equation? Sample answer: The denominator of a fraction can be thought of as the number of pieces each whole being shared is divided into.

Fractions as Quotients

Mathematical Context

In this module, understanding fractions as quotients will help students use fractions to solve real-world problems involving comparisons and grouping. Seeing fractional expressions as division helps students later recognize that ratios and percentages can be expressed using fractions. Expressing ratios as fractions is useful because equivalent ratios can be found by writing equivalent fractions.

Vocabulary

denominator divide (division)
fraction
numerator
quotient

Standards

See *online module information or Program Overview for standards alignment.*

Lesson Title	Goal	Prerequisite Skills
Quotient Fractions (Models)	Students will interpret a fraction as the division of the numerator by the denominator by using models.	To be successful with interpreting fractions as quotients using models, students need to be able to apply division to solving word problems. They also need to know the meaning of a fraction and of an improper fraction.
Quotient Fractions (Models & Equations)	Students will interpret a fraction as the division of the numerator by the denominator by using models and equations.	To be successful with interpreting fractions as equations involving division of the numerator by the denominator, students need to be able to use models to represent division and apply division to solving word problems.
Quotient Fractions (Word Problems)	Students will solve word problems involving division of whole numbers resulting in answers in the form of fractions or mixed numbers by using equations.	To be successful with interpreting fractions as quotients, students need to understand what the numerator and the denominator represent in a fraction. Students also need to have a basic understanding of the construction of a division equation.

Quotient Fractions (Models)

Goal Students will interpret a fraction as the division of the numerator by the denominator by using models.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Fraction circles: halves, fourths, sixths (2 of each per student)

Begin the Activity

Give each student 1 fraction circle divided into halves. Tell students to imagine this is an orange. Explain that they need to share this orange between 2 people. Tell students to split the orange into 2 groups. **How much of the orange is in each group?** $\frac{1}{2}$ Point out the numerator. **What does 1 tell us? 1 tells us that there is 1 orange or 1 whole.** Point out the denominator. **What does 2 tell us? 2 tells us that there are 2 groups or 2 people sharing the object.** Explain that students took 1 orange and divided it into 2 groups. Write “1 orange divided into 2 groups. $\frac{1}{2}$ per group.”

Give each student 1 fraction circle divided into fourths. Explain that they need to share this orange among 4 people. Tell students to split the orange into 4 groups. **How much of the orange is in each group?** $\frac{1}{4}$ Point out the numerator. **What does 1 tell us? 1 tells us that there is 1 whole.** Point out the denominator. **What does 4 tell us? 4 tells us that there are 4 groups.** Explain that students took 1 orange and divided it into 4 groups. Write “1 orange divided into 4 groups. $\frac{1}{4}$ per group.”

Give each student 1 fraction circle divided into sixths. Explain that they need to share this orange among 6 people. Tell students to split the orange into 6 groups. **How much of the orange is in each group?** $\frac{1}{6}$ Point out the numerator. **What does 1 tell us? 1 tells us that there is 1 whole.** Point out the denominator. **What does 6 tell us? 6 tells us that there are 6 groups.** Explain that students took 1 orange and divided it into 6 groups. Write “1 orange divided into 6 groups. $\frac{1}{6}$ per group.”

Discuss with students the patterns between the statements on the board and the fraction. Guide students to understand that the numerator always matches the number of wholes and denominator always matches the number of groups the wholes were divided into.

Give each student 2 fraction circles divided into fourths. Explain that they need to share 2 oranges among 4 people. Tell students to split the oranges into 4 groups. **How much of the orange is in each group?** $\frac{2}{4}$ Point out the numerator. **What does 2 tells us? 2 tells us that there are 2 wholes.** Point out the denominator. **What does 4 tell us? 4 tells us that there are 4 groups.** Explain that students took 2 oranges and divided them into 4 groups. Write “2 oranges divided into 4 groups. $\frac{2}{4}$ per group.” Compare this example to the other models.

Conclude the Activity

Review the models students created during the activity and how they related to the fraction each group received.

Questions

- **How is the last model different from the rest?** **Sample answer: In the last model, we divided 2 wholes into groups.**
- **How did this change the fraction?** **Sample answer: The numerator was 2, instead of 1 as it was in the other models.**
- **How would the numerator change if we divided 3 wholes into groups instead of 2?** **Sample answer: The numerator would be 3 instead of 2.**

Quotient Fractions (Models & Equations)

Goal Students will interpret a fraction as the division of the numerator by the denominator by using models and equations.

Guided Support 15 MINUTES



Materials

- 3-inch diameter circles cut from construction paper (3 per student)
- Scissors (1 per student)

Begin the Activity

Give each student 3 construction paper circles. Tell the students that they have 3 muffins that need to be shared equally among 4 people. The circles represent the muffins. Tell the students that they will divide each of the muffins equally among the people by cutting the circles. Each person will get one piece from each muffin. Have students draw a horizontal and vertical line on each circle. Students should then number the 4 parts with 1, 2, 3, and 4. Students should then cut apart the circles and arrange the pieces according to the number labels. Guide students to see that each person gets $\frac{3}{4}$ of a muffin. Write the following equation for the situation on the board.

$$3 \div 4 = \frac{3}{4}$$

Continue to use models or drawings to solve word problems involving fractions as quotients. For example: There are 5 bags of seed to plant in a community garden. Three students volunteer to help plant. Each student will plant an equal amount of seeds. How much of the bags of seed will each student plant? Guide students to draw 5 rectangles to represent the bags of seeds. Point out that there are 5 bags to be shared among 3 students. **How can we divide the 5 bags equally among the 3 students? Sample answer: Divide each bag into 3 parts, 1 part for each student.** Have students divide the first rectangle into 3 equal parts and label the parts with the numbers 1, 2, and 3. Have them continue to divide and label the remaining 4 bags this way. When the students have labeled all of the bags, have them count the parts of each bag that each student will plant. **How much will each volunteer plant? $\frac{5}{3}$** Guide students to see that $\frac{5}{3}$ is the same as $1\frac{2}{3}$.

Conclude the Activity

Review with students how the equation is found for each situation represented in the activity. Emphasize that when a smaller number is divided by a larger number, the answer is a fraction. Also, when a larger number is divided by a smaller number, the answer is a mixed number.

Questions

- If each person gets $\frac{3}{4}$ of a muffin, why is the denominator 4? Sample answer: Each muffin is divided into 4 parts – one for each member of the group.
- How does making a model help you find the amount each person will get? Sample answer: The model shows how many pieces, or fraction of the whole, each person will get.

Quotient Fractions (Word Problems)

Goal Students will solve word problems involving division of whole numbers resulting in answers in the form of fractions or mixed numbers by using equations.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Blackline Master: *Fractions and Division Equations* (1 per student)

Begin the Activity

Explain to students that they will be solving division word problems with fractions as quotients. Go over the lesson vocabulary, division, numerator, and denominator, before beginning the activity.

Tell students that they will be solving four separate word problems. Read the following example problem to students.

There is 1 glass of lemonade that 3 people want to share. How much of the lemonade can each person have?

Work through the problem using *Fractions and Division Equations*. How much lemonade is there? 1 glass One is the numerator of the fraction. How many people are sharing the lemonade? 3 This number is the denominator of the fraction. Continue to explain the problem using the completed example.

Read and solve each of the following word problems with students. Encourage students to use the example on *Fractions and Division Equations* as a guide.

1. There are 5 bags of bird food and 4 bird feeders to fill. How much will each bird feeder get if the food is evenly split? $5 \div 4 = \frac{5}{4}$ or $1\frac{1}{4}$ bags of bird food
2. There are 3 slices of an apple left. 2 friends decide to equally share the slices. How much does each person get? $3 \div 2 = \frac{3}{2}$ or $1\frac{1}{2}$ slices of apple
3. There are 2 bottles of juice left for 5 people to share at a school party. How much juice does each person get? $2 \div 5 = \frac{2}{5}$ of a bottle
4. There are 3 cookies left after a birthday party. The 4 members of the Jones family decide to split them up evenly. How much does each family member get? $3 \div 4 = \frac{3}{4}$ of a cookie

Conclude the Activity

After students have completed the activity, have them explain their answers to two of the word problems.

Questions

- What does the numerator represent in each equation? Sample answer: The numerator of a fraction can be thought of as the total starting amount being split or shared.
- What does the denominator represent in each equation? Sample answer: The denominator of a fraction can be thought of as the number of pieces each whole being shared is divided into.

Division of Positive Fractions

Mathematical Context

Fraction division is often one of the more difficult concepts for students to grasp. The reason it is important to demonstrate division of fractions with both bar models and number lines is that these models provide visual explanations that help students make sense of the operations. Since fraction division also involves the use of the reciprocal, it is important that students both understand the meaning of a reciprocal and demonstrate how to find it whether they are presented with a fraction, mixed number, or whole number.

Vocabulary

dividend
divisor
quotient
reciprocal

Standards

See *online module information* or *Program Overview for standards alignment*.

Lesson Title	Goal	Prerequisite Skills
Divide Fractions and Whole Numbers—Model	Students will model the division of positive fractions by whole numbers and whole numbers by positive fractions.	To be successful with representing the division of positive fractions and whole numbers with models, students need to be able to show the location of fractions on number lines and model fractions using bar models.
Divide Fractions—Model	Students will represent the division of positive fractions.	To be successful with modeling the division of fractions, students need to understand that a fraction represents a certain number of parts out of a whole. They need to be able to divide a unit of one into fractional parts.
Find Reciprocals	Students will find the reciprocals of positive fractions and whole numbers.	To be successful with finding reciprocals of positive fractions and whole numbers, students need to be able to identify the parts of fractions. They also need to know how to write a whole number and a mixed number as a fraction.
Divide Fractions	Students will divide two positive fractions.	To be successful with dividing positive fractions, students need to be able to multiply fractions. They also need to be able to identify the reciprocal of a fraction.
Divide Fractions and Whole Numbers	Students will divide positive fractions and whole numbers.	To be successful with dividing positive fractions and whole numbers, students need to be able to multiply fractions. They also need to be able to identify the reciprocal of a fraction.

Divide Fractions and Whole Numbers—Model

Goal Students will model the division of positive fractions by whole numbers and whole numbers by positive fractions.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Blackline Master: *Fraction Strips* (1 per student)
- Scissors (1 per student)

Begin the Activity

Display $2 \div \frac{1}{3}$. Say: We will find the quotient of $2 \div \frac{1}{3}$ by first placing two fraction strips side by side and tracing them and then determining how many strips of length $\frac{1}{3}$ fill the two fraction strips.

Using the models from *Fraction Strips*, have students create two strips marked in thirds. Direct students to count how many $\frac{1}{3}$ -length sections there are. Students should determine that there are 6 sections. So $2 \div \frac{1}{3} = 6$.



The additional problems that follow will give students further practice and will allow for assessment of student understanding of dividing whole numbers by fractions.

$$4 \div \frac{2}{3} \quad 6$$

$$4 \div \frac{1}{2} \quad 8$$

$$6 \div \frac{3}{4} \quad 8$$

Display $\frac{2}{3} \div 4$. Say: We will find the quotient of $\frac{2}{3} \div 4$ by shading two-thirds of one fraction strip to represent $\frac{2}{3}$.

Have students shade from the left of the whole strip. Direct students to split the shaded section into 4 equal groups. Ask students to determine the length of each group by comparing their strip to those on *Fraction Strips*. Students should determine that the length of each section will be $\frac{1}{6}$ of the whole fraction strip. Have students work on these additional problems:

$$\frac{2}{3} \div 2 \frac{1}{3}$$

$$\frac{3}{4} \div 3 \frac{1}{4}$$

$$\frac{2}{3} \div 3 \frac{2}{9}$$

Conclude the Activity

Make sure students understand that when dividing whole numbers by positive fractions, they are representing the whole numbers as whole lengths, and counting smaller lengths within each whole length to find the total number of smaller lengths that fit into the total length.

Students should understand that when dividing positive fractions by whole numbers, they are representing a fraction of one whole length and dividing it into an equal number of groups. They can measure the length of one individual group to determine its length.

Questions

- How is dividing whole numbers by fractions like dividing whole numbers by whole numbers? Sample answer: You are looking at a total group, then determining how many items of the same size fit into the whole group.
- How is dividing fractions by whole numbers like dividing whole numbers by whole numbers? Sample answer: You are looking at a total group, then determining, based on the number of equal groups the fraction should be split into, the length of one group.

Divide Fractions—Model

Goal Students will represent the division of positive fractions.

Guided Support 15 MINUTES



Materials

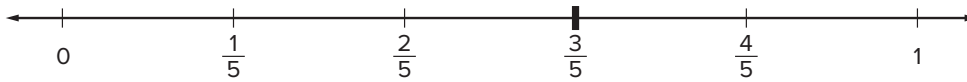
- Blackline Master: *Number Lines and Grids* (1 per student)
- Colored pencils

Begin the Activity

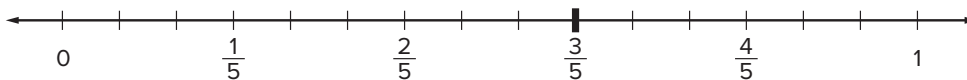
Write $\frac{3}{5} \div \frac{1}{15}$ on the board. Say: We can model $\frac{3}{5}$ on a number line and divide the number line into sections of $\frac{1}{15}$. The number of sections will be the quotient.

Distribute *Number Lines and Grids* to students.

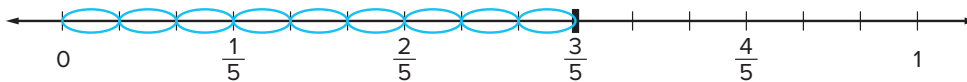
Ask: What is the dividend? $\frac{3}{5}$ Say: First, we need to label each $\frac{1}{5}$ section of a number line. Instruct students to label each $\frac{1}{5}$ section of the first number line and place a bold mark at $\frac{3}{5}$.



Ask: What is the divisor? $\frac{1}{15}$ Say: Now we need to divide the number line into 15 equal sections. Instruct students to divide the first number line into 15 equal sections.

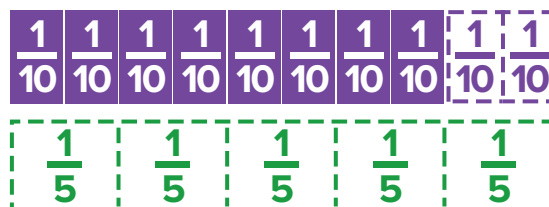


Say: Now, let's see how many $\frac{1}{15}$ sections it takes to get to $\frac{3}{5}$. Instruct students to circle each $\frac{1}{15}$ section up to the $\frac{3}{5}$ mark.



Say: There are 9 circled sections, so $\frac{3}{5}$ divided by $\frac{1}{15} = 9$.

Say: You can also model division problems using a grid, also called a fraction strip. Next, we will model $\frac{8}{10}$ divided by $\frac{1}{5}$. Write $\frac{8}{10} \div \frac{1}{5}$ on the board.



Ask: What is the dividend? $\frac{8}{10}$ Say: We can show a model in tenths and shade 8 of those tenths. Instruct students to shade $\frac{8}{10}$ of the first model.

Ask: What is the divisor? $\frac{1}{5}$ Say: Look at the grid underneath that is divided in fifths. Ask: How many $\frac{1}{5}$ sections does it take to reach $\frac{8}{10}$? 4

Say: It takes four $\frac{1}{5}$ sections, so $\frac{8}{10}$ divided by $\frac{1}{5} = 4$.

Have students complete the following problems, using the number lines for the first 2 problems and the grids for the last one. Monitor students as they work and offer support as needed.

$$\frac{5}{6} \div \frac{1}{12} \quad 10 \qquad \frac{3}{4} \div \frac{3}{8} \quad 2 \qquad \frac{6}{8} \div \frac{1}{4} \quad 3$$

Conclude the Activity

Make sure students can explain how to model division on a number line. Have them explain how they found the solution to one of the problems they completed on their own.

Questions

- How do you decide what increments to use for the number line? **Sample answer:** First use the denominator of the dividend. Then add more tick marks to show the denominator of the divisor.
- How do you find the quotient when using the number line? **Sample answer:** Count the divisor-sized circles it takes to reach the dividend.
- How do you find the quotient when using the fraction-strip model? **Sample answer:** Count the divisor-sized sections it takes to reach the dividend.

Find Reciprocals

Goal Students will find the reciprocals of positive fractions and whole numbers.

Guided Support 15 MINUTES



Materials

- Blackline Master: *Reciprocals of Whole Numbers and Positive Fractions* (1 per student)

Begin the Activity

Review with students the process for finding reciprocals of whole numbers and fractions.

- To find the reciprocal of a whole number, first write the whole number in the numerator of a fraction and write 1 as the denominator. Then flip the numerator and the denominator in the fraction.
- To find the reciprocal of a fraction, write the numerator of the fraction as the denominator and write the denominator as the numerator.
- To find the reciprocal of a mixed number, first write the mixed number as an improper fraction. Then flip the numerator and the denominator in the improper fraction.

Distribute *Reciprocals of Whole Numbers and Positive Fractions* to students. Instruct them to write the number 7 in the left-hand column.

Ask: *If I wanted to write 7 as a fraction, I would write 7 in the numerator and what number in the denominator? 1 Why? 7 divided by 1 is the same as 7.*

Have students fill in the next column as shown. Then tell students: *To find the reciprocal of a fraction, flip the numerator and the denominator in the fraction. What is the reciprocal of $\frac{7}{1}$? $\frac{1}{7}$* Have students fill in the third column as shown.

Number	Equivalent Fraction (if Whole Number)	Reciprocal
7	$\frac{7}{1}$	$\frac{1}{7}$

Say: *The product of a number and its reciprocal will always be 1.* Ask: *How can I check that $\frac{1}{7}$ is the reciprocal of 7? Sample answer: Multiply 7 and $\frac{1}{7}$. If the product equals 1, then 7 and $\frac{1}{7}$ are reciprocals. $7 \times \frac{1}{7} = \frac{7}{1} \times \frac{1}{7} = \frac{7}{7} = 1$*

Provide another example with the fraction $\frac{6}{11}$. Students should realize that since the fraction already has a numerator and denominator, they simply need to flip the numerator and denominator.

Finally, show an example with a mixed number, $4\frac{1}{5}$. Ask: **How can this mixed number be written as an improper fraction?** **Sample answer:** Multiply the denominator by the whole number and add the numerator. Write that over the denominator. **What is $4\frac{1}{5}$ as an improper fraction?** $\frac{21}{5}$ Now that this number is in fraction form, students can flip the numerator and denominator to find the reciprocal.

Have students find the reciprocal of the following numbers.

$$\frac{5}{8} \quad \frac{8}{5}$$

$$9 \quad \frac{1}{9}$$

$$2\frac{2}{3} \quad \frac{3}{8}$$

Conclude the Activity

Check students' answers. If any discrepancies in the answers arise, students should discuss their work and make a final decision on the correct answer. Make sure students understand that when finding the reciprocal, they are simply flipping the numerator and denominator. The product of a number and its reciprocal is 1.

Questions

- **How do you determine the reciprocal of a whole number?** **Sample answer:** Write the whole number over 1, then flip the numerator and denominator.
- **How do you determine the reciprocal of a fraction?** **Sample answer:** Flip the numerator and denominator.
- **How can you check that a reciprocal is correct?** **Sample answer:** Multiply by the original whole number or fraction. If the product is 1, the answer is correct.

Divide Fractions

Goal Students will divide two positive fractions.

Guided Support 15 MINUTES



HANDS-ON

Begin the Activity

Tell students they will find the quotient of two fractions.

Present the process for dividing fractions. You may want to display the following:

Dividing Positive Fractions:

1. Write the original division problem.
2. Rewrite the division problem by keeping the first quantity, replacing the division sign with a multiplication sign, and then writing the reciprocal of the second quantity.
3. Multiply the rewritten expression.

Demonstrate using the process of dividing positive fractions by finding the quotient of $\frac{4}{7} \div \frac{3}{8}$.

Ask: **Do we need to find the reciprocal of $\frac{4}{7}$ or $\frac{3}{8}$?** $\frac{3}{8}$ Make sure students understand they only need to find the reciprocal of the divisor and not the dividend. Ask: **What is the reciprocal of $\frac{3}{8}$?** $\frac{8}{3}$

Demonstrate how to rewrite the division expression as an equivalent multiplication expression.

$$\text{Display: } \frac{4}{7} \div \frac{3}{8} = \frac{4}{7} \times \frac{8}{3}$$

Ask: **What is the product of the numerators?** 32 **What is the product of the denominators?** 21

Demonstrate how to write the product.

$$\text{Display: } \frac{4}{7} \times \frac{8}{3} = \frac{32}{21}$$

Ask: **Can the fraction be rewritten as a mixed number?** yes

Demonstrate how to rewrite the fraction as a mixed number.

$$\text{Display: } \frac{32}{21} = 1\frac{11}{21}$$

Emphasize that each division problem must be rewritten as multiplication by the reciprocal of the divisor before any calculations are made.

These additional problems will give students more practice and will allow for assessment of student understanding of dividing positive fractions.

$$\frac{1}{5} \div \frac{3}{7} \quad \frac{1}{5} \div \frac{3}{7} = \frac{1}{5} \times \frac{7}{3} = \frac{7}{15}$$

$$\frac{2}{9} \div \frac{6}{5} \quad \frac{2}{9} \div \frac{6}{5} = \frac{2}{9} \times \frac{5}{6} = \frac{10}{54} = \frac{5}{27}$$

$$\frac{5}{9} \div \frac{1}{3} \quad \frac{5}{9} \div \frac{1}{3} = \frac{5}{9} \times \frac{3}{1} = \frac{15}{9} = 1\frac{6}{9} = 1\frac{2}{3}$$

$$\frac{7}{8} \div \frac{3}{4} \quad \frac{7}{8} \div \frac{3}{4} = \frac{7}{8} \times \frac{4}{3} = \frac{28}{24} = 1\frac{4}{24} = 1\frac{1}{6}$$

Conclude the Activity

Monitor students as they check their work. If any discrepancies in the answers arise, students should review their work and correct any errors. Make sure students understand that when dividing positive fractions, they are always multiplying the first fraction by the reciprocal of the second fraction.

Questions

- When dividing fractions, do I take the reciprocal of the first fraction or the reciprocal of the second fraction? **the reciprocal of the second fraction**
- What happens if my answer is an improper fraction? **Sample answer: You can rename the answer as a mixed number.**
- How can I be sure my answer is in lowest terms? **Sample answer: You can divide both the numerator and denominator by any common factors they share until they do not share any common factors.**

Divide Fractions and Whole Numbers

Goal Students will divide positive fractions and whole numbers.

Guided Support 15 MINUTES



HANDS-ON

Begin the Activity

Work with students individually. Tell students they will find the quotient of $\frac{4}{7} \div 6$. Say: We will find the quotient of $\frac{4}{7} \div 6$ by rewriting $\frac{4}{7} \div 6$ as $\frac{4}{7} \times \frac{1}{6}$, then finding the product. Division is equivalent to multiplying by the divisor's reciprocal.

Present the process for dividing whole numbers and fractions. You may want to display the following:

Dividing Whole Numbers and Fractions:

1. Write the original division problem.
2. Use a colored pencil or highlighter to identify the number for which you need to find the reciprocal.
3. Rewrite the division problem by keeping the dividend, replacing the division sign with a multiplication sign, and then writing the reciprocal of the divisor.
4. Multiply the rewritten expression.

Demonstrate the problem $\frac{4}{7} \div 6$, showing how to divide a fraction by a whole number.

Original problem: $\frac{4}{7} \div 6$

Replace \div with \times . Write the reciprocal of the divisor. $\frac{4}{7} \times \frac{1}{6}$

Multiply the fractions. $\frac{4 \times 1}{7 \times 6}$

Simplify. $\frac{4}{42}$

Write the answer in lowest terms. $\frac{2}{21}$

Demonstrate another problem, $2 \div \frac{4}{5}$, in which the whole number is divided by a fraction.

Original problem: $2 \div \frac{4}{5}$

Replace \div with \times . Write the reciprocal of the divisor. $\frac{2}{1} \times \frac{5}{4}$

Multiply the fractions. $\frac{2 \times 5}{1 \times 4}$

Simplify. $\frac{10}{4}$

Write the answer in lowest terms. $\frac{5}{2} = 2\frac{1}{2}$

Help students realize that the process for either case is the same. They are always multiplying the first quantity by the reciprocal of the divisor. Emphasize that the problem must be rewritten using multiplication and the reciprocal before any further calculations are made. For further practice, have students solve the following division problems.

$$\frac{4}{5} \div 3 = \frac{4}{15}$$

$$\frac{2}{9} \div 6 = \frac{1}{27}$$

$$9 \div \frac{3}{4} = 12$$

$$5 \div \frac{1}{3} = 15$$

Conclude the Activity

Remind students that they keep the dividend, change the operation to multiplication, and write the reciprocal of the divisor.

Questions

- How can I set up a division problem as a multiplication problem? **Sample answer:** Multiply the dividend by the reciprocal of the divisor.
- Explain why dividing by 3 and multiplying by $\frac{1}{3}$ are equivalent. **Sample answer:** A fraction can be thought of as a division problem. $3 = \frac{3}{1}$ because $3 \div 1 = 3$. Dividing by 3 separates a whole into thirds, so one part is $\frac{1}{3}$. This means dividing by 3 is the same as multiplying by $\frac{1}{3}$.
- If a whole number is being divided by $\frac{1}{5}$, what can you multiply the whole number by to get the correct quotient? Explain. **Sample answer:** 5; 5 is the reciprocal of $\frac{1}{5}$.

Rational Numbers

Mathematical Context

Rational numbers are a subset of real numbers. A rational number is any number that can be written as the ratio of two integers. Students may think of rational numbers as fractions. To change a ratio or fraction to decimal form, the numerator is divided by the denominator. If the division comes to an end so that there are a finite number of digits in the result, the decimal is a “terminating decimal.” If the division does not come to an end, the digits will repeat in a pattern. These are called “repeating decimals.” The pattern may be shown by placing a line over the digits that are repeating or by using an ellipsis once the pattern is clear. Plotting rational numbers on a number line helps show their relative magnitude. The inequality symbols can also show the relative values of two rational numbers.

Vocabulary

inequality symbols
rational number
repeating decimal
terminating decimal

Standards

See *online module information* or *Program Overview for standards alignment*.

Lesson Title	Goal	Prerequisite Skills
Convert Fractional and Decimal Numbers	Students will convert rational numbers written as common fractions to decimal numbers.	To be successful with converting fractions to decimal numbers, students need to know which number in a fraction is the numerator and which is the denominator. Students must also be able to solve long division problems that result in quotients that are decimal numbers.
Terminating & Repeating Decimal Numbers	Students will determine whether a rational number is terminating or repeating.	To be successful with determining whether a rational number is terminating or repeating, students need to be able to solve long division problems that result in quotients that are decimal numbers. Students should also have a strong understanding of place value, including tenths, hundredths, thousandths, and so on. Experience in working with number patterns is also important.
Approximate Rational Numbers	Students will demonstrate understanding of approximating rational numbers.	To be successful with approximating rational numbers, students need to know that rational numbers include fractions and decimals. They need to understand place value including tenths, hundredths, and thousandths, and be able to plot decimals on a number line and convert fractions to decimals.
Compare Rational Numbers	Students will compare rational numbers, including fractions and decimal numbers.	To be successful with comparing rational numbers, students need to be able to use inequality symbols correctly and have a basic understanding of number lines and rational numbers.

Convert Fractional and Decimal Numbers

Goal Students will convert rational numbers written as common fractions to decimal numbers.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Pennies (100 per student/group)
- Colored pencils or markers (1 per student)
- Blackline Master: *Converting Fractions to Decimals* (1 per student)

Begin the Activity

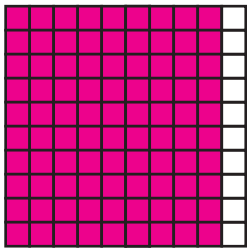
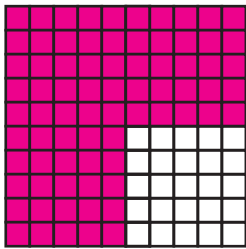
Place 100 pennies on a table in a 10-by-10 array. Ask: **How many pennies have the same value as one dollar? 100 pennies** Tell students that the group of pennies also represents 1 (or 1 dollar). Ask: **Since 100 pennies represent 1 dollar, how many pennies represent $\frac{1}{2}$ of a dollar? 50 pennies** Ask: **How do we represent 50 pennies using decimal notation? 0.50** Have students use pennies to model $\frac{9}{10}$ and $\frac{3}{4}$. Say: **Now, let's try to use pennies to model $\frac{8}{11}$.** Students will express difficulty. Say: **I also have difficulty expressing $\frac{8}{11}$ with pennies. Let's try another way to convert fractions to decimals.**

Give each student a copy of *Converting Fractions to Decimals*. Have students write $\frac{9}{10}$, $\frac{3}{4}$, and $\frac{8}{11}$ in the top row. $\frac{9}{10}$ should be written in the second column, $\frac{3}{4}$ in the third column, and $\frac{8}{11}$ in the far right column. Say: **We can use a 10-by-10 grid to represent 100 pennies. Since we had some trouble with the pennies representing $\frac{8}{11}$, let's not use a grid for that fraction. Instead, we can use division.**

Starting with $\frac{9}{10}$, work through the ways of representing the fraction as a decimal. Have students shade $\frac{9}{10}$ of the grid and ask: **What decimal is modeled by the grid? 0.90** Let's check that by dividing. **How can we write the fraction $\frac{9}{10}$ as a division problem? $9 \div 10$** To divide 9 by 10, which number is the divisor? **10** Demonstrate writing the divisor outside of the division house and the dividend (9) inside the division house. Ask: **Since 10 is greater than 9, how can we divide 9 by 10? Sample answer: In the dividend, place a decimal point after 9 and then write 0. Then, we can divide 9.0 by 10.** Complete working through the division of 9 by 10, showing the remainder of 0. Point out that the decimal answer is the same as the decimal they found by shading the grid. Have students record the decimal 0.9 in the last row of the table.

Work through the remaining rows and columns with students, making sure they understand each step of the process. Say: **The grid helps us see and find the decimal number. Long division helps us find the decimal number, but we cannot see it. If it is difficult or too time consuming to use a grid to find the decimal number, we can always go straight to division. Remember to read the fraction from top to bottom. Divide the numerator by the denominator.**

As students divide to find the decimal number for $\frac{8}{11}$, say: Sometimes a decimal representation stops after one or more digits and sometimes it does not. When you divide and start to see a pattern, you can stop and say that you found a decimal approximately equal to the fraction.

Write your fraction.	$\frac{9}{10}$	$\frac{3}{4}$	$\frac{8}{11}$
Shade a 10×10 (hundredths) grid to represent the fraction.			
Long Division	$\begin{array}{r} 0.9 \\ 10 \overline{)9.0} \\ \underline{-9\ 0} \\ 0\ 0 \end{array}$	$\begin{array}{r} 0.75 \\ 4 \overline{)3.00} \\ \underline{-2\ 8} \\ 2\ 0 \\ \underline{-2\ 0} \\ 0\ 0 \end{array}$	$\begin{array}{r} 0.7272\dots \\ 11 \overline{)8.0000} \\ \underline{-7\ 7} \\ 3\ 0 \\ \underline{-2\ 2} \\ 8\ 0 \\ \underline{-7\ 7} \\ 3\ 0 \\ \underline{-2\ 2} \\ 8 \end{array}$
Write the decimal equivalent.	0.9	0.75	0.72...

Conclude the Activity

To make the connection between the penny model, the grid, and long division, have students model a fraction using pennies, write the money amount it represents as a decimal, and shade a grid for the fraction. Have students compare the representations. Finally, have students use long division for the same fraction and compare the quotient with their models and decimal numbers.

Questions

- What method can you always use to convert a fraction to a decimal number?
long division
- Is division always the easiest and quickest way to find the decimal that is equivalent to a fraction? Sample answer: No, sometimes it is easier to see how the fraction is represented by a hundredths grid.
- When you use division to write a fraction as a decimal, which number is the divisor? the denominator
- How can you write the decimal number when the quotient continues for many digits? Sample answer: Write the decimal to just three decimal places and say that the decimal is an approximation of the fraction.

Terminating & Repeating Decimal Numbers

Goal Students will determine whether a rational number is terminating or repeating.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Blackline Master: *Fraction Cards 2* (1 per student or pair)
- Calculators (optional) (1 per student)

Begin the Activity

Prior to this activity, you may want to cut out *Fraction Cards 2*.

Display $\frac{1}{2} = 0.5$. Say: All rational numbers have a decimal number equivalent. The decimal number equivalent of one-half is five-tenths. One-half of a dollar is equivalent to 5 dimes, or 5 tenths of a dollar. Five-tenths is a terminating decimal number. It terminates, or ends, with the digit 5. Have students find the decimal number equivalent to $\frac{3}{11}$. Keep dividing until students recognize the pattern of digits. Say: The decimal number equivalent to $\frac{3}{11}$ is a repeating decimal number. If we continue dividing, we will continue to repeat 2 and 7 in the quotient. Since this pattern will never end, the decimal number is called a “repeating decimal number.”

Say: Every rational number is equivalent to a decimal number that is either terminating or repeating. A terminating decimal number is a decimal number that ends. A repeating decimal number is a decimal number that has one or more digits that keep repeating. Write 0.3333 . . . , 0.090909 . . . , and 0.037037 . . . on the board. Point to 0.3333 . . . Ask: In this decimal number, which number repeats? **3** Point to 0.090909 . . . Ask: What numbers repeat in this decimal number? **0 and 9** Point to 0.037037 . . . Ask: What numbers repeat in this decimal number? **0, 3, and 7**

Provide each student or pair of students with *Fraction Cards 2*. Have students look at each fraction, and predict whether it will be a terminating or a repeating decimal number. On the back of each card, students should write *T* for terminating or *R* for repeating.

Tell students to choose a fraction card, convert its number to a decimal number, and write the equivalent decimal number below the fraction. If students struggle with division, you may wish to provide calculators for this activity. Students should determine whether the decimal number is terminating or repeating. Have students create two stacks of cards—one for terminating decimal numbers and one for repeating decimal numbers. After students have placed their cards into two stacks, gather the cards, but keep them separated.

Conclude the Activity

Tell students that they will now check their predictions. Sort through the cards, and choose one with a correct equivalent decimal number. Display the fraction and equivalent decimal number. Ask: **Is this a repeating or a terminating decimal number?** Continue this process with several more that are correct. Then, display a fraction with an incorrect decimal number (or create one). Tell students that the decimal number is not equivalent to the fraction. Work through the division algorithm to find the correct equivalent decimal number. Point out where a mistake may have been made. Then, have students determine whether the decimal number is terminating or repeating and whether their prediction was correct. Continue the process for all the fraction cards.

Questions

- Can you tell by looking at a fraction if its equivalent decimal number will be terminating or repeating? Why or why not? **Sample answer:** If I have seen a fraction before and remember its equivalent decimal number, I can tell. Otherwise, it is not something you can determine just by looking at a fraction.
- What is the difference between terminating and repeating decimal numbers? **Sample answer:** Terminating decimal numbers eventually end. When you divide, you get a remainder of 0. Repeating decimal numbers do not end. When you divide, the quotient has a repeating pattern of digits.
- Can zeros be included in the pattern of a repeating decimal number? Explain your answer. Give an example. **Sample answer:** Yes. Zeros can be included, but zeros cannot be the only digit in the repeating pattern. For example, $\frac{1}{11}$ can be written as $1 \div 11$. This is $0.090909 \dots$, which is a repeating decimal number with a 0 in the pattern. But $0.5000000 \dots$ is not a repeating decimal number, because all the digits after 5 are zeros.

Approximate Rational Numbers

Goal Students will demonstrate understanding of approximating rational numbers.

Guided Support 15 MINUTES



Materials

- Blackline Master: *Approximate Real Numbers* (1 per student)

Begin the Activity

Work with students individually or in pairs. Give each student a copy of *Approximate Real Numbers*. Tell students they are going to place fractions on a number line labeled with decimals. To find the location of a fraction on a number line, they will first convert each fraction to a decimal, and then compare the decimal with other decimals.

First, have students convert the fraction $\frac{5}{6}$ to a decimal using long division. Ask: **What do you need to do to convert $\frac{5}{6}$ to a decimal? Divide the numerator of the fraction by the denominator.** Ask: **When you set up the division problem, which number goes inside the division house? 5, or the numerator** Display the problem, and work through it step by step. Ask the following questions as you work:

- **Since we cannot divide 5 objects into 6 whole parts, what do we need to do first? In the dividend, place a decimal point and a 0 after 5.**
- **Why do we place a decimal point and 0 after 5 in the dividend? Sample answer: to provide tenths to divide**
- **There is a remainder after we divide 50 by 6. What do we do next? Record another 0 in the dividend, carry it down, and keep dividing.**

Continue dividing until there are three decimal places. Then say: **This decimal will continue, but we have enough places to determine where to place the number on the number line.** Have students record 0.833 in the second column of their table.

Work with students to convert the remaining four fractions to decimals using long division and to round them to the nearest hundredth. Have students record the equivalent decimal for each fraction. **0.57; 0.77; 0.73; 0.81** Give students more independence when you can see that they have an understanding of the process.

Then ask: How can decimals help you find the location of each fraction on the number line? Sample answer: Find the interval for the decimal and put a point in that interval. Ask: When you know in which interval to place the point, how do you know where along the interval to place it? Sample answer: Look at the second digit to the right of the decimal point. If that digit is less than 5, then the point should be closer to the lesser number. If the second digit is greater than 5, then the point should be closer to the greater number. Have students use the decimal equivalents to plot a point for each fraction. Make sure that each point is in the correct interval, and that it is closer to the correct tenth.

Conclude the Activity

Encourage students to discuss how to plot rational numbers on a number line labeled by decimals when the numbers are given as fractions. Have students compare their methods and share any differences.

Questions

- How can you compare a fraction with decimal numbers? Sample answer: I can convert a number from a fraction to a decimal, and then compare that decimal with other decimals.
- How can you determine where to place a fraction on a number line labeled by decimals? Sample answer: I can convert the fraction to an equivalent decimal, determine the interval where the equivalent decimal belongs, and plot a point in that interval.
- If the number line were labeled by hundredths, would the process be the same? Explain your reasoning. Sample answer: For a number line labeled in tenths, I only need two decimal places to determine in which interval to place the number. But for a number line labeled in hundredths, I would need three decimal places.

Compare Rational Numbers

Goal Students will compare rational numbers, including fractions and decimal numbers.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Blackline Master: *Number Line Divided into Fourths* (1 per student)
- Blank cubes with labels (4 per student)

Begin the Activity

Before starting the activity, have students label their number cubes with the following numbers. Provide each student with a copy of *Number Line Divided into Fourths*.

Cube 1: $-\frac{2}{5}$, -1.6 , $-1\frac{4}{5}$, 1.19 , 0.75 , $\frac{3}{5}$

Cube 2: 1.9 , $-\frac{3}{5}$, -0.05 , $\frac{2}{5}$, $-1\frac{1}{5}$, 1.42

Cube 3: 1.51 , 0.32 , 1.8 , 1.2 , 0.77 , 0.24

Cube 4: $\frac{3}{8}$, $\frac{7}{4}$, $\frac{3}{10}$, $\frac{8}{5}$, $\frac{7}{16}$, $\frac{3}{4}$

Review the inequality symbols with students. Ask: **How do you know whether to use the *greater than* or *less than* symbol when comparing two rational numbers?** **Sample answer:** The inequality symbol always points to the lesser number. If the first number in the comparison is the greater number, the symbol points to the second number. If the first number is the lesser number, the symbol points to the first number.

Have students decide whether to use a horizontal number line or a vertical number line. Then, have them label the number line from -2 to 2 .

Tell students they will use the number cubes and a number line to compare rational numbers. Say: **Toss the first pair of number cubes (Cube 1 and Cube 2). Choose one number cube and place it on the number line. Record the number below the number line. Next, place the second number cube on the number line. Record the number next to the first number, leaving space for an inequality symbol.**

Remind students that the numbers increase in value from left to right on a horizontal number line and from bottom to top on a vertical number line.

Next, have students put their finger on the number cube they rolled first. Ask: **Is this number the greater number or the lesser number?**

Remind students that the greater number is to the right or above the other number. The lesser number is to the left or below the other number. After students determine whether the first number is the greater or lesser number, have them record the inequality symbol between the numbers.

Remind students that they may need to use long division to find the decimal number equivalent to the fraction. Make sure that students understand how to perform the long division. Once students convert the fraction to a decimal number, have them determine which number is greater. Tell students to write an inequality statement for the pair of numbers using the correct inequality symbol.

Have students continue rolling the number cubes, using Cube 3 and Cube 4. Students will now roll one fraction and one decimal number, and compare the pair of numbers rolled. After several comparisons, have students add a negative sign to each fraction and decimal number on both number cubes and roll them again. Students will now compare a negative decimal number and a negative fraction.

Conclude the Activity

Have students choose two positive numbers from the number cubes. Then have them write a comparison statement. Have them do this exercise for two negative numbers and for one positive and one negative number. Discuss how students made their comparisons for each pair of numbers.

Questions

- When comparing two numbers, what do you notice when one number is positive and one is negative? **Sample answer:** The positive number is always the greater number, and the negative number is always the lesser number.
- Which inequality symbol would you use to compare $2\frac{1}{2}$ and 3.14? Explain your reasoning. **Sample answer:** The “less than” symbol because $2\frac{1}{2}$ is to the left of 3.14 on a horizontal number line or below 3.14 on a vertical number line.
- Is $-\frac{7}{12}$ greater than or less than $-\frac{11}{12}$? How do you know? **Sample answer:** $-\frac{7}{12}$ is greater than $-\frac{11}{12}$ because it is closer to 0 on a number line.
- How do you compare a fraction and a decimal that are so close that it is difficult to determine which is greater? **Sample answer:** I can write the fraction as a decimal number and compare decimal places.

Irrational Numbers

Mathematical Context

The set of real numbers is divided into the subset of rational numbers and the subset of irrational numbers. A rational number is any number that can be written as the ratio of two integers. This includes numbers expressed as integers, fractions, and decimals. All rational numbers either terminate with a finite number of digits or repeat in a predictable pattern. Irrational numbers cannot be written as ratios of two integers. They have a non-ending, non-repeating string of digits to the right of the decimal point. Commonly used irrational numbers are π and roots of rational numbers. Although it is not possible to specify the exact value of an irrational number, it is possible to estimate it. For example, $\sqrt{2}$ is irrational. However, its value is somewhere between 1.4 and 1.5 because $1.4^2 = 1.96$, $1.5^2 = 2.25$, and 2 is between 1.96 and 2.25. As students continue their study of mathematics, especially geometry and trigonometry, it is important that they are able to recognize irrational numbers and estimate their values.

Vocabulary

irrational numbers
rational numbers
real numbers

Standards

See *online module information or Program Overview for standards alignment.*

Lesson Title	Goal	Prerequisite Skills
What Is an Irrational Number?	Students will determine irrational numbers.	To be successful with classifying rational and irrational numbers, students need to understand how whole numbers and decimals can be written as fractions. They should be able to recognize terminating and repeating rational numbers.
Irrational Numbers on a Number Line	Students will use rational approximations of irrational numbers.	To be successful with approximating irrational numbers on a number line, students need to know perfect squares. Students should know the difference between rational and irrational numbers, and they should have experience with discovering irrational numbers as numbers that are non-repeating and non-terminating decimals.
Approximate Irrational Numbers	Students will determine and compare the value of irrational numbers.	To be successful with comparing approximate values of irrational numbers, students need to be able to identify rational numbers that are close in value to irrational numbers. They also need to be able to use the greater than ($>$) and less than ($<$) symbols accurately.

What Is an Irrational Number?

Goal Students will determine irrational numbers.

Guided Support 15 MINUTES



Materials

- Blackline Master: *Real Numbers Are Rational or Irrational* (1 per student)

Begin the Activity

Work with students individually or in pairs. Say: **Real numbers consist of rational and irrational numbers. A rational number is any number that could be written as a fraction. Decimal numbers that end (terminate) or have a repeating decimal pattern are also rational numbers.** Display examples of terminating and repeating decimals such as 0.45, 0.44444 . . . , and 0.454545 Point out that decimals that terminate stop after a few digits. Have students identify the patterns in the repeating decimals. Then have students write their own examples of rational numbers. Students may name any fraction, mixed number, percentage, or decimal that terminates or repeats. Say: **All numbers that cannot be written as fractions are called irrational numbers. If a number is not rational, it is an irrational number. Irrational numbers go on forever. They are non-terminating and do not have a repeating decimal pattern.** Direct students to the graphic organizer on *Real Numbers Are Rational or Irrational*. Have them write a definition of rational numbers in their own words. Check for accuracy.

Below the graphic organizer is a list of numbers. Have students record each number in one of the sections of the graphic organizer. Ask students to think aloud as they work through the activity and explain how they decide where to place each number. When students complete placing all of the numbers in the graphic organizer, have them record one more number of their own in each section.

Conclude the Activity

Check the numbers that students added to the graphic organizer to make sure they were placed accurately. Have students explain how they chose where to place their numbers. Ask questions to help students identify any errors. Students should recognize that rational numbers consist of fractions, mixed numbers, percentages, and decimals that terminate or have a repeating decimal pattern. For each irrational number, ask: **Is there a fraction that is equivalent to this number? How do you know?** For numbers that are correctly classified as irrational, students should point out that decimals that do not end and do not have a repeating pattern cannot be written as fractions, so they are irrational. If a number is incorrectly classified as irrational, show students how to write an equivalent value as a fraction.

Questions

- How would you define a rational number? Sample answer: A rational number is any number that can be written as a fraction or ratio, including percentages, mixed numbers, terminating decimals, and decimals with repeating patterns.
- How would you define an irrational number? Sample answer: An irrational number is any number that is not rational; an irrational number is any number that never ends and does not have a repeating decimal pattern.
- How do you know that all percentages are rational numbers? Sample answer: All percentages can be written as fractions with a denominator of 100.
- Are integers rational or irrational numbers? How do you know? Sample answer: They are rational numbers because all integers can be written as fractions with a denominator of 1.

Irrational Numbers on a Number Line

Goal Students will use rational approximations of irrational numbers.

Guided Support 15 MINUTES



Materials

- Colored pencils or markers (2 colors per student)
- Scissors (1 pair per student)
- Blackline Master: *Plotting an Irrational Number on a Number Line* (1 per student)

Begin the Activity

Explain to students that they will be approximating irrational numbers using a number line. Remind them that irrational numbers are numbers that cannot be written as fractions, but can be approximated.

The goal of this activity is for students to be able to identify which two perfect squares an irrational number falls between. Have students write the squares of the numbers 1–10 on the back of *Plotting an Irrational Number on a Number Line*. Then, give students a square root such as $\sqrt{75}$. Walk students through each step of approximating the irrational number.

For step 1: Teach students how to appropriately guess until they find the perfect squares the irrational number is between. For example, $3 \cdot 3 = 9$; this would be too small. $9 \cdot 9 = 81$ is close, but it is greater than 75. Try $8 \cdot 8 = 64$ as the other closest perfect square. So, $\sqrt{75}$ is between the whole numbers 8 and 9 on a number line. Have students circle the perfect squares that are closest to 75.

For step 2: Let students know that they are now going to find approximately where $\sqrt{75}$ would be on the number line to the nearest tenth. Start by squaring the numbers closest to the square root of 81, since 75 is closer to 81 than to 64. For example, try $8.9 \cdot 8.9 \approx 79.2$. That is too far from 75. However, $8.7 \cdot 8.7 \approx 75.7$, which is very close to 75.

For step 3: Label the number line and place the approximate point. Remind students that the point will be slightly to the left of 8.7 since 75.7 is greater than 75.

If students need extra practice with this concept, give them other examples that they can do on their own papers.

Conclude the Activity

Ask: How did you find the square of each number from 1 to 10? Sample answer: I multiplied each number by itself. How did you find the perfect square that is closest to the number? Sample answer: by first finding the two perfect squares closest to the number, and then using the number line to find the one that is closest

Questions

- How can you approximate the square root of an irrational number? Sample answer: by finding the closest perfect square and using its square root as an approximation
- Why do you need to find two perfect square roots to approximate the square root of an irrational number? Sample answer: You need to find the perfect square roots that the irrational number is between to help you narrow down the options of the value of the irrational number.
- Why is it necessary to approximate square roots? Sample answer: Unless a number is a perfect square, most square roots have irrational values. These are decimals that never end. We can never know their exact value, so we have to approximate.

Approximate Irrational Numbers

Goal Students will determine and compare the value of irrational numbers.

Guided Support 15 MINUTES



HANDS-ON

Materials

- Scissors (1 per student pair)
- Blackline Master: *2–6 Number Line* (1 per student pair)
- Blackline Master: *Irrational Number Comparisons* (1 per student pair)

Begin the Activity

Work with students individually or in pairs. Give students copies of *2–6 Number Line* and *Irrational Number Comparisons*. Have students cut out the cards from *Irrational Number Comparisons* and set aside the rest of the sheet for later.

Review the number line with students by asking: **What value should be under the square root sign if its square root is 2?** After establishing that $\sqrt{4} = 2$, have students write 4 beneath the square root sign. Then ask: **What value should be under the cube root sign if its cube root is 2?** After establishing that $\sqrt[3]{8} = 2$, have students write 8 under the cube root sign. Continue completing the expressions beneath the number line. Ask: **What do you know about the values of each of the square roots and cube roots you wrote below the number line?** **Sample answer:** They are equal to the whole number above them.

Tell students they will compare two values printed on the cards using the number line. Have students pick two of the irrational number cards. Walk students through finding the location of each root. Suppose a student picks the cards $\sqrt{19}$ and $\sqrt{32}$. Ask: **Between which two square roots does the square root of 19 belong? 16 and 25. How do you know?** **Sample answer:** It belongs between the square root of 16 and the square root of 25 because 19 is greater than 16 but less than 25. Ask similar questions about the square root of 32.

Ask: **Which number is greater, $\sqrt{19}$ or $\sqrt{32}$?** **Sample answer:** $\sqrt{32}$ is farther to the right from 0 on the number line.

Have students place the cards in the first and last rectangles remaining on *Irrational Number Comparison*.

Ask students which symbol, $<$ or $>$, should be placed between the numbers to make a correct comparison. In the case of the example using $\sqrt{19}$ and $\sqrt{32}$, the statement should be $\sqrt{19} < \sqrt{32}$, or $\sqrt{32} > \sqrt{19}$.

Repeat the comparison activity as many times as necessary, drawing two new cards each time. When students appear to have grasped the concept of comparing, have them compare two numbers independently. If more attempts are needed after all the cards have been used, shuffle all the cards together so the activity can be repeated as necessary.

Conclude the Activity

Discuss the activity with students. Ask them to describe how they decided which inequality symbol to use.

Questions

- **Is the square root of 50 greater than the cube root of 200? Explain your reasoning.** Sample answer: The square root of 50 is between 7 and 8, since 50 is between 49 and 64. The cube root of 200 is between 5 and 6, since 200 is between 125 and 216. So, the square root of 50 is greater than the cube root of 200.
- **Name an irrational number that has a radical symbol that is greater than 4 but less than 5.** Students could name any square root of any number between 16 and 25 or any cube root of any number between 64 and 125.
- **What steps do you take to compare two irrational numbers?** Sample answer: Find the two perfect squares or two perfect cubes between which each irrational number lies. Then compare the whole numbers.

